Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/1.1.4.3-e-x-^m-a-x^j+b-x^k-^p-c+d-x^n-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [298]. This is test number [31].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (298)	% 0.00 (0)
Mathematica	% 99.33 (296)	% 0.67 (2)
Maple	% 92.28 (275)	% 7.72 (23)
Maxima	% 71.14 (212)	% 28.86 (86)
Fricas	% 76.51 (228)	% 23.49 (70)
Sympy	% 42.28 (126)	% 57.72 (172)
Giac	% 71.48 (213)	% 28.52 (85)
Mupad	% 66.11 (197)	% 33.89 (101)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

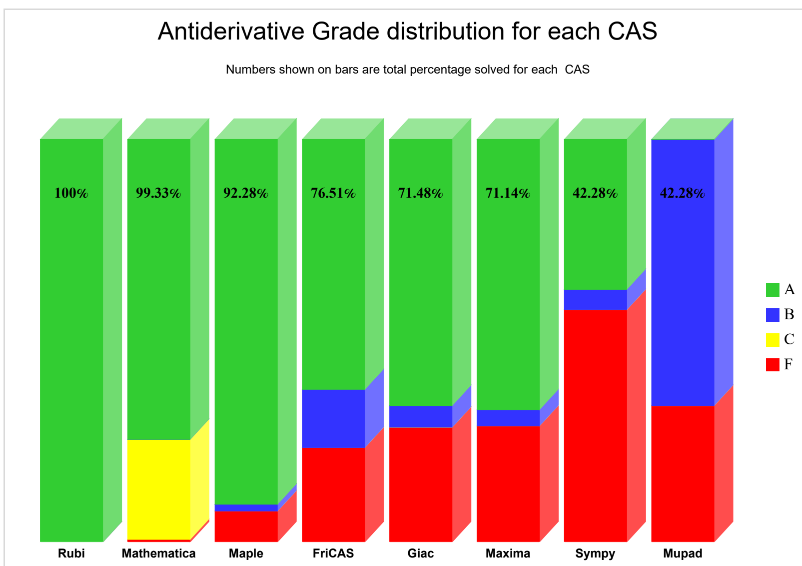
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

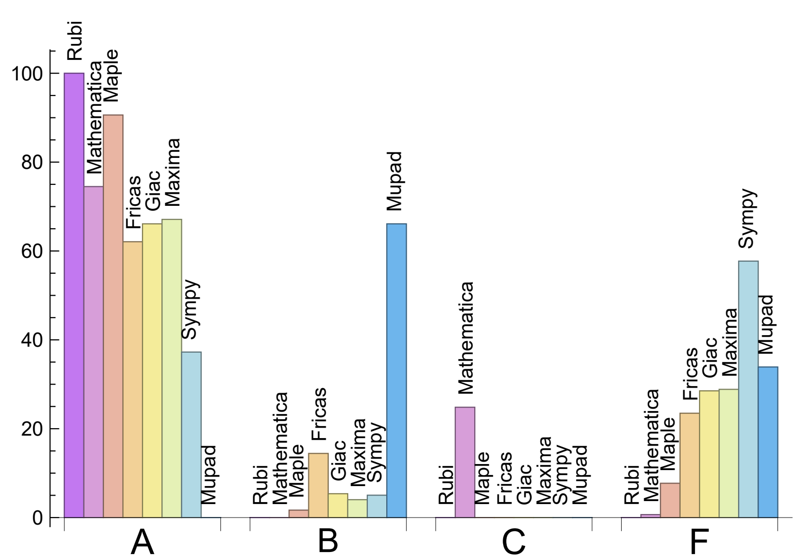
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	74.50	0.00	24.83	0.67
Maple	90.60	1.68	0.00	7.72
Maxima	67.11	4.03	0.00	28.86
Fricas	62.08	14.43	0.00	23.49
Sympy	37.25	5.03	0.00	57.72
Giac	66.11	5.37	0.00	28.52
Mupad	0.00	66.11	0.00	33.89

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	23	100.00 %	0.00 %	0.00 %
Maxima	86	100.00 %	0.00 %	0.00 %
Fricas	70	100.00 %	0.00 %	0.00 %
Sympy	172	67.44 %	32.56 %	0.00 %
Giac	85	95.29 %	0.00 %	4.71 %
Mupad	101	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

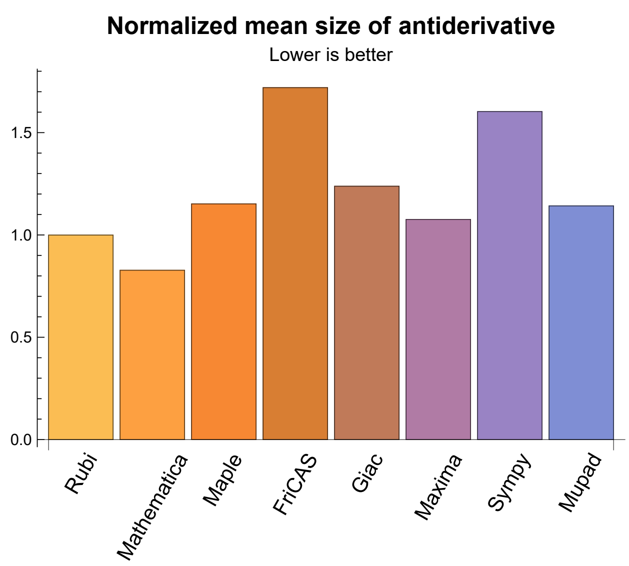
1.3 Performance

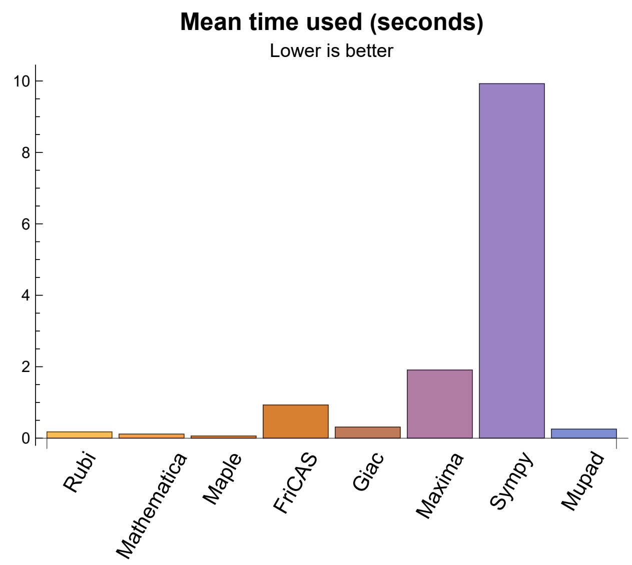
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	145.03	1.00	103.50	1.00
Mathematica	0.11	99.23	0.83	85.00	0.89
Maple	0.06	174.17	1.15	118.00	1.15
Maxima	1.91	124.24	1.08	84.00	0.99
Fricas	0.93	250.76	1.72	130.00	1.50
Sympy	9.92	130.07	1.60	80.00	1.18
Giac	0.31	144.20	1.24	97.00	1.03
Mupad	0.25	151.83	1.14	76.00	0.96

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {277}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

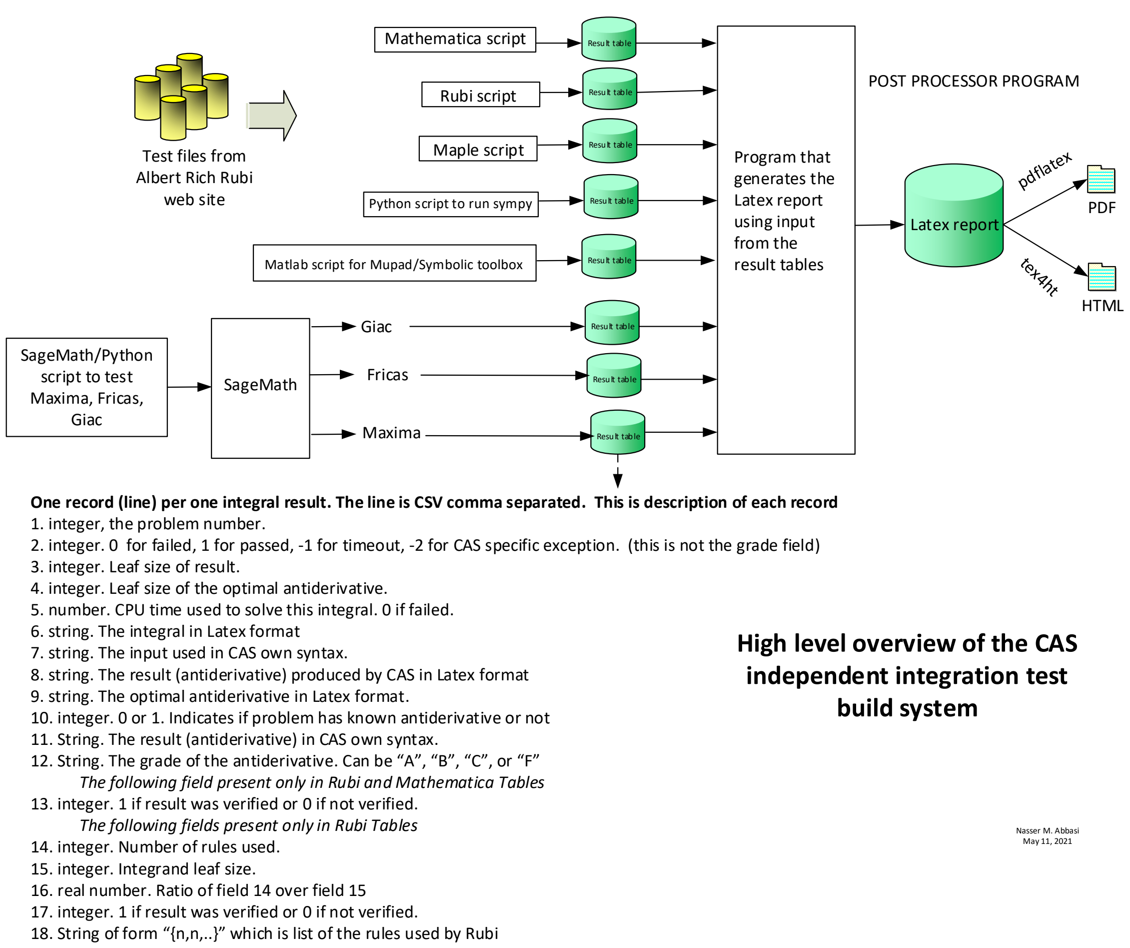
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298 }

B grade: { }

C grade: { 112, 113, 125, 127, 128, 129, 157, 158, 191, 192, 193, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 283 }

F grade: { 284, 292 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 278, 280, 281 }

B grade: { 126, 269, 270, 271, 279 }

C grade: { }

F grade: { 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 106, 109, 110, 111, 112, 119, 120, 121, 122, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 275, 278, 280, 281 }

B grade: { 91, 96, 107, 108, 113, 114, 115, 116, 117, 118, 279, 283 }

C grade: { }

F grade: { 103, 104, 105, 123, 124, 125, 126, 127, 128, 129, 142, 143, 144, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 275, 278, 279, 280, 281 }

B grade: { 84, 86, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 283 }

C grade: { }

F grade: { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 47, 48, 50, 53, 55, 57, 58, 59, 61, 62, 63, 64, 65, 67, 68, 69, 71, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 86, 87, 88, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 269, 270, 271, 278, 279, 280, 281 }

B grade: { 28, 43, 45, 49, 51, 52, 54, 56, 60, 66, 70, 72, 77, 83, 85 }

C grade: { }

F grade: { 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 183, 184, 185, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 145, 146, 147, 149, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 279, 280, 281 }

B grade: { 28, 95, 96, 97, 98, 99, 113, 114, 115, 116, 117, 118, 269, 270, 271, 283 }

C grade: { }

F grade: { 139, 140, 141, 143, 144, 148, 150, 151, 152, 153, 154, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 103, 108, 114, 115, 116, 117, 118, 119, 120, 121, 122, 133, 134, 135, 136, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 278, 279, 280, 281 }

C grade: { }

F grade: { 94, 95, 104, 105, 106, 107, 109, 110, 111, 112, 113, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 142, 143, 144, 145, 146, 147, 156, 157, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	29	29	29	28
normalized size	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85
time (sec)	N/A	0.027	0.007	0.043	1.403	0.812	0.074	0.150	0.179

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	29	29	29	28
normalized size	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85
time (sec)	N/A	0.041	0.010	0.044	1.259	0.996	0.066	0.149	0.059

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	29	29	29	28
normalized size	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85
time (sec)	N/A	0.024	0.006	0.049	1.348	0.537	0.066	0.193	0.040

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
normalized size	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.038	0.008	0.043	1.307	0.558	0.067	0.148	0.059

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
normalized size	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.019	0.019	0.041	1.406	0.697	0.066	0.146	0.060

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	28	25	27	30	26
normalized size	1	1.00	1.00	0.97	0.97	0.86	0.93	1.03	0.90
time (sec)	N/A	0.027	0.009	0.041	1.334	0.988	0.117	0.147	0.061

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	20	23	24
normalized size	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92
time (sec)	N/A	0.023	0.009	0.059	1.325	0.834	0.115	0.158	0.073

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	28	30	26	42	25
normalized size	1	1.00	1.00	0.90	0.97	1.03	0.90	1.45	0.86
time (sec)	N/A	0.028	0.015	0.049	1.316	0.785	0.193	0.162	0.042

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	26	29	27	28	26
normalized size	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00
time (sec)	N/A	0.022	0.034	0.049	1.345	0.771	0.219	0.169	0.057

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	28	30	31	29	39	29
normalized size	1	1.00	1.07	0.97	1.03	1.07	1.00	1.34	1.00
time (sec)	N/A	0.027	0.027	0.049	1.396	0.819	0.374	0.153	0.074

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	29	29	32	31	29
normalized size	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94
time (sec)	N/A	0.022	0.012	0.050	1.347	0.767	0.387	0.148	0.038

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	53	56	53	51
normalized size	1	1.00	1.00	0.95	0.93	0.96	1.02	0.96	0.93
time (sec)	N/A	0.048	0.010	0.043	1.333	0.617	0.076	0.145	0.085

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	53	53	51
normalized size	1	1.00	1.00	0.95	0.93	0.93	0.96	0.96	0.93
time (sec)	N/A	0.068	0.009	0.044	1.374	0.584	0.080	0.150	0.047

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
normalized size	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.039	0.009	0.044	1.322	0.967	0.080	0.167	0.049

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	51	53	53	51
normalized size	1	1.00	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.070	0.016	0.039	1.363	0.855	0.082	0.162	0.045

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
normalized size	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.032	0.009	0.048	1.244	0.918	0.080	0.174	0.047

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	52	49	49	53	48
normalized size	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.040	0.017	0.049	1.337	0.830	0.162	0.146	0.042

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
normalized size	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.037	0.035	0.047	1.294	0.941	0.144	0.149	0.051

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	50	52	54	48	70	48
normalized size	1	1.00	0.96	0.98	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.053	0.030	0.050	1.345	0.815	0.236	0.187	0.047

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
normalized size	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.037	0.021	0.047	1.319	0.882	0.255	0.143	0.049

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	51	54	55	51	72	51
normalized size	1	1.00	0.98	1.00	1.06	1.08	1.00	1.41	1.00
time (sec)	N/A	0.047	0.041	0.056	1.372	0.849	0.551	0.188	0.081

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	51	53	54	53	50
normalized size	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04
time (sec)	N/A	0.037	0.027	0.055	1.389	1.005	0.609	0.155	0.068

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	52	55	55	56	66	51
normalized size	1	1.00	1.04	1.02	1.08	1.08	1.10	1.29	1.00
time (sec)	N/A	0.045	0.047	0.053	1.273	0.859	0.995	0.191	0.091

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	59	48	53	53	58	55	52
normalized size	1	1.00	1.11	0.91	1.00	1.00	1.09	1.04	0.98
time (sec)	N/A	0.036	0.021	0.049	1.370	1.237	1.074	0.150	0.038

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	80	77	69
normalized size	1	1.00	1.00	1.01	0.97	0.97	1.07	1.03	0.92
time (sec)	N/A	0.064	0.026	0.037	1.344	0.772	0.085	0.188	0.060

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	76	73	73	82	77	69
normalized size	1	1.00	1.01	1.12	1.07	1.07	1.21	1.13	1.01
time (sec)	N/A	0.134	0.031	0.039	1.317	0.932	0.086	0.212	0.030

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	82	77	69
normalized size	1	1.00	1.00	1.01	0.97	0.97	1.09	1.03	0.92
time (sec)	N/A	0.055	0.014	0.045	1.367	0.827	0.086	0.153	0.030

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	69	76	73	73	80	77	69
normalized size	1	1.00	1.64	1.81	1.74	1.74	1.90	1.83	1.64
time (sec)	N/A	0.071	0.024	0.043	1.445	0.881	0.085	0.153	0.031

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	76	73	65
normalized size	1	1.00	1.00	1.04	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.042	0.012	0.043	1.281	0.830	0.087	0.166	0.031

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	71	76	74	71	80	78	67
normalized size	1	1.00	1.18	1.27	1.23	1.18	1.33	1.30	1.12
time (sec)	N/A	0.053	0.022	0.045	1.315	0.908	0.180	0.152	0.035

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	71	69	75	68	70	65
normalized size	1	1.00	1.00	1.09	1.06	1.15	1.05	1.08	1.00
time (sec)	N/A	0.045	0.029	0.050	1.337	0.970	0.171	0.156	0.033

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	73	75	74	77	78	97	67
normalized size	1	1.00	1.03	1.06	1.04	1.08	1.10	1.37	0.94
time (sec)	N/A	0.079	0.044	0.051	1.301	0.562	0.265	0.173	0.039

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	71	70	73	75	75	74	68
normalized size	1	1.00	1.03	1.01	1.06	1.09	1.09	1.07	0.99
time (sec)	N/A	0.050	0.035	0.049	1.331	0.860	0.291	0.151	0.057

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	76	76	76	75	98	76
normalized size	1	1.00	1.01	1.06	1.06	1.06	1.04	1.36	1.06
time (sec)	N/A	0.072	0.034	0.049	1.284	0.686	0.589	0.153	0.074

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	64	73	75	78	75	73
normalized size	1	1.00	1.00	0.94	1.07	1.10	1.15	1.10	1.07
time (sec)	N/A	0.050	0.036	0.047	1.326	0.931	0.676	0.164	0.057

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	75	77	77	78	99	75
normalized size	1	1.00	1.00	1.06	1.08	1.08	1.10	1.39	1.06
time (sec)	N/A	0.064	0.047	0.052	1.442	0.913	1.294	0.152	0.087

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	63	73	75	80	77	71
normalized size	1	1.00	1.00	0.95	1.11	1.14	1.21	1.17	1.08
time (sec)	N/A	0.050	0.036	0.046	1.301	0.704	1.497	0.210	0.080

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	77	76	77	77	82	90	75
normalized size	1	1.00	1.22	1.21	1.22	1.22	1.30	1.43	1.19
time (sec)	N/A	0.047	0.059	0.053	1.322	0.826	2.367	0.167	0.103

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	75	75	83	79	74
normalized size	1	1.00	1.00	0.90	1.03	1.03	1.14	1.08	1.01
time (sec)	N/A	0.049	0.039	0.046	1.383	0.820	2.446	0.152	0.070

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	66	75	75	83	79	76
normalized size	1	1.00	1.59	1.35	1.53	1.53	1.69	1.61	1.55
time (sec)	N/A	0.040	0.021	0.048	1.328	0.550	3.426	0.157	0.071

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	140	124	274	204	133	144
normalized size	1	1.00	1.00	1.18	1.04	2.30	1.71	1.12	1.21
time (sec)	N/A	0.087	0.085	0.048	3.005	0.826	0.444	0.170	0.171

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	110	97	98	94	101	100
normalized size	1	1.00	0.96	1.15	1.01	1.02	0.98	1.05	1.04
time (sec)	N/A	0.126	0.041	0.043	1.380	1.020	0.352	0.152	0.058

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	116	100	228	180	108	118
normalized size	1	1.00	1.00	1.18	1.02	2.33	1.84	1.10	1.20
time (sec)	N/A	0.076	0.068	0.049	2.940	0.986	0.408	0.182	0.043

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	86	74	75	70	77	76
normalized size	1	1.00	0.95	1.15	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.094	0.032	0.045	1.337	0.844	0.319	0.154	0.087

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	92	78	178	153	85	96
normalized size	1	1.00	1.00	1.19	1.01	2.31	1.99	1.10	1.25
time (sec)	N/A	0.066	0.057	0.049	2.864	0.961	0.372	0.178	0.066

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	50	51	46	52	52
normalized size	1	1.00	0.87	1.15	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.069	0.021	0.051	1.326	0.850	0.286	0.152	0.074

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	68	53	129	90	57	70
normalized size	1	1.00	0.98	1.17	0.91	2.22	1.55	0.98	1.21
time (sec)	N/A	0.050	0.043	0.047	2.859	0.932	0.337	0.158	0.107

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	31	30	27	32	31
normalized size	1	1.00	0.89	1.14	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.043	0.013	0.044	1.299	0.763	0.250	0.232	0.056

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	45	34	99	82	34	31
normalized size	1	1.00	1.00	1.12	0.85	2.48	2.05	0.85	0.78
time (sec)	N/A	0.028	0.025	0.044	2.924	0.899	0.288	0.160	0.054

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	35	32	26	34	32
normalized size	1	1.00	1.00	1.09	1.03	0.94	0.76	1.00	0.94
time (sec)	N/A	0.041	0.013	0.047	1.343	0.990	0.755	0.200	0.085

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	48	36	105	82	36	35
normalized size	1	1.00	1.00	1.14	0.86	2.50	1.95	0.86	0.83
time (sec)	N/A	0.032	0.049	0.048	2.944	1.045	0.423	0.165	0.092

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	51	103	75	38	33
normalized size	1	1.00	1.00	0.95	1.24	2.51	1.83	0.93	0.80
time (sec)	N/A	0.034	0.026	0.049	3.062	0.822	0.443	0.152	0.145

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	56	48	47	41	71	46
normalized size	1	1.00	1.00	1.14	0.98	0.96	0.84	1.45	0.94
time (sec)	N/A	0.056	0.023	0.055	1.319	0.707	0.756	0.154	0.137

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	60	72	56	135	129	57	53
normalized size	1	1.00	0.98	1.18	0.92	2.21	2.11	0.93	0.87
time (sec)	N/A	0.056	0.087	0.057	2.815	0.822	0.437	0.158	0.106

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	81	70	73	61	100	70
normalized size	1	1.00	1.00	1.16	1.00	1.04	0.87	1.43	1.00
time (sec)	N/A	0.072	0.038	0.054	1.377	0.738	0.918	0.190	0.137

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	96	79	184	163	81	70
normalized size	1	1.00	1.00	1.23	1.01	2.36	2.09	1.04	0.90
time (sec)	N/A	0.068	0.089	0.048	3.048	1.132	0.583	0.156	0.106

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	96	107	96	98	88	126	92
normalized size	1	1.00	1.04	1.16	1.04	1.07	0.96	1.37	1.00
time (sec)	N/A	0.090	0.046	0.046	1.280	0.559	1.412	0.155	0.153

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	134	155	136	350	238	139	203
normalized size	1	1.00	1.01	1.17	1.02	2.63	1.79	1.05	1.53
time (sec)	N/A	0.166	0.114	0.053	2.940	0.870	1.012	0.169	0.104

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	122	107	148	104	135	121
normalized size	1	1.00	0.89	1.16	1.02	1.41	0.99	1.29	1.15
time (sec)	N/A	0.135	0.078	0.054	1.380	0.953	0.930	0.154	0.097

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	111	132	112	298	211	115	141
normalized size	1	1.00	1.01	1.20	1.02	2.71	1.92	1.05	1.28
time (sec)	N/A	0.118	0.091	0.053	3.020	1.151	0.761	0.156	0.085

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	98	82	121	78	106	86
normalized size	1	1.00	0.87	1.18	0.99	1.46	0.94	1.28	1.04
time (sec)	N/A	0.098	0.058	0.054	1.322	0.791	0.703	0.167	0.072

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	105	85	240	129	88	104
normalized size	1	1.00	1.00	1.18	0.96	2.70	1.45	0.99	1.17
time (sec)	N/A	0.089	0.080	0.064	3.026	0.936	0.689	0.157	0.110

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	74	60	81	56	70	62
normalized size	1	1.00	0.82	1.21	0.98	1.33	0.92	1.15	1.02
time (sec)	N/A	0.070	0.038	0.056	1.310	0.883	0.600	0.156	0.116

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	82	61	208	114	59	59
normalized size	1	1.00	1.00	1.21	0.90	3.06	1.68	0.87	0.87
time (sec)	N/A	0.061	0.056	0.053	2.999	0.723	0.553	0.191	0.126

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	40	44	36	37	37
normalized size	1	1.00	1.00	1.15	0.98	1.07	0.88	0.90	0.90
time (sec)	N/A	0.047	0.014	0.054	1.311	0.870	0.374	0.187	0.086

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	57	182	112	57	51
normalized size	1	1.00	1.00	1.08	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.033	0.051	0.056	2.965	0.655	0.417	0.182	0.120

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	51	70	46	52	47
normalized size	1	1.00	0.90	1.04	1.00	1.37	0.90	1.02	0.92
time (sec)	N/A	0.055	0.034	0.055	1.314	0.878	0.441	0.157	0.159

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	85	63	210	114	62	63
normalized size	1	1.00	1.00	1.21	0.90	3.00	1.63	0.89	0.90
time (sec)	N/A	0.074	0.036	0.056	2.977	1.035	0.506	0.213	0.129

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	86	76	117	70	80	78
normalized size	1	1.00	0.88	1.18	1.04	1.60	0.96	1.10	1.07
time (sec)	N/A	0.080	0.060	0.070	1.332	0.800	0.880	0.159	0.147

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	110	93	250	184	85	83
normalized size	1	1.00	1.00	1.22	1.03	2.78	2.04	0.94	0.92
time (sec)	N/A	0.117	0.082	0.074	3.004	0.893	0.603	0.157	0.137

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	114	106	154	100	150	100
normalized size	1	1.00	0.88	1.18	1.09	1.59	1.03	1.55	1.03
time (sec)	N/A	0.108	0.106	0.056	1.328	0.928	1.040	0.322	0.138

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	112	136	119	308	218	112	104
normalized size	1	1.00	1.01	1.23	1.07	2.77	1.96	1.01	0.94
time (sec)	N/A	0.198	0.099	0.056	2.942	0.812	0.699	0.168	0.158

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	133	174	147	416	252	138	177
normalized size	1	1.00	0.95	1.24	1.05	2.97	1.80	0.99	1.26
time (sec)	N/A	0.235	0.144	0.059	2.897	1.054	1.399	0.181	0.120

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	94	134	116	179	119	132	118
normalized size	1	1.00	0.85	1.21	1.05	1.61	1.07	1.19	1.06
time (sec)	N/A	0.136	0.095	0.060	1.412	0.773	1.519	0.177	0.118

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	113	147	120	358	214	111	138
normalized size	1	1.00	0.96	1.25	1.02	3.03	1.81	0.94	1.17
time (sec)	N/A	0.162	0.134	0.055	3.017	0.878	1.297	0.163	0.078

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	109	94	142	94	93	95
normalized size	1	1.00	1.03	1.22	1.06	1.60	1.06	1.04	1.07
time (sec)	N/A	0.102	0.048	0.055	1.360	0.993	1.294	0.162	0.091

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	122	94	328	194	80	92
normalized size	1	1.00	0.97	1.28	0.99	3.45	2.04	0.84	0.97
time (sec)	N/A	0.099	0.109	0.052	2.938	0.820	1.110	0.177	0.152

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	80	72	89	70	55	70
normalized size	1	1.00	0.96	1.19	1.07	1.33	1.04	0.82	1.04
time (sec)	N/A	0.076	0.026	0.054	1.335	0.779	0.932	0.189	0.108

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	83	89	92	301	155	78	82
normalized size	1	1.00	0.92	0.99	1.02	3.34	1.72	0.87	0.91
time (sec)	N/A	0.076	0.122	0.058	2.980	0.990	0.762	0.160	0.146

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	42	42	42	28	44
normalized size	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.031	0.014	0.053	1.348	0.740	0.545	0.161	0.076

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	90	92	300	150	78	82
normalized size	1	1.00	0.91	0.98	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.045	0.083	0.056	2.949	1.058	0.605	0.159	0.140

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	68	77	119	75	76	71
normalized size	1	1.00	0.87	1.00	1.13	1.75	1.10	1.12	1.04
time (sec)	N/A	0.073	0.068	0.060	1.416	0.692	0.594	0.158	0.178

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	125	96	324	194	82	113
normalized size	1	1.00	1.00	1.30	1.00	3.38	2.02	0.85	1.18
time (sec)	N/A	0.118	0.097	0.056	2.967	1.084	0.727	0.166	0.176

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	118	109	197	107	105	107
normalized size	1	1.00	0.89	1.22	1.12	2.03	1.10	1.08	1.10
time (sec)	N/A	0.117	0.100	0.063	1.373	0.883	1.113	0.183	0.152

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	119	152	128	368	226	108	114
normalized size	1	1.00	1.02	1.30	1.09	3.15	1.93	0.92	0.97
time (sec)	N/A	0.177	0.077	0.062	3.111	0.762	0.828	0.162	0.179

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	108	150	137	229	136	132	131
normalized size	1	1.00	0.89	1.24	1.13	1.89	1.12	1.09	1.08
time (sec)	N/A	0.131	0.088	0.065	1.429	1.030	1.251	0.169	0.174

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	177	154	426	260	135	135
normalized size	1	1.00	1.00	1.26	1.10	3.04	1.86	0.96	0.96
time (sec)	N/A	0.332	0.085	0.072	3.024	0.969	0.920	0.162	0.195

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	135	180	170	267	165	201	155
normalized size	1	1.00	0.91	1.22	1.15	1.80	1.11	1.36	1.05
time (sec)	N/A	0.173	0.136	0.060	1.433	0.713	1.334	0.166	0.185

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	193	290	321	368	0	245	289
normalized size	1	1.00	0.89	1.33	1.47	1.69	0.00	1.12	1.33
time (sec)	N/A	0.381	0.320	0.078	1.622	1.478	0.000	0.230	1.470

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	173	248	273	321	0	211	233
normalized size	1	1.00	0.96	1.37	1.51	1.77	0.00	1.17	1.29
time (sec)	N/A	0.334	0.302	0.061	1.514	0.903	0.000	0.203	0.886

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	206	225	272	0	177	177
normalized size	1	1.00	1.21	1.65	1.80	2.18	0.00	1.42	1.42
time (sec)	N/A	0.199	0.227	0.053	1.490	0.884	0.000	0.221	0.736

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	129	164	177	223	0	140	140
normalized size	1	1.00	1.21	1.53	1.65	2.08	0.00	1.31	1.31
time (sec)	N/A	0.154	0.199	0.053	1.424	0.983	0.000	0.193	0.752

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	91	124	128	172	0	103	117
normalized size	1	1.00	0.91	1.24	1.28	1.72	0.00	1.03	1.17
time (sec)	N/A	0.197	0.155	0.054	1.451	1.063	0.000	0.185	0.609

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	130	105	161	0	92	-1
normalized size	1	1.00	0.80	1.34	1.08	1.66	0.00	0.95	-0.01
time (sec)	N/A	0.213	0.154	0.055	1.479	0.977	0.000	0.274	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	109	96	160	0	163	-1
normalized size	1	1.00	1.08	1.36	1.20	2.00	0.00	2.04	-0.01
time (sec)	N/A	0.196	0.130	0.059	1.418	0.980	0.000	0.498	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	48	111	59	0	250	113
normalized size	1	1.00	0.72	0.79	1.82	0.97	0.00	4.10	1.85
time (sec)	N/A	0.162	0.019	0.047	1.495	0.983	0.000	0.920	0.469

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	70	161	85	0	310	160
normalized size	1	1.00	0.69	0.73	1.68	0.89	0.00	3.23	1.67
time (sec)	N/A	0.211	0.027	0.044	1.479	1.019	0.000	1.355	0.676

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	88	94	209	109	0	370	210
normalized size	1	1.00	0.66	0.71	1.57	0.82	0.00	2.78	1.58
time (sec)	N/A	0.245	0.030	0.053	1.494	0.998	0.000	1.970	1.039

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	94	118	257	133	0	430	260
normalized size	1	1.00	0.55	0.69	1.51	0.78	0.00	2.53	1.53
time (sec)	N/A	0.300	0.073	0.049	1.565	1.337	0.000	2.973	1.406

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	82	91	106	106	0	140	103
normalized size	1	1.00	0.63	0.69	0.81	0.81	0.00	1.07	0.79
time (sec)	N/A	0.220	0.064	0.048	1.419	1.394	0.000	0.170	0.252

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	64	67	83	82	0	105	83
normalized size	1	1.00	0.68	0.71	0.88	0.87	0.00	1.12	0.88
time (sec)	N/A	0.166	0.046	0.051	1.535	1.044	0.000	0.209	0.191

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	51	57	0	72	60
normalized size	1	1.00	0.67	0.74	0.84	0.93	0.00	1.18	0.98
time (sec)	N/A	0.019	0.028	0.047	1.460	0.847	0.000	0.160	0.153

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	84	85	0	159	0	116	99
normalized size	1	1.00	1.08	1.09	0.00	2.04	0.00	1.49	1.27
time (sec)	N/A	0.146	0.086	0.050	0.000	0.744	0.000	0.196	0.498

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	94	135	0	169	0	76	-1
normalized size	1	1.00	0.94	1.35	0.00	1.69	0.00	0.76	-0.01
time (sec)	N/A	0.161	0.049	0.054	0.000	1.136	0.000	0.218	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	95	174	0	198	0	132	-1
normalized size	1	1.00	0.92	1.69	0.00	1.92	0.00	1.28	-0.01
time (sec)	N/A	0.158	0.119	0.058	0.000	1.006	0.000	0.304	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	215	328	363	418	0	280	-1
normalized size	1	1.00	0.96	1.47	1.63	1.87	0.00	1.26	-0.00
time (sec)	N/A	0.404	0.363	0.081	1.581	1.566	0.000	0.337	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	193	286	315	369	0	246	-1
normalized size	1	1.00	1.16	1.71	1.89	2.21	0.00	1.47	-0.01
time (sec)	N/A	0.248	0.319	0.066	1.556	1.291	0.000	0.232	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	171	244	267	316	0	207	236
normalized size	1	1.00	1.16	1.65	1.80	2.14	0.00	1.40	1.59
time (sec)	N/A	0.191	0.267	0.060	1.508	1.116	0.000	0.217	1.011

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	151	202	216	275	0	178	-1
normalized size	1	1.00	1.05	1.40	1.50	1.91	0.00	1.24	-0.01
time (sec)	N/A	0.266	0.243	0.057	1.493	0.894	0.000	0.202	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	130	162	168	224	0	142	-1
normalized size	1	1.00	0.95	1.18	1.23	1.64	0.00	1.04	-0.01
time (sec)	N/A	0.296	0.126	0.052	1.429	1.280	0.000	0.207	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	96	174	148	209	0	126	-1
normalized size	1	1.00	0.75	1.36	1.16	1.63	0.00	0.98	-0.01
time (sec)	N/A	0.277	0.205	0.059	1.494	1.265	0.000	0.347	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	219	167	189	0	225	-1
normalized size	1	1.00	0.72	1.61	1.23	1.39	0.00	1.65	-0.01
time (sec)	N/A	0.274	0.054	0.059	1.503	1.102	0.000	0.526	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	153	177	207	0	254	-1
normalized size	1	1.00	0.90	1.47	1.70	1.99	0.00	2.44	-0.01
time (sec)	N/A	0.251	0.057	0.055	1.381	0.763	0.000	1.024	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	48	193	82	0	370	156
normalized size	1	1.00	0.72	0.79	3.16	1.34	0.00	6.07	2.56
time (sec)	N/A	0.173	0.026	0.046	1.555	1.067	0.000	1.889	1.031

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	70	241	109	0	430	206
normalized size	1	1.00	0.69	0.73	2.51	1.14	0.00	4.48	2.15
time (sec)	N/A	0.233	0.032	0.048	1.532	1.089	0.000	2.863	1.439

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	89	94	289	134	0	490	256
normalized size	1	1.00	0.67	0.71	2.17	1.01	0.00	3.68	1.92
time (sec)	N/A	0.279	0.038	0.054	1.580	1.340	0.000	3.235	1.925

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	95	118	337	157	0	550	306
normalized size	1	1.00	0.56	0.69	1.98	0.92	0.00	3.24	1.80
time (sec)	N/A	0.321	0.085	0.044	1.624	1.239	0.000	4.405	2.503

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	89	142	385	181	0	582	356
normalized size	1	1.00	0.43	0.69	1.86	0.87	0.00	2.81	1.72
time (sec)	N/A	0.352	0.077	0.046	1.650	1.865	0.000	5.435	3.210

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	113	115	150	154	0	175	143
normalized size	1	1.00	0.67	0.68	0.89	0.92	0.00	1.04	0.85
time (sec)	N/A	0.297	0.090	0.052	1.553	0.867	0.000	0.340	0.349

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	92	91	128	131	0	140	124
normalized size	1	1.00	0.70	0.69	0.98	1.00	0.00	1.07	0.95
time (sec)	N/A	0.241	0.076	0.047	1.527	0.928	0.000	0.211	0.292

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	71	67	105	106	0	105	103
normalized size	1	1.00	0.74	0.70	1.09	1.10	0.00	1.09	1.07
time (sec)	N/A	0.070	0.054	0.052	1.557	1.060	0.000	0.168	0.255

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	45	80	80	0	72	83
normalized size	1	1.00	0.79	0.74	1.31	1.31	0.00	1.18	1.36
time (sec)	N/A	0.159	0.035	0.050	1.551	0.910	0.000	0.213	0.229

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	99	0	206	0	140	-1
normalized size	1	1.00	1.07	0.97	0.00	2.02	0.00	1.37	-0.01
time (sec)	N/A	0.205	0.097	0.048	0.000	0.872	0.000	0.176	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	109	172	0	195	0	115	-1
normalized size	1	1.00	0.82	1.29	0.00	1.47	0.00	0.86	-0.01
time (sec)	N/A	0.220	0.070	0.057	0.000	1.133	0.000	0.276	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	63	213	0	217	0	145	-1
normalized size	1	1.00	0.47	1.58	0.00	1.61	0.00	1.07	-0.01
time (sec)	N/A	0.216	0.040	0.062	0.000	1.096	0.000	0.257	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	121	259	0	250	0	175	-1
normalized size	1	1.00	0.86	1.85	0.00	1.79	0.00	1.25	-0.01
time (sec)	N/A	0.223	0.143	0.075	0.000	0.943	0.000	0.272	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	66	302	0	299	0	214	-1
normalized size	1	1.00	0.37	1.71	0.00	1.69	0.00	1.21	-0.01
time (sec)	N/A	0.281	0.037	0.079	0.000	1.003	0.000	0.325	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	65	344	0	345	0	234	-1
normalized size	1	1.00	0.30	1.61	0.00	1.61	0.00	1.09	-0.00
time (sec)	N/A	0.339	0.036	0.090	0.000	1.041	0.000	0.309	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	66	386	0	393	0	294	-1
normalized size	1	1.00	0.26	1.54	0.00	1.57	0.00	1.17	-0.00
time (sec)	N/A	0.390	0.040	0.151	0.000	1.327	0.000	0.374	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	145	211	231	275	0	149	-1
normalized size	1	1.00	0.82	1.20	1.31	1.56	0.00	0.85	-0.01
time (sec)	N/A	0.328	0.262	0.058	1.379	1.000	0.000	0.223	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	123	169	183	226	0	119	-1
normalized size	1	1.00	0.88	1.22	1.32	1.63	0.00	0.86	-0.01
time (sec)	N/A	0.275	0.182	0.055	1.454	0.978	0.000	0.250	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	127	134	177	0	91	-1
normalized size	1	1.00	1.17	1.53	1.61	2.13	0.00	1.10	-0.01
time (sec)	N/A	0.171	0.134	0.054	1.457	0.772	0.000	0.216	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	81	88	88	131	0	67	89
normalized size	1	1.00	1.23	1.33	1.33	1.98	0.00	1.02	1.35
time (sec)	N/A	0.123	0.051	0.065	1.437	0.656	0.000	0.215	0.814

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	67	56	136	0	66	57
normalized size	1	1.00	1.30	1.18	0.98	2.39	0.00	1.16	1.00
time (sec)	N/A	0.151	0.041	0.058	1.412	0.882	0.000	0.224	0.509

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	43	47	70	38	0	88	39
normalized size	1	1.00	0.70	0.77	1.15	0.62	0.00	1.44	0.64
time (sec)	N/A	0.168	0.041	0.050	1.469	0.910	0.000	0.201	0.197

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	64	70	119	62	0	153	62
normalized size	1	1.00	0.67	0.73	1.24	0.65	0.00	1.59	0.65
time (sec)	N/A	0.209	0.029	0.051	1.489	0.684	0.000	0.209	0.251

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	89	94	167	86	0	219	121
normalized size	1	1.00	0.67	0.71	1.26	0.65	0.00	1.65	0.91
time (sec)	N/A	0.253	0.051	0.054	1.488	0.949	0.000	0.308	0.301

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	94	118	215	110	0	287	156
normalized size	1	1.00	0.55	0.69	1.26	0.65	0.00	1.69	0.92
time (sec)	N/A	0.303	0.097	0.047	1.542	1.238	0.000	0.268	0.322

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	85	89	106	83	0	0	87
normalized size	1	1.00	0.65	0.68	0.81	0.63	0.00	0.00	0.66
time (sec)	N/A	0.240	0.105	0.048	1.530	0.818	0.000	0.000	0.265

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	63	65	83	59	0	0	64
normalized size	1	1.00	0.67	0.69	0.88	0.63	0.00	0.00	0.68
time (sec)	N/A	0.193	0.071	0.047	1.540	1.121	0.000	0.000	0.228

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	40	42	50	36	0	0	41
normalized size	1	1.00	0.68	0.71	0.85	0.61	0.00	0.00	0.69
time (sec)	N/A	0.135	0.052	0.049	1.458	0.682	0.000	0.000	0.187

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	73	72	0	138	0	60	-1
normalized size	1	1.00	1.33	1.31	0.00	2.51	0.00	1.09	-0.02
time (sec)	N/A	0.020	0.039	0.056	0.000	0.704	0.000	0.326	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	87	105	0	152	0	0	-1
normalized size	1	1.00	1.28	1.54	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.089	0.055	0.000	1.126	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	146	0	199	0	0	-1
normalized size	1	1.00	1.01	1.42	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.223	0.056	0.000	0.983	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	136	166	237	340	0	176	-1
normalized size	1	1.00	0.74	0.90	1.29	1.85	0.00	0.96	-0.01
time (sec)	N/A	0.333	0.258	0.057	1.488	0.682	0.000	0.311	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	113	140	187	289	0	147	-1
normalized size	1	1.00	0.77	0.95	1.27	1.97	0.00	1.00	-0.01
time (sec)	N/A	0.280	0.209	0.061	1.545	1.017	0.000	0.280	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	91	115	138	230	0	116	-1
normalized size	1	1.00	0.81	1.03	1.23	2.05	0.00	1.04	-0.01
time (sec)	N/A	0.242	0.165	0.058	1.514	1.008	0.000	0.263	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	75	75	79	188	0	0	78
normalized size	1	1.00	1.12	1.12	1.18	2.81	0.00	0.00	1.16
time (sec)	N/A	0.177	0.106	0.053	1.481	0.922	0.000	0.000	0.633

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	47	65	49	0	36	53
normalized size	1	1.00	1.00	1.27	1.76	1.32	0.00	0.97	1.43
time (sec)	N/A	0.121	0.020	0.050	1.489	1.165	0.000	0.211	0.181

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	66	112	72	0	0	70
normalized size	1	1.00	0.97	1.00	1.70	1.09	0.00	0.00	1.06
time (sec)	N/A	0.164	0.043	0.048	1.385	0.911	0.000	0.000	0.274

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	85	94	160	98	0	0	95
normalized size	1	1.00	0.84	0.93	1.58	0.97	0.00	0.00	0.94
time (sec)	N/A	0.220	0.037	0.049	1.501	0.890	0.000	0.000	0.424

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	75	118	208	121	0	0	173
normalized size	1	1.00	0.54	0.86	1.51	0.88	0.00	0.00	1.25
time (sec)	N/A	0.267	0.054	0.050	1.478	1.112	0.000	0.000	0.559

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	82	91	82	93	0	0	92
normalized size	1	1.00	0.59	0.65	0.59	0.67	0.00	0.00	0.66
time (sec)	N/A	0.248	0.076	0.051	1.576	0.909	0.000	0.000	0.397

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	60	66	59	68	0	0	67
normalized size	1	1.00	0.58	0.63	0.57	0.65	0.00	0.00	0.64
time (sec)	N/A	0.200	0.041	0.052	1.559	1.165	0.000	0.000	0.280

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	35	44	39	45	0	60	44
normalized size	1	1.00	0.51	0.64	0.57	0.65	0.00	0.87	0.64
time (sec)	N/A	0.150	0.027	0.050	1.569	0.694	0.000	0.292	0.210

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	73	79	0	199	0	0	-1
normalized size	1	1.00	1.14	1.23	0.00	3.11	0.00	0.00	-0.02
time (sec)	N/A	0.139	0.035	0.057	0.000	0.934	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	61	129	0	260	0	0	-1
normalized size	1	1.00	0.43	0.91	0.00	1.83	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.027	0.060	0.000	0.844	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	64	157	0	315	0	0	89
normalized size	1	1.00	0.47	1.15	0.00	2.30	0.00	0.00	0.65
time (sec)	N/A	0.192	0.048	0.056	0.000	0.923	0.000	0.000	1.263

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.023	0.018	0.046	1.344	0.985	20.137	0.147	0.056

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.023	0.018	0.059	1.354	0.967	11.253	0.148	0.106

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.023	0.017	0.049	1.330	0.680	5.592	0.151	0.044

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	37	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	0.95	0.74	0.79
time (sec)	N/A	0.021	0.017	0.046	1.307	0.973	2.340	0.161	0.042

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.021	0.017	0.050	1.366	0.700	2.060	0.209	0.044

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	30	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.021	0.016	0.050	1.383	0.934	2.263	0.162	0.043

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	27	29	44	29	31
normalized size	1	1.00	0.89	0.86	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.021	0.016	0.049	1.337	0.848	2.624	0.183	0.042

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	32	27	29	44	29	31
normalized size	1	1.00	0.95	0.86	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.021	0.017	0.045	1.328	0.843	3.918	0.187	0.103

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	51	56	80	53	51
normalized size	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.041	0.036	0.048	1.297	0.870	57.866	0.164	0.130

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	56	80	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.039	0.034	0.046	1.336	0.819	39.816	0.152	0.050

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	51	56	80	53	51
normalized size	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.038	0.031	0.049	1.300	0.796	20.946	0.172	0.048

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	56	66	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.89	1.05	0.84	0.81
time (sec)	N/A	0.038	0.032	0.049	1.383	0.883	3.729	0.178	0.049

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	56	80	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.039	0.032	0.050	1.350	0.880	9.942	0.155	0.048

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	51	56	80	53	51
normalized size	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.039	0.031	0.055	1.364	0.829	10.479	0.208	0.048

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	56	80	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.038	0.032	0.057	1.366	0.803	12.055	0.151	0.049

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	54	80	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.038	0.034	0.054	1.332	0.961	15.463	0.173	0.051

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.052	0.052	0.050	1.325	0.600	134.768	0.155	0.102

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.050	0.050	0.049	1.361	0.759	92.322	0.170	0.033

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.050	0.046	0.049	1.377	0.971	54.010	0.153	0.035

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	95	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.12	0.91	0.81
time (sec)	N/A	0.050	0.043	0.052	1.334	0.841	5.510	0.171	0.033

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.052	0.043	0.051	1.360	0.895	29.884	0.151	0.033

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.050	0.043	0.052	1.321	0.891	31.769	0.156	0.033

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.052	0.040	0.053	1.403	0.768	35.910	0.163	0.036

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.051	0.051	0.047	1.338	0.912	53.837	0.154	0.033

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	133	336	237	920	0	298	115
normalized size	1	1.00	0.48	1.21	0.85	3.31	0.00	1.07	0.41
time (sec)	N/A	0.274	0.259	0.079	3.070	0.914	0.000	0.244	0.227

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	227	330	259	714	0	298	788
normalized size	1	1.00	0.82	1.20	0.94	2.59	0.00	1.08	2.86
time (sec)	N/A	0.247	0.309	0.052	3.010	0.891	0.000	0.200	0.252

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	110	308	214	899	0	264	92
normalized size	1	1.00	0.43	1.20	0.83	3.50	0.00	1.03	0.36
time (sec)	N/A	0.218	0.146	0.050	3.134	1.019	0.000	0.209	0.236

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	208	299	235	660	393	263	789
normalized size	1	1.00	0.82	1.17	0.92	2.59	1.54	1.03	3.09
time (sec)	N/A	0.209	0.224	0.062	2.990	1.005	164.461	0.196	0.275

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	95	280	194	834	459	251	71
normalized size	1	1.00	0.40	1.18	0.82	3.52	1.94	1.06	0.30
time (sec)	N/A	0.188	0.080	0.053	3.019	1.127	65.599	0.227	0.153

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	166	277	218	645	355	251	739
normalized size	1	1.00	0.71	1.18	0.93	2.74	1.51	1.07	3.14
time (sec)	N/A	0.184	0.146	0.056	3.159	0.716	27.828	0.183	0.292

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	74	277	194	843	456	251	71
normalized size	1	1.00	0.31	1.18	0.83	3.59	1.94	1.07	0.30
time (sec)	N/A	0.189	0.097	0.062	3.011	1.111	18.455	0.194	0.222

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	168	280	218	653	364	251	811
normalized size	1	1.00	0.71	1.18	0.92	2.76	1.54	1.06	3.42
time (sec)	N/A	0.188	0.154	0.060	3.067	0.932	27.101	0.228	0.301

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	46	299	213	883	366	268	90
normalized size	1	1.00	0.18	1.17	0.84	3.46	1.44	1.05	0.35
time (sec)	N/A	0.215	0.022	0.061	3.080	0.913	108.599	0.241	0.226

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	47	308	247	707	405	257	555
normalized size	1	1.00	0.18	1.20	0.96	2.75	1.58	1.00	2.16
time (sec)	N/A	0.214	0.020	0.060	2.930	1.150	109.298	0.187	0.320

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	47	330	237	931	0	291	107
normalized size	1	1.00	0.17	1.20	0.86	3.37	0.00	1.05	0.39
time (sec)	N/A	0.245	0.017	0.059	3.105	0.982	0.000	0.199	0.234

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	47	336	276	734	0	291	563
normalized size	1	1.00	0.17	1.21	0.99	2.64	0.00	1.05	2.03
time (sec)	N/A	0.241	0.019	0.059	3.117	0.760	0.000	0.195	0.351

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	417	372	298	804	0	335	857
normalized size	1	1.00	1.26	1.12	0.90	2.42	0.00	1.01	2.58
time (sec)	N/A	0.277	0.674	0.065	3.071	0.777	0.000	0.220	0.219

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	154	348	247	989	0	299	127
normalized size	1	1.00	0.50	1.12	0.80	3.19	0.00	0.96	0.41
time (sec)	N/A	0.245	0.297	0.063	3.109	1.276	0.000	0.270	0.171

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	385	339	271	748	0	298	823
normalized size	1	1.00	1.24	1.09	0.87	2.41	0.00	0.96	2.65
time (sec)	N/A	0.253	0.548	0.068	3.027	0.970	0.000	0.192	0.259

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	136	317	223	925	0	283	106
normalized size	1	1.00	0.47	1.10	0.77	3.20	0.00	0.98	0.37
time (sec)	N/A	0.229	0.265	0.065	3.312	1.054	0.000	0.209	0.247

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	354	323	250	725	0	283	744
normalized size	1	1.00	1.22	1.12	0.87	2.51	0.00	0.98	2.57
time (sec)	N/A	0.234	0.528	0.064	3.108	1.089	0.000	0.195	0.212

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	95	305	217	912	0	273	91
normalized size	1	1.00	0.36	1.17	0.83	3.49	0.00	1.05	0.35
time (sec)	N/A	0.200	0.182	0.062	3.110	1.033	0.000	0.209	0.229

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	203	305	241	717	0	273	750
normalized size	1	1.00	0.78	1.17	0.92	2.75	0.00	1.05	2.87
time (sec)	N/A	0.200	0.366	0.056	3.110	0.984	0.000	0.207	0.331

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	117	323	222	920	0	278	104
normalized size	1	1.00	0.41	1.14	0.78	3.24	0.00	0.98	0.37
time (sec)	N/A	0.230	0.316	0.060	3.036	1.161	0.000	0.205	0.232

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	355	317	251	741	0	283	859
normalized size	1	1.00	1.23	1.10	0.87	2.56	0.00	0.98	2.97
time (sec)	N/A	0.225	0.582	0.058	3.111	1.094	0.000	0.195	0.372

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	151	339	250	974	0	303	121
normalized size	1	1.00	0.49	1.09	0.81	3.14	0.00	0.98	0.39
time (sec)	N/A	0.271	0.637	0.063	3.026	1.033	0.000	0.209	0.246

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	385	348	286	795	0	292	595
normalized size	1	1.00	1.24	1.12	0.92	2.56	0.00	0.94	1.92
time (sec)	N/A	0.255	0.733	0.065	3.125	1.117	0.000	0.246	0.399

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	176	372	276	1024	0	328	142
normalized size	1	1.00	0.53	1.12	0.83	3.08	0.00	0.99	0.43
time (sec)	N/A	0.286	0.593	0.070	3.014	1.057	0.000	0.211	0.200

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	435	381	306	817	0	321	865
normalized size	1	1.00	1.27	1.11	0.89	2.38	0.00	0.94	2.52
time (sec)	N/A	0.278	0.628	0.076	3.080	1.011	0.000	0.234	0.249

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	176	357	256	993	0	304	138
normalized size	1	1.00	0.55	1.11	0.80	3.08	0.00	0.94	0.43
time (sec)	N/A	0.247	0.469	0.067	3.095	0.782	0.000	0.232	0.261

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	403	363	283	793	0	304	760
normalized size	1	1.00	1.25	1.13	0.88	2.46	0.00	0.94	2.36
time (sec)	N/A	0.254	0.627	0.072	3.060	0.844	0.000	0.202	0.228

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	137	325	251	990	0	293	122
normalized size	1	1.00	0.47	1.11	0.86	3.38	0.00	1.00	0.42
time (sec)	N/A	0.223	0.319	0.065	3.109	0.941	0.000	0.231	0.174

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	389	334	280	806	0	298	799
normalized size	1	1.00	1.31	1.12	0.94	2.70	0.00	1.00	2.68
time (sec)	N/A	0.236	0.707	0.065	2.965	0.880	0.000	0.194	0.383

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	62	335	253	1005	0	298	124
normalized size	1	1.00	0.21	1.12	0.85	3.37	0.00	1.00	0.42
time (sec)	N/A	0.229	0.087	0.070	3.014	0.837	0.000	0.233	0.165

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	230	325	276	793	0	293	780
normalized size	1	1.00	0.78	1.11	0.94	2.71	0.00	1.00	2.66
time (sec)	N/A	0.231	0.378	0.064	3.054	0.901	0.000	0.195	0.392

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	147	363	255	988	0	300	133
normalized size	1	1.00	0.47	1.15	0.81	3.13	0.00	0.95	0.42
time (sec)	N/A	0.256	0.294	0.069	3.074	0.987	0.000	0.212	0.261

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	400	357	285	809	0	304	888
normalized size	1	1.00	1.24	1.11	0.89	2.51	0.00	0.94	2.76
time (sec)	N/A	0.251	0.496	0.068	3.089	0.853	0.000	0.205	0.446

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	189	381	285	1043	0	326	152
normalized size	1	1.00	0.55	1.11	0.83	3.04	0.00	0.95	0.44
time (sec)	N/A	0.275	0.517	0.070	3.038	0.928	0.000	0.221	0.194

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	433	390	321	864	0	315	626
normalized size	1	1.00	1.26	1.14	0.94	2.52	0.00	0.92	1.83
time (sec)	N/A	0.278	0.571	0.068	3.145	1.025	0.000	0.219	0.449

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	216	414	311	1093	0	351	173
normalized size	1	1.00	0.59	1.13	0.85	2.99	0.00	0.96	0.47
time (sec)	N/A	0.320	0.552	0.074	3.083	1.250	0.000	0.260	0.289

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	467	420	353	894	0	351	639
normalized size	1	1.00	1.28	1.15	0.97	2.45	0.00	0.96	1.75
time (sec)	N/A	0.321	0.628	0.072	3.097	0.831	0.000	0.281	0.507

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	136	307	0	0	0	0	-1
normalized size	1	1.00	0.56	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.177	0.201	0.000	1.084	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	111	446	0	0	0	0	-1
normalized size	1	1.00	0.30	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.436	0.145	0.096	0.000	0.847	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	111	283	0	0	0	0	-1
normalized size	1	1.00	0.54	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.139	0.097	0.000	0.950	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	94	422	0	0	0	0	-1
normalized size	1	1.00	0.29	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	0.095	0.088	0.000	0.789	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	94	257	0	0	0	0	-1
normalized size	1	1.00	0.57	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.062	0.092	0.000	0.922	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	97	399	0	0	0	0	-1
normalized size	1	1.00	0.30	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.048	0.109	0.000	0.937	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	97	239	0	0	0	0	-1
normalized size	1	1.00	0.60	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.048	0.085	0.000	0.864	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	96	422	0	0	0	0	-1
normalized size	1	1.00	0.29	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.374	0.052	0.104	0.000	0.948	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	98	255	0	0	0	0	-1
normalized size	1	1.00	0.59	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.063	0.092	0.000	0.804	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	99	452	0	0	0	0	-1
normalized size	1	1.00	0.27	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.436	0.055	0.097	0.000	0.966	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	98	283	0	0	0	0	-1
normalized size	1	1.00	0.48	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.063	0.099	0.000	0.765	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	160	518	0	0	0	0	-1
normalized size	1	1.00	0.33	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.671	0.305	0.107	0.000	0.987	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	160	355	0	0	0	0	-1
normalized size	1	1.00	0.50	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	0.229	0.102	0.000	1.015	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	138	494	0	0	0	0	-1
normalized size	1	1.00	0.31	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	0.192	0.079	0.000	0.954	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	138	331	0	0	0	0	-1
normalized size	1	1.00	0.49	1.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	0.185	0.069	0.000	1.049	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	115	470	0	0	0	0	-1
normalized size	1	1.00	0.28	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.515	0.159	0.069	0.000	0.818	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	115	307	0	0	0	0	-1
normalized size	1	1.00	0.48	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.154	0.068	0.000	0.976	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	98	446	0	0	0	0	-1
normalized size	1	1.00	0.27	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	0.108	0.071	0.000	1.176	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	97	283	0	0	0	0	-1
normalized size	1	1.00	0.48	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.073	0.068	0.000	1.285	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	85	429	0	0	0	0	-1
normalized size	1	1.00	0.24	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.080	0.078	0.000	1.104	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	101	260	0	0	0	0	-1
normalized size	1	1.00	0.50	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	0.086	0.088	0.000	1.070	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	99	427	0	0	0	0	-1
normalized size	1	1.00	0.28	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.068	0.072	0.000	1.034	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	101	254	0	0	0	0	-1
normalized size	1	1.00	0.50	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.058	0.076	0.000	0.668	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	100	452	0	0	0	0	-1
normalized size	1	1.00	0.27	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.061	0.073	0.000	1.058	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	143	298	0	0	0	0	-1
normalized size	1	1.00	0.59	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.177	0.091	0.000	1.031	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	122	437	0	0	0	0	-1
normalized size	1	1.00	0.33	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	0.152	0.092	0.000	1.065	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	122	274	0	0	0	0	-1
normalized size	1	1.00	0.60	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.146	0.071	0.000	0.868	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	97	413	0	0	0	0	-1
normalized size	1	1.00	0.29	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.122	0.069	0.000	0.696	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	97	248	0	0	0	0	-1
normalized size	1	1.00	0.58	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.120	0.068	0.000	1.014	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	81	378	0	0	0	0	-1
normalized size	1	1.00	0.28	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.100	0.067	0.000	0.844	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	80	216	0	0	0	0	-1
normalized size	1	1.00	0.62	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.067	0.066	0.000	0.908	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	82	377	0	0	0	0	-1
normalized size	1	1.00	0.29	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.052	0.069	0.000	0.992	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	82	219	0	0	0	0	-1
normalized size	1	1.00	0.63	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.070	0.072	0.000	0.925	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	83	413	0	0	0	0	-1
normalized size	1	1.00	0.25	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.068	0.075	0.000	1.152	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	85	247	0	0	0	0	-1
normalized size	1	1.00	0.51	1.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.052	0.071	0.000	1.307	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	84	443	0	0	0	0	-1
normalized size	1	1.00	0.23	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	0.055	0.083	0.000	1.160	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	84	274	0	0	0	0	-1
normalized size	1	1.00	0.41	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.066	0.086	0.000	1.324	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	134	281	0	0	0	0	-1
normalized size	1	1.00	0.53	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.245	0.149	0.000	0.888	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	110	420	0	0	0	0	-1
normalized size	1	1.00	0.29	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	0.181	0.122	0.000	1.219	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	110	255	0	0	0	0	-1
normalized size	1	1.00	0.51	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.175	0.089	0.000	0.966	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	85	394	0	0	0	0	-1
normalized size	1	1.00	0.25	1.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	0.133	0.092	0.000	0.963	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	86	230	0	0	0	0	-1
normalized size	1	1.00	0.48	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.130	0.082	0.000	0.933	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	78	388	0	0	0	0	-1
normalized size	1	1.00	0.26	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	0.111	0.081	0.000	1.023	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	76	222	0	0	0	0	-1
normalized size	1	1.00	0.55	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.079	0.087	0.000	0.966	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	77	392	0	0	0	0	-1
normalized size	1	1.00	0.24	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.082	0.098	0.000	1.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	92	235	0	0	0	0	-1
normalized size	1	1.00	0.55	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.071	0.090	0.000	1.333	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	79	420	0	0	0	0	-1
normalized size	1	1.00	0.21	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	0.062	0.087	0.000	1.027	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	79	254	0	0	0	0	-1
normalized size	1	1.00	0.39	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	0.052	0.093	0.000	0.959	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	79	450	0	0	0	0	-1
normalized size	1	1.00	0.20	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	0.058	0.094	0.000	0.931	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	89	474	129	381	2077	603	291
normalized size	1	1.00	0.93	4.94	1.34	3.97	21.64	6.28	3.03
time (sec)	N/A	0.070	0.142	0.052	1.366	0.914	9.055	0.215	0.477

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	91	217	1051	340	179
normalized size	1	1.00	0.93	3.69	1.28	3.06	14.80	4.79	2.52
time (sec)	N/A	0.052	0.071	0.052	1.371	0.969	4.422	0.189	0.340

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	110	53	94	415	149	97
normalized size	1	1.00	0.93	2.44	1.18	2.09	9.22	3.31	2.16
time (sec)	N/A	0.030	0.054	0.049	1.300	1.279	1.621	0.163	0.247

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	55	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.076	0.546	0.000	0.767	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	92	80	0	0	0	0	0	-1
normalized size	1	0.94	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.105	0.532	0.000	0.890	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	126	135	0	0	0	0	0	-1
normalized size	1	0.90	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.122	0.530	0.000	0.989	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	0	112	140	0	0	-1
normalized size	1	1.00	0.66	0.00	1.18	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.075	0.799	2.042	0.918	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.176	2.311	0.000	0.846	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	210	0	0	0	0	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.357	0.246	1.081	0.000	0.690	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	17	13
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.89	0.89	0.68
time (sec)	N/A	0.026	0.006	0.046	2.909	0.863	0.113	0.155	0.171

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	35	11	10	16	11
normalized size	1	1.00	1.00	2.40	2.33	0.73	0.67	1.07	0.73
time (sec)	N/A	0.022	0.006	0.050	3.010	0.928	0.121	0.152	0.153

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	16	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.94	0.76
time (sec)	N/A	0.025	0.006	0.048	2.939	1.265	0.146	0.191	0.139

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	10	16	14
normalized size	1	1.00	1.00	0.94	0.88	1.12	0.62	1.00	0.88
time (sec)	N/A	0.014	0.004	0.053	1.623	0.687	0.102	0.183	0.129

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.021	0.920	0.000	1.025	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	116	0	37	43	0	48	-1
normalized size	1	1.00	6.44	0.00	2.06	2.39	0.00	2.67	-0.06
time (sec)	N/A	0.038	0.221	1.104	2.150	0.788	0.000	0.369	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.109	0.589	0.000	0.720	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	157	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.174	0.510	0.000	1.001	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	119	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.079	0.539	0.000	1.056	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.036	0.520	0.000	0.783	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.017	0.525	0.000	0.655	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.026	0.556	0.000	0.697	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	112	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.144	0.529	0.000	1.207	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	184	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.239	0.526	0.000	0.744	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.108	0.518	0.000	1.011	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	179	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.242	0.534	0.000	0.815	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	120	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.151	0.539	0.000	0.913	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.040	0.510	0.000	1.153	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.124	0.563	0.000	0.950	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	125	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.111	0.511	0.000	0.867	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	182	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	0.346	0.540	0.000	1.025	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [207] had the largest ratio of [.4231]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	22	0.091
2	A	4	3	1.00	20	0.150
3	A	3	2	1.00	19	0.105
4	A	4	3	1.00	22	0.136
5	A	3	2	1.00	22	0.091
6	A	4	3	1.00	22	0.136
7	A	3	2	1.00	22	0.091
8	A	4	3	1.00	22	0.136
9	A	3	2	1.00	22	0.091
10	A	4	3	1.00	22	0.136
11	A	3	2	1.00	22	0.091
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	24	0.125
14	A	3	2	1.00	24	0.083
15	A	4	3	1.00	24	0.125
16	A	3	2	1.00	24	0.083
17	A	5	4	1.00	24	0.167
18	A	3	2	1.00	24	0.083
19	A	4	3	1.00	24	0.125
20	A	3	2	1.00	24	0.083
21	A	4	3	1.00	24	0.125
22	A	3	2	1.00	24	0.083
23	A	4	3	1.00	24	0.125
24	A	3	2	1.00	24	0.083
25	A	3	2	1.00	24	0.083
26	A	4	3	1.00	24	0.125
27	A	3	2	1.00	24	0.083
28	A	4	3	1.00	24	0.125
29	A	3	2	1.00	24	0.083
30	A	5	4	1.00	24	0.167
31	A	3	2	1.00	24	0.083
32	A	4	3	1.00	24	0.125
33	A	3	2	1.00	24	0.083
34	A	4	3	1.00	24	0.125
35	A	3	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	4	3	1.00	24	0.125
37	A	3	2	1.00	24	0.083
38	A	5	4	1.00	24	0.167
39	A	3	2	1.00	24	0.083
40	A	4	4	1.00	24	0.167
41	A	5	4	1.00	24	0.167
42	A	4	3	1.00	24	0.125
43	A	5	4	1.00	24	0.167
44	A	4	3	1.00	24	0.125
45	A	5	4	1.00	24	0.167
46	A	4	3	1.00	24	0.125
47	A	4	4	1.00	24	0.167
48	A	4	3	1.00	24	0.125
49	A	3	3	1.00	24	0.125
50	A	4	3	1.00	22	0.136
51	A	3	3	1.00	21	0.143
52	A	3	3	1.00	22	0.136
53	A	4	3	1.00	24	0.125
54	A	4	4	1.00	24	0.167
55	A	4	3	1.00	24	0.125
56	A	5	4	1.00	24	0.167
57	A	4	3	1.00	24	0.125
58	A	5	4	1.00	24	0.167
59	A	4	3	1.00	24	0.125
60	A	5	4	1.00	24	0.167
61	A	4	3	1.00	24	0.125
62	A	5	4	1.00	24	0.167
63	A	4	3	1.00	24	0.125
64	A	4	4	1.00	24	0.167
65	A	4	3	1.00	24	0.125
66	A	3	3	1.00	24	0.125
67	A	4	3	1.00	24	0.125
68	A	4	4	1.00	24	0.167
69	A	4	3	1.00	22	0.136
70	A	5	4	1.00	21	0.190
71	A	4	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	5	4	1.00	24	0.167
73	A	6	5	1.00	24	0.208
74	A	4	3	1.00	24	0.125
75	A	6	5	1.00	24	0.208
76	A	4	3	1.00	24	0.125
77	A	5	5	1.00	24	0.208
78	A	4	3	1.00	24	0.125
79	A	4	4	1.00	24	0.167
80	A	3	3	1.00	24	0.125
81	A	4	4	1.00	24	0.167
82	A	4	3	1.00	24	0.125
83	A	5	4	1.00	24	0.167
84	A	4	3	1.00	24	0.125
85	A	6	5	1.00	24	0.208
86	A	4	3	1.00	22	0.136
87	A	6	5	1.00	21	0.238
88	A	4	3	1.00	24	0.125
89	A	8	7	1.00	26	0.269
90	A	7	7	1.00	26	0.269
91	A	5	5	1.00	26	0.192
92	A	5	5	1.00	24	0.208
93	A	5	5	1.00	26	0.192
94	A	5	5	1.00	26	0.192
95	A	5	5	1.00	26	0.192
96	A	3	3	1.00	26	0.115
97	A	4	4	1.00	26	0.154
98	A	5	4	1.00	26	0.154
99	A	6	4	1.00	26	0.154
100	A	4	3	1.00	26	0.115
101	A	3	3	1.00	26	0.115
102	A	2	2	1.00	23	0.087
103	A	4	4	1.00	26	0.154
104	A	4	4	1.00	26	0.154
105	A	4	4	1.00	26	0.154
106	A	8	7	1.00	26	0.269
107	A	6	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	6	5	1.00	24	0.208
109	A	6	6	1.00	26	0.231
110	A	6	6	1.00	26	0.231
111	A	6	5	1.00	26	0.192
112	A	6	6	1.00	26	0.231
113	A	6	5	1.00	26	0.192
114	A	3	3	1.00	26	0.115
115	A	4	4	1.00	26	0.154
116	A	5	4	1.00	26	0.154
117	A	6	4	1.00	26	0.154
118	A	7	4	1.00	26	0.154
119	A	5	4	1.00	26	0.154
120	A	4	4	1.00	26	0.154
121	A	3	3	1.00	23	0.130
122	A	2	2	1.00	26	0.077
123	A	5	4	1.00	26	0.154
124	A	5	4	1.00	26	0.154
125	A	5	5	1.00	26	0.192
126	A	5	4	1.00	26	0.154
127	A	6	5	1.00	26	0.192
128	A	7	5	1.00	26	0.192
129	A	8	5	1.00	26	0.192
130	A	7	6	1.00	26	0.231
131	A	6	6	1.00	26	0.231
132	A	4	4	1.00	26	0.154
133	A	4	4	1.00	24	0.167
134	A	4	4	1.00	26	0.154
135	A	3	3	1.00	26	0.115
136	A	4	4	1.00	26	0.154
137	A	5	4	1.00	26	0.154
138	A	6	4	1.00	26	0.154
139	A	4	3	1.00	26	0.115
140	A	3	3	1.00	26	0.115
141	A	2	2	1.00	26	0.077
142	A	3	3	1.00	23	0.130
143	A	3	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	4	4	1.00	26	0.154
145	A	7	6	1.00	26	0.231
146	A	6	6	1.00	26	0.231
147	A	5	5	1.00	26	0.192
148	A	4	4	1.00	26	0.154
149	A	2	2	1.00	24	0.083
150	A	3	3	1.00	26	0.115
151	A	4	4	1.00	26	0.154
152	A	5	4	1.00	26	0.154
153	A	4	3	1.00	26	0.115
154	A	3	3	1.00	26	0.115
155	A	2	2	1.00	26	0.077
156	A	3	3	1.00	26	0.115
157	A	5	5	1.00	23	0.217
158	A	5	5	1.00	26	0.192
159	A	3	2	1.00	24	0.083
160	A	3	2	1.00	24	0.083
161	A	3	2	1.00	24	0.083
162	A	3	2	1.00	24	0.083
163	A	3	2	1.00	24	0.083
164	A	3	2	1.00	24	0.083
165	A	3	2	1.00	24	0.083
166	A	3	2	1.00	24	0.083
167	A	3	2	1.00	26	0.077
168	A	3	2	1.00	26	0.077
169	A	3	2	1.00	26	0.077
170	A	3	2	1.00	26	0.077
171	A	3	2	1.00	26	0.077
172	A	3	2	1.00	26	0.077
173	A	3	2	1.00	26	0.077
174	A	3	2	1.00	26	0.077
175	A	3	2	1.00	26	0.077
176	A	3	2	1.00	26	0.077
177	A	3	2	1.00	26	0.077
178	A	3	2	1.00	26	0.077
179	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	3	2	1.00	26	0.077
181	A	3	2	1.00	26	0.077
182	A	3	2	1.00	26	0.077
183	A	14	10	1.00	26	0.385
184	A	14	10	1.00	26	0.385
185	A	13	10	1.00	26	0.385
186	A	13	10	1.00	26	0.385
187	A	12	9	1.00	26	0.346
188	A	12	9	1.00	26	0.346
189	A	12	9	1.00	26	0.346
190	A	12	9	1.00	26	0.346
191	A	13	10	1.00	26	0.385
192	A	13	10	1.00	26	0.385
193	A	14	10	1.00	26	0.385
194	A	14	10	1.00	26	0.385
195	A	15	10	1.00	26	0.385
196	A	14	10	1.00	26	0.385
197	A	14	10	1.00	26	0.385
198	A	13	10	1.00	26	0.385
199	A	13	10	1.00	26	0.385
200	A	12	9	1.00	26	0.346
201	A	12	9	1.00	26	0.346
202	A	13	10	1.00	26	0.385
203	A	13	10	1.00	26	0.385
204	A	14	10	1.00	26	0.385
205	A	14	10	1.00	26	0.385
206	A	15	10	1.00	26	0.385
207	A	15	11	1.00	26	0.423
208	A	14	11	1.00	26	0.423
209	A	14	11	1.00	26	0.423
210	A	13	10	1.00	26	0.385
211	A	13	10	1.00	26	0.385
212	A	13	10	1.00	26	0.385
213	A	13	10	1.00	26	0.385
214	A	14	11	1.00	26	0.423
215	A	14	11	1.00	26	0.423

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	15	11	1.00	26	0.423
217	A	15	11	1.00	26	0.423
218	A	16	11	1.00	26	0.423
219	A	16	11	1.00	26	0.423
220	A	7	6	1.00	28	0.214
221	A	8	8	1.00	28	0.286
222	A	6	6	1.00	28	0.214
223	A	7	7	1.00	28	0.250
224	A	5	5	1.00	28	0.179
225	A	7	7	1.00	28	0.250
226	A	5	5	1.00	28	0.179
227	A	7	7	1.00	28	0.250
228	A	5	5	1.00	28	0.179
229	A	8	8	1.00	28	0.286
230	A	6	6	1.00	28	0.214
231	A	11	8	1.00	28	0.286
232	A	9	6	1.00	28	0.214
233	A	10	8	1.00	28	0.286
234	A	8	6	1.00	28	0.214
235	A	9	8	1.00	28	0.286
236	A	7	6	1.00	28	0.214
237	A	8	7	1.00	28	0.250
238	A	6	5	1.00	28	0.179
239	A	8	7	1.00	28	0.250
240	A	6	5	1.00	28	0.179
241	A	8	8	1.00	28	0.286
242	A	6	6	1.00	28	0.214
243	A	8	7	1.00	28	0.250
244	A	7	5	1.00	28	0.179
245	A	8	7	1.00	28	0.250
246	A	6	5	1.00	28	0.179
247	A	7	7	1.00	28	0.250
248	A	5	5	1.00	28	0.179
249	A	6	6	1.00	28	0.214
250	A	4	4	1.00	28	0.143
251	A	6	6	1.00	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	4	4	1.00	28	0.143
253	A	7	7	1.00	28	0.250
254	A	5	5	1.00	28	0.179
255	A	8	7	1.00	28	0.250
256	A	6	5	1.00	28	0.179
257	A	7	5	1.00	28	0.179
258	A	8	7	1.00	28	0.250
259	A	6	5	1.00	28	0.179
260	A	7	7	1.00	28	0.250
261	A	5	5	1.00	28	0.179
262	A	6	6	1.00	28	0.214
263	A	4	4	1.00	28	0.143
264	A	7	7	1.00	28	0.250
265	A	5	5	1.00	28	0.179
266	A	8	8	1.00	28	0.286
267	A	6	6	1.00	28	0.214
268	A	9	8	1.00	28	0.286
269	A	3	2	1.00	24	0.083
270	A	3	2	1.00	24	0.083
271	A	3	2	1.00	22	0.091
272	A	3	3	1.00	24	0.125
273	A	3	3	0.94	24	0.125
274	A	4	4	0.90	24	0.167
275	A	2	2	1.00	32	0.062
276	A	4	3	1.00	30	0.100
277	A	4	3	1.00	34	0.088
278	A	4	3	1.00	19	0.158
279	A	4	3	1.00	15	0.200
280	A	4	3	1.00	19	0.158
281	A	3	2	1.00	17	0.118
282	A	4	4	1.00	25	0.160
283	A	2	2	1.00	39	0.051
284	A	4	4	1.00	20	0.200
285	A	7	7	1.00	20	0.350
286	A	6	6	1.00	18	0.333
287	A	5	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	3	3	1.00	20	0.150
289	A	4	4	1.00	20	0.200
290	A	5	4	1.00	20	0.200
291	A	5	4	1.00	20	0.200
292	A	4	4	1.00	20	0.200
293	A	7	7	1.00	20	0.350
294	A	6	6	1.00	18	0.333
295	A	3	3	1.00	17	0.176
296	A	4	4	1.00	20	0.200
297	A	5	5	1.00	20	0.250
298	A	5	5	1.00	20	0.250

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(A+Bx^2\right) \left(bx^2+cx^4\right) \, dx$

Optimal. Leaf size=33

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

```
[Out] 1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(A+Bx^2\right)\left(bx^2+cx^4\right) \, dx &= \int x^4 \left(A+Bx^2\right)\left(b+cx^2\right) \, dx \\
&= \int \left(Abx^4+(bB+Ac)x^6+Bcx^8\right) \, dx \\
&= \frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

fricas [A] time = 0.81, size = 29, normalized size = 0.88

$$\frac{1}{9}x^9cB+\frac{1}{7}x^7bB+\frac{1}{7}x^7cA+\frac{1}{5}x^5bA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/9*x^9*c*B + 1/7*x^7*b*B + 1/7*x^7*c*A + 1/5*x^5*b*A
```

giac [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{9}\,Bcx^9+\frac{1}{7}\,Bbx^7+\frac{1}{7}\,Acx^7+\frac{1}{5}\,Abx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/9*B*c*x^9 + 1/7*B*b*x^7 + 1/7*A*c*x^7 + 1/5*A*b*x^5
```

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bc\,x^9}{9}+\frac{Ab\,x^5}{5}+\frac{(Ac+bB)\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2),x)
[Out] 1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

maxima [A] time = 1.40, size = 27, normalized size = 0.82

$$\frac{1}{9}\,Bcx^9+\frac{1}{7}\,(Bb+Ac)x^7+\frac{1}{5}\,Abx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

mupad [B] time = 0.18, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^9}{9}+\left(\frac{A\,c}{7}+\frac{B\,b}{7}\right)\,x^7+\frac{A\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] x^7*((A*c)/7 + (B*b)/7) + (A*b*x^5)/5 + (B*c*x^9)/9
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^5}{5} + \frac{Bcx^9}{9} + x^7\left(\frac{Ac}{7} + \frac{Bb}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**5/5 + B*c*x**9/9 + x**7*(A*c/7 + B*b/7)
```

3.2 $\int x \left(A+Bx^2\right) \left(bx^2+cx^4\right) \, dx$

Optimal. Leaf size=33

$$\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

[Out] 1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1584, 446, 76}

$$\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4),x]

[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x \left(A+Bx^2\right)\left(bx^2+cx^4\right) dx &= \int x^3 \left(A+Bx^2\right)\left(b+cx^2\right) dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x(A+Bx)(b+cx)\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(Abx+(bB+Ac)x^2+Bcx^3\right) dx, x, x^2\right)\\
&= \frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

fricas [A] time = 1.00, size = 29, normalized size = 0.88

$$\frac{1}{8}x^8cB + \frac{1}{6}x^6bB + \frac{1}{6}x^6cA + \frac{1}{4}x^4bA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/8*x^8*c*B + 1/6*x^6*b*B + 1/6*x^6*c*A + 1/4*x^4*b*A
```

giac [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{8}Bcx^8 + \frac{1}{6}Bbx^6 + \frac{1}{6}Acx^6 + \frac{1}{4}Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/6*A*c*x^6 + 1/4*A*b*x^4
```

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bc\,x^8}{8} + \frac{Ab\,x^4}{4} + \frac{(Ac + bB)\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2),x)
[Out] 1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

maxima [A] time = 1.26, size = 27, normalized size = 0.82

$$\frac{1}{8}Bcx^8 + \frac{1}{6}(Bb + Ac)x^6 + \frac{1}{4}Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

mupad [B] time = 0.06, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^8}{8} + \left(\frac{A\,c}{6} + \frac{B\,b}{6}\right)\,x^6 + \frac{A\,b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] x^6*((A*c)/6 + (B*b)/6) + (A*b*x^4)/4 + (B*c*x^8)/8
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^4}{4} + \frac{Bcx^8}{8} + x^6\left(\frac{Ac}{6} + \frac{Bb}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**4/4 + B*c*x**8/8 + x**6*(A*c/6 + B*b/6)
```

3.3 $\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

```
[Out] 1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1593, 448}

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^2\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^2+(bB+Ac)x^4+Bcx^6\right)\,dx\\ &= \frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

fricas [A] time = 0.54, size = 29, normalized size = 0.88

$$\frac{1}{7}x^7cB+\frac{1}{5}x^5bB+\frac{1}{5}x^5cA+\frac{1}{3}x^3bA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*B + 1/5*x^5*b*B + 1/5*x^5*c*A + 1/3*x^3*b*A
```

giac [A] time = 0.19, size = 29, normalized size = 0.88

$$\frac{1}{7}Bcx^7 + \frac{1}{5}Bbx^5 + \frac{1}{5}Acx^5 + \frac{1}{3}Abx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/5*A*c*x^5 + 1/3*A*b*x^3
```

maple [A] time = 0.05, size = 28, normalized size = 0.85

$$\frac{Bc\,x^7}{7} + \frac{Ab\,x^3}{3} + \frac{(Ac + bB)\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

maxima [A] time = 1.35, size = 27, normalized size = 0.82

$$\frac{1}{7}Bcx^7 + \frac{1}{5}(Bb + Ac)x^5 + \frac{1}{3}Abx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

mupad [B] time = 0.04, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^7}{7} + \left(\frac{A\,c}{5} + \frac{B\,b}{5}\right)\,x^5 + \frac{A\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] x^5*((A*c)/5 + (B*b)/5) + (A*b*x^3)/3 + (B*c*x^7)/7
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^3}{3} + \frac{Bcx^7}{7} + x^5\left(\frac{Ac}{5} + \frac{Bb}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**3/3 + B*c*x**7/7 + x**5*(A*c/5 + B*b/5)
```

3.4 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x} \, dx$

Optimal. Leaf size=33

$$\frac{1}{4}x^4(Ac+bB)+\frac{1}{2}Abx^2+\frac{1}{6}Bcx^6$$

```
[Out] 1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 444, 43}

$$\frac{1}{4}x^4(Ac+bB)+\frac{1}{2}Abx^2+\frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

```
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x} \, dx &= \int x\left(A+Bx^2\right)\left(b+cx^2\right) dx \\
&= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)\, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(Ab+(bB+Ac)x+Bcx^2\right) dx, x, x^2\right) \\
&= \frac{1}{2}Abx^2+\frac{1}{4}(bB+Ac)x^4+\frac{1}{6}Bcx^6
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{4}x^4(Ac+bB)+\frac{1}{2}Abx^2+\frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

```
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

fricas [A] time = 0.56, size = 27, normalized size = 0.82

$$\frac{1}{6}\,Bcx^{6} + \frac{1}{4}\,(Bb + Ac)x^{4} + \frac{1}{2}\,Abx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

giac [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{6}\,Bcx^{6} + \frac{1}{4}\,Bbx^{4} + \frac{1}{4}\,Acx^{4} + \frac{1}{2}\,Abx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/4*A*c*x^4 + 1/2*A*b*x^2
```

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bc\,x^{6}}{6} + \frac{Ab\,x^{2}}{2} + \frac{(Ac + bB)\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x,x)
```

```
[Out] 1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

maxima [A] time = 1.31, size = 27, normalized size = 0.82

$$\frac{1}{6}\,Bcx^{6} + \frac{1}{4}\,(Bb + Ac)x^{4} + \frac{1}{2}\,Abx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

mupad [B] time = 0.06, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^{6}}{6} + \left(\frac{A\,c}{4} + \frac{B\,b}{4}\right)\,x^{4} + \frac{A\,b\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x,x)
```

```
[Out] x^4*((A*c)/4 + (B*b)/4) + (A*b*x^2)/2 + (B*c*x^6)/6
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Abx^{2}}{2} + \frac{Bcx^{6}}{6} + x^{4}\left(\frac{Ac}{4} + \frac{Bb}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x,x)
```

```
[Out] A*b*x**2/2 + B*c*x**6/6 + x**4*(A*c/4 + B*b/4)
```

3.5 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

```
[Out] A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 373}

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
```

```
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx &= \int (A+Bx^2)(b+cx^2)\,dx\\ &= \int \left(Ab+(bB+Ac)x^2+Bcx^4\right)dx\\ &= Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
```

```
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

fricas [A] time = 0.70, size = 24, normalized size = 0.86

$$\frac{1}{5}Bcx^5+\frac{1}{3}(Bb+Ac)x^3+Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{3}Acx^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + A*b*x
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{Bc\,x^5}{5} + Abx + \frac{(Ac + bB)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^2,x)
```

```
[Out] A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

maxima [A] time = 1.41, size = 24, normalized size = 0.86

$$\frac{1}{5}Bcx^5 + \frac{1}{3}(Bb + Ac)x^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

mupad [B] time = 0.06, size = 25, normalized size = 0.89

$$\frac{B\,c\,x^5}{5} + \left(\frac{A\,c}{3} + \frac{B\,b}{3}\right)\,x^3 + A\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x^3*((A*c)/3 + (B*b)/3) + A*b*x + (B*c*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.93

$$Abx + \frac{Bcx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Bb}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**2,x)
```

```
[Out] A*b*x + B*c*x**5/5 + x**3*(A*c/3 + B*b/3)
```

3.6 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}x^2(Ac+bB)+Ab\log(x)+\frac{1}{4}Bcx^4$$

```
[Out] 1/2*(A*c+B*b)*x^2+1/4*B*c*x^4+A*b*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 76}

$$\frac{1}{2}x^2(Ac+bB)+Ab\log(x)+\frac{1}{4}Bcx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(bB+Ac+\frac{Ab}{x}+Bcx\right)dx,x,x^2\right)\\ &= \frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{2}x^2(Ac+bB)+Ab\log(x)+\frac{1}{4}Bcx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

fricas [A] time = 0.99, size = 25, normalized size = 0.86

$$\frac{1}{4}\,Bcx^4 + \frac{1}{2}\,(Bb + Ac)x^2 + Ab\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="fricas")
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + A*b*log(x)
```

giac [A] time = 0.15, size = 30, normalized size = 1.03

$$\frac{1}{4}\,Bcx^4 + \frac{1}{2}\,Bbx^2 + \frac{1}{2}\,Acx^2 + \frac{1}{2}\,Ab\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="giac")
[Out] 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/2*A*c*x^2 + 1/2*A*b*log(x^2)
```

maple [A] time = 0.04, size = 28, normalized size = 0.97

$$\frac{Bc\,x^4}{4} + \frac{Ac\,x^2}{2} + \frac{Bb\,x^2}{2} + Ab\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^3,x)
[Out] 1/4*B*c*x^4+1/2*A*x^2*c+1/2*B*b*x^2+A*b*ln(x)
```

maxima [A] time = 1.33, size = 28, normalized size = 0.97

$$\frac{1}{4}\,Bcx^4 + \frac{1}{2}\,(Bb + Ac)x^2 + \frac{1}{2}\,Ab\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="maxima")
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + 1/2*A*b*log(x^2)
```

mupad [B] time = 0.06, size = 26, normalized size = 0.90

$$x^2\,\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)+\frac{B\,c\,x^4}{4}+A\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x)
[Out] x^2*((A*c)/2 + (B*b)/2) + (B*c*x^4)/4 + A*b*log(x)
```

sympy [A] time = 0.12, size = 27, normalized size = 0.93

$$Ab\log\left(x\right) + \frac{Bcx^4}{4} + x^2\left(\frac{Ac}{2} + \frac{Bb}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**3,x)
```

```
[Out] A*b*log(x) + B*c*x**4/4 + x**2*(A*c/2 + B*b/2)
```

3.7 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx$

Optimal. Leaf size=26

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

```
[Out] -A*b/x+(A*c+B*b)*x+1/3*B*c*x^3
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^2}\,dx\\ &= \int \left(bB\left(1+\frac{Ac}{bB}\right)+\frac{Ab}{x^2}+Bcx^2\right)dx\\ &= -\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

fricas [A] time = 0.83, size = 28, normalized size = 1.08

$$\frac{Bcx^4+3\,(Bb+Ac)x^2-3\,Ab}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="fricas")
[Out] 1/3*(B*c*x^4 + 3*(B*b + A*c)*x^2 - 3*A*b)/x
```

giac [A] time = 0.16, size = 23, normalized size = 0.88

$$\frac{1}{3}\,Bcx^3 + Bbx + Acx - \frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="giac")
[Out] 1/3*B*c*x^3 + B*b*x + A*c*x - A*b/x
```

maple [A] time = 0.06, size = 24, normalized size = 0.92

$$\frac{Bc\,x^3}{3} + Acx + Bbx - \frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^4,x)
[Out] 1/3*B*c*x^3+A*c*x+B*b*x-A*b/x
```

maxima [A] time = 1.33, size = 24, normalized size = 0.92

$$\frac{1}{3}\,Bcx^3 + (Bb + Ac)x - \frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="maxima")
[Out] 1/3*B*c*x^3 + (B*b + A*c)*x - A*b/x
```

mupad [B] time = 0.07, size = 24, normalized size = 0.92

$$x\,\left(A\,c + B\,b\right) - \frac{A\,b}{x} + \frac{B\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x)
[Out] x*(A*c + B*b) - (A*b)/x + (B*c*x^3)/3
```

sympy [A] time = 0.11, size = 20, normalized size = 0.77

$$-\frac{Ab}{x} + \frac{Bcx^3}{3} + x\,(Ac + Bb)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**4,x)
[Out] -A*b/x + B*c*x**3/3 + x*(A*c + B*b)
```

3.8 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} dx$

Optimal. Leaf size=29

$$\log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

[Out] -1/2*A*b/x^2+1/2*B*c*x^2+(A*c+B*b)*ln(x)

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 76}

$$\log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]

[Out] -(A*b)/(2*x^2) + (B*c*x^2)/2 + (b*B + A*c)*Log[x]

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^3}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(Bc+\frac{Ab}{x^2}+\frac{bB+Ac}{x}\right)dx,x,x^2\right)\\
&= -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
[Out] -1/2*(A*b)/x^2 + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

fricas [A] time = 0.79, size = 30, normalized size = 1.03

$$\frac{Bcx^4 + 2\,(Bb + Ac)x^2\log(x) - Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="fricas")
[Out] 1/2*(B*c*x^4 + 2*(B*b + A*c)*x^2*log(x) - A*b)/x^2
```

giac [A] time = 0.16, size = 42, normalized size = 1.45

$$\frac{1}{2}\,Bcx^2 + \frac{1}{2}\,(Bb + Ac)\log\left(x^2\right) - \frac{Bbx^2 + Acx^2 + Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="giac")
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*(B*b*x^2 + A*c*x^2 + A*b)/x^2
```

maple [A] time = 0.05, size = 26, normalized size = 0.90

$$\frac{Bc\,x^2}{2} + Ac\ln(x) + Bb\ln(x) - \frac{Ab}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^5,x)
[Out] 1/2*B*c*x^2-1/2*A*b/x^2+A*ln(x)*c+B*b*ln(x)
```

maxima [A] time = 1.32, size = 28, normalized size = 0.97

$$\frac{1}{2}\,Bcx^2 + \frac{1}{2}\,(Bb + Ac)\log\left(x^2\right) - \frac{Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="maxima")
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*A*b/x^2
```

mupad [B] time = 0.04, size = 25, normalized size = 0.86

$$\ln(x)\ (A\,c + B\,b) - \frac{A\,b}{2\,x^2} + \frac{B\,c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x)
[Out] log(x)*(A*c + B*b) - (A*b)/(2*x^2) + (B*c*x^2)/2
```

sympy [A] time = 0.19, size = 26, normalized size = 0.90

$$-\frac{Ab}{2x^2} + \frac{Bcx^2}{2} + (Ac + Bb)\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**5,x)
[Out] -A*b/(2*x**2) + B*c*x**2/2 + (A*c + B*b)*log(x)
```

3.9 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx$

Optimal. Leaf size=26

$$-\frac{Ac+bB}{x}-\frac{Ab}{3x^3}+Bcx$$

[Out] -1/3*A*b/x^3+(-A*c-B*b)/x+B*c*x

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$-\frac{Ac+bB}{x}-\frac{Ab}{3x^3}+Bcx$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]

[Out] -(A*b)/(3*x^3) - (b*B + A*c)/x + B*c*x

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^4}\,dx\\
&= \int \left(Bc+\frac{Ab}{x^4}+\frac{bB+Ac}{x^2}\right)dx\\
&= -\frac{Ab}{3x^3}-\frac{bB+Ac}{x}+Bcx
\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.04

$$\frac{-Ac-bB}{x}-\frac{Ab}{3x^3}+Bcx$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]

[Out] -1/3*(A*b)/x^3 + (-(b*B) - A*c)/x + B*c*x

fricas [A] time = 0.77, size = 29, normalized size = 1.12

$$\frac{3\,Bcx^4-3\,(Bb+Ac)x^2-Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="fricas")
[Out] 1/3*(3*B*c*x^4 - 3*(B*b + A*c)*x^2 - A*b)/x^3
```

giac [A] time = 0.17, size = 28, normalized size = 1.08

$$Bcx - \frac{3\,Bbx^2 + 3\,Acx^2 + Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="giac")
[Out] B*c*x - 1/3*(3*B*b*x^2 + 3*A*c*x^2 + A*b)/x^3
```

maple [A] time = 0.05, size = 25, normalized size = 0.96

$$Bcx - \frac{Ab}{3x^3} - \frac{Ac + bB}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^6,x)
[Out] B*c*x-1/3*A*b/x^3-(A*c+B*b)/x
```

maxima [A] time = 1.34, size = 26, normalized size = 1.00

$$Bcx - \frac{3\,(Bb + Ac)x^2 + Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="maxima")
[Out] B*c*x - 1/3*(3*(B*b + A*c)*x^2 + A*b)/x^3
```

mupad [B] time = 0.06, size = 26, normalized size = 1.00

$$B\,c\,x - \frac{(A\,c + B\,b)\;x^2 + \frac{A\,b}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x)
[Out] B*c*x - ((A*b)/3 + x^2*(A*c + B*b))/x^3
```

sympy [A] time = 0.22, size = 27, normalized size = 1.04

$$Bcx + \frac{-Ab + x^2\,(-3Ac - 3Bb)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**6,x)
[Out] B*c*x + (-A*b + x**2*(-3*A*c - 3*B*b))/(3*x**3)
```

3.10 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx$

Optimal. Leaf size=29

$$-\frac{Ac+bB}{2x^2}-\frac{Ab}{4x^4}+Bc\log(x)$$

```
[Out] -1/4*A*b/x^4+1/2*(-A*c-B*b)/x^2+B*c*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 76}

$$-\frac{Ac+bB}{2x^2}-\frac{Ab}{4x^4}+Bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(A*b)/(4*x^4) - (b*B + A*c)/(2*x^2) + B*c*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)}{x^7}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)}{x^5}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab}{x^3}+\frac{bB+Ac}{x^2}+\frac{Bc}{x}\right)dx,x,x^2\right)\\
&= -\frac{Ab}{4x^4}-\frac{bB+Ac}{2x^2}+Bc\log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.07

$$\frac{-Ac-bB}{2x^2}-\frac{Ab}{4x^4}+Bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
[Out] -1/4*(A*b)/x^4 + (-(b*B) - A*c)/(2*x^2) + B*c*Log[x]
```

fricas [A] time = 0.82, size = 31, normalized size = 1.07

$$\frac{4\,Bcx^4\log(x) - 2\,(Bb + Ac)x^2 - Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="fricas")
[Out] 1/4*(4*B*c*x^4*log(x) - 2*(B*b + A*c)*x^2 - A*b)/x^4
```

giac [A] time = 0.15, size = 39, normalized size = 1.34

$$\frac{1}{2}\,Bc\log\left(x^2\right) - \frac{3\,Bcx^4 + 2\,Bbx^2 + 2\,Acx^2 + Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="giac")
[Out] 1/2*B*c*log(x^2) - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*b)/x^4
```

maple [A] time = 0.05, size = 28, normalized size = 0.97

$$Bc\ln(x) - \frac{Ac}{2x^2} - \frac{Bb}{2x^2} - \frac{Ab}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^7,x)
[Out] -1/4*A*b/x^4-1/2/x^2*A*c-1/2/x^2*b*B+B*c*ln(x)
```

maxima [A] time = 1.40, size = 30, normalized size = 1.03

$$\frac{1}{2}\,Bc\log\left(x^2\right) - \frac{2\,(Bb + Ac)x^2 + Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="maxima")
[Out] 1/2*B*c*log(x^2) - 1/4*(2*(B*b + A*c)*x^2 + A*b)/x^4
```

mupad [B] time = 0.07, size = 29, normalized size = 1.00

$$B\,c\,\ln(x) - \frac{\left(\frac{A\,c}{2} + \frac{B\,b}{2}\right)\,x^2 + \frac{A\,b}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x)
[Out] B*c*log(x) - ((A*b)/4 + x^2*((A*c)/2 + (B*b)/2))/x^4
```

sympy [A] time = 0.37, size = 29, normalized size = 1.00

$$Bc\log\left(x\right) + \frac{-Ab + x^2\,(-2Ac - 2Bb)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**7,x)
```

```
[Out] B*c*log(x) + (-A*b + x**2*(-2*A*c - 2*B*b))/(4*x**4)
```

3.11 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8} \, dx$

Optimal. Leaf size=31

$$-\frac{Ac+bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

```
[Out] -1/5*A*b/x^5+1/3*(-A*c-B*b)/x^3-B*c/x
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$-\frac{Ac+bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

```
[Out] -(A*b)/(5*x^5) - (b*B + A*c)/(3*x^3) - (B*c)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^6} \, dx \\
&= \int \left(\frac{Ab}{x^6}+\frac{bB+Ac}{x^4}+\frac{Bc}{x^2}\right) dx \\
&= -\frac{Ab}{5x^5}-\frac{bB+Ac}{3x^3}-\frac{Bc}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.06

$$\frac{-Ac-bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

```
[Out] -1/5*(A*b)/x^5 + (-(b*B) - A*c)/(3*x^3) - (B*c)/x
```

fricas [A] time = 0.77, size = 29, normalized size = 0.94

$$-\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="fricas")
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

giac [A] time = 0.15, size = 31, normalized size = 1.00

$$-\frac{15\,Bcx^4 + 5\,Bbx^2 + 5\,Acx^2 + 3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="giac")
[Out] -1/15*(15*B*c*x^4 + 5*B*b*x^2 + 5*A*c*x^2 + 3*A*b)/x^5
```

maple [A] time = 0.05, size = 28, normalized size = 0.90

$$-\frac{Bc}{x} - \frac{Ab}{5x^5} - \frac{Ac + bB}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^8,x)
[Out] -1/5*A*b/x^5-1/3*(A*c+B*b)/x^3-B*c/x
```

maxima [A] time = 1.35, size = 29, normalized size = 0.94

$$-\frac{15\,Bcx^4 + 5\,(Bb + Ac)x^2 + 3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="maxima")
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

mupad [B] time = 0.04, size = 29, normalized size = 0.94

$$-\frac{B\,c\,x^4 + \left(\frac{A\,c}{3} + \frac{B\,b}{3}\right)\,x^2 + \frac{A\,b}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x)
[Out] -((A*b)/5 + x^2*((A*c)/3 + (B*b)/3) + B*c*x^4)/x^5
```

sympy [A] time = 0.39, size = 32, normalized size = 1.03

$$\frac{-3Ab - 15Bcx^4 + x^2\,(-5Ac - 5Bb)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**8,x)
[Out] (-3*A*b - 15*B*c*x**4 + x**2*(-5*A*c - 5*B*b))/(15*x**5)
```

3.12 $\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

```
[Out] 1/5*A*b^2*x^5+1/7*b*(2*A*c+B*b)*x^7+1/9*c*(A*c+2*B*b)*x^9+1/11*B*c^2*x^11
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1593, 448}

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx &= \int x^4\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^4+b(bB+2Ac)x^6+c(2bB+Ac)x^8+Bc^2x^{10}\right)dx\\ &= \frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

fricas [A] time = 0.62, size = 53, normalized size = 0.96

$$\frac{1}{11}x^{11}c^2B+\frac{2}{9}x^9cbB+\frac{1}{9}x^9c^2A+\frac{1}{7}x^7b^2B+\frac{2}{7}x^7cbA+\frac{1}{5}x^5b^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2*B + 2/9*x^9*c*b*B + 1/9*x^9*c^2*A + 1/7*x^7*b^2*B + 2/7*x^7*c
*b*A + 1/5*x^5*b^2*A
```

giac [A] time = 0.15, size = 53, normalized size = 0.96

$$\frac{1}{11}\,Bc^2x^{11} + \frac{2}{9}\,Bbcx^9 + \frac{1}{9}\,Ac^2x^9 + \frac{1}{7}\,Bb^2x^7 + \frac{2}{7}\,Abcx^7 + \frac{1}{5}\,Ab^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/9*A*c^2*x^9 + 1/7*B*b^2*x^7 + 2/7*A*b*c
*x^7 + 1/5*A*b^2*x^5
```

maple [A] time = 0.04, size = 52, normalized size = 0.95

$$\frac{B\,c^2x^{11}}{11} + \frac{\left(A\,c^2 + 2bBc\right)x^9}{9} + \frac{A\,b^2x^5}{5} + \frac{\left(2Abc + B\,b^2\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/11*B*c^2*x^11+1/9*(A*c^2+2*B*b*c)*x^9+1/7*(2*A*b*c+B*b^2)*x^7+1/5*A*b^2*x
^5
```

maxima [A] time = 1.33, size = 51, normalized size = 0.93

$$\frac{1}{11}\,Bc^2x^{11} + \frac{1}{9}\left(2\,Bbc + Ac^2\right)x^9 + \frac{1}{5}\,Ab^2x^5 + \frac{1}{7}\left(Bb^2 + 2\,Abc\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
2*A*b*c)*x^7
```

mupad [B] time = 0.09, size = 51, normalized size = 0.93

$$x^7\left(\frac{B\,b^2}{7} + \frac{2\,A\,c\,b}{7}\right) + x^9\left(\frac{A\,c^2}{9} + \frac{2\,B\,b\,c}{9}\right) + \frac{A\,b^2\,x^5}{5} + \frac{B\,c^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((B*b^2)/7 + (2*A*b*c)/7) + x^9*((A*c^2)/9 + (2*B*b*c)/9) + (A*b^2*x^5)
/5 + (B*c^2*x^11)/11
```

sympy [A] time = 0.08, size = 56, normalized size = 1.02

$$\frac{Ab^2x^5}{5} + \frac{Bc^2x^{11}}{11} + x^9\left(\frac{Ac^2}{9} + \frac{2Bbc}{9}\right) + x^7\left(\frac{2Abc}{7} + \frac{Bb^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] A*b**2*x**5/5 + B*c**2*x**11/11 + x**9*(A*c**2/9 + 2*B*b*c/9) + x**7*(2*A*b
*c/7 + B*b**2/7)
```

3.13 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x} \, dx$

Optimal. Leaf size=55

$$\frac{1}{4}Ab^2x^4+\frac{1}{8}cx^8(Ac+2bB)+\frac{1}{6}bx^6(2Ac+bB)+\frac{1}{10}Bc^2x^{10}$$

```
[Out] 1/4*A*b^2*x^4+1/6*b*(2*A*c+B*b)*x^6+1/8*c*(A*c+2*B*b)*x^8+1/10*B*c^2*x^10
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$\frac{1}{4}Ab^2x^4+\frac{1}{8}cx^8(Ac+2bB)+\frac{1}{6}bx^6(2Ac+bB)+\frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x} \, dx &= \int x^3\left(A+Bx^2\right)\left(b+cx^2\right)^2 \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)(b+cx)^2 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(Ab^2x+b(bB+2Ac)x^2+c(2bB+Ac)x^3+Bc^2x^4\right) dx, x, x^2\right)\\
&= \frac{1}{4}Ab^2x^4+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{8}c(2bB+Ac)x^8+\frac{1}{10}Bc^2x^{10}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac + 2bB) + \frac{1}{6}bx^6(2Ac + bB) + \frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

fricas [A] time = 0.58, size = 51, normalized size = 0.93

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{8}\left(2Bbc + Ac^2\right)x^8 + \frac{1}{4}Ab^2x^4 + \frac{1}{6}\left(Bb^2 + 2Abc\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
2*A*b*c)*x^6
```

giac [A] time = 0.15, size = 53, normalized size = 0.96

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{4}Bbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Abcx^6 + \frac{1}{4}Ab^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/8*A*c^2*x^8 + 1/6*B*b^2*x^6 + 1/3*A*b*c
*x^6 + 1/4*A*b^2*x^4
```

maple [A] time = 0.04, size = 52, normalized size = 0.95

$$\frac{Bc^2x^{10}}{10} + \frac{\left(Ac^2 + 2bBc\right)x^8}{8} + \frac{Ab^2x^4}{4} + \frac{\left(2Abc + Bb^2\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x,x)
```

```
[Out] 1/10*B*c^2*x^10+1/8*(A*c^2+2*B*b*c)*x^8+1/6*(2*A*b*c+B*b^2)*x^6+1/4*A*b^2*x
^4
```

maxima [A] time = 1.37, size = 51, normalized size = 0.93

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{8}\left(2Bbc + Ac^2\right)x^8 + \frac{1}{4}Ab^2x^4 + \frac{1}{6}\left(Bb^2 + 2Abc\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
2*A*b*c)*x^6
```

mupad [B] time = 0.05, size = 51, normalized size = 0.93

$$x^6\left(\frac{Bb^2}{6} + \frac{Acb}{3}\right) + x^8\left(\frac{Ac^2}{8} + \frac{Bbc}{4}\right) + \frac{Ab^2x^4}{4} + \frac{Bc^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x)
```

```
[Out] x^6*((B*b^2)/6 + (A*b*c)/3) + x^8*((A*c^2)/8 + (B*b*c)/4) + (A*b^2*x^4)/4 +
 (B*c^2*x^10)/10
```

sympy [A] time = 0.08, size = 53, normalized size = 0.96

$$\frac{Ab^2x^4}{4} + \frac{Bc^2x^{10}}{10} + x^8\left(\frac{Ac^2}{8} + \frac{Bbc}{4}\right) + x^6\left(\frac{Abc}{3} + \frac{Bb^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x,x)
```

```
[Out] A*b**2*x**4/4 + B*c**2*x**10/10 + x**8*(A*c**2/8 + B*b*c/4) + x**6*(A*b*c/3
 + B*b**2/6)
```

3.14 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^2}\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}Ab^2x^3+\frac{1}{7}cx^7(Ac+2bB)+\frac{1}{5}bx^5(2Ac+bB)+\frac{1}{9}Bc^2x^9$$

```
[Out] 1/3*A*b^2*x^3+1/5*b*(2*A*c+B*b)*x^5+1/7*c*(A*c+2*B*b)*x^7+1/9*B*c^2*x^9
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{1}{3}Ab^2x^3+\frac{1}{7}cx^7(Ac+2bB)+\frac{1}{5}bx^5(2Ac+bB)+\frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^2}\,dx &= \int x^2\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^2+b(bB+2Ac)x^4+c(2bB+Ac)x^6+Bc^2x^8\right)dx\\ &= \frac{1}{3}Ab^2x^3+\frac{1}{5}b(bB+2Ac)x^5+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{9}Bc^2x^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{3}Ab^2x^3+\frac{1}{7}cx^7(Ac+2bB)+\frac{1}{5}bx^5(2Ac+bB)+\frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

fricas [A] time = 0.97, size = 51, normalized size = 0.93

$$\frac{1}{9}\,Bc^2x^9 + \frac{1}{7}\left(2\,Bbc + Ac^2\right)x^7 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{5}\left(Bb^2 + 2\,Abc\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

giac [A] time = 0.17, size = 53, normalized size = 0.96

$$\frac{1}{9}\,Bc^2x^9 + \frac{2}{7}\,Bbcx^7 + \frac{1}{7}\,Ac^2x^7 + \frac{1}{5}\,Bb^2x^5 + \frac{2}{5}\,Abcx^5 + \frac{1}{3}\,Ab^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/7*A*c^2*x^7 + 1/5*B*b^2*x^5 + 2/5*A*b*c*x
^5 + 1/3*A*b^2*x^3
```

maple [A] time = 0.04, size = 52, normalized size = 0.95

$$\frac{B\,c^2x^9}{9} + \frac{\left(A\,c^2 + 2bBc\right)x^7}{7} + \frac{A\,b^2x^3}{3} + \frac{\left(2Abc + B\,b^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x)
```

```
[Out] 1/9*B*c^2*x^9+1/7*(A*c^2+2*B*b*c)*x^7+1/5*(2*A*b*c+B*b^2)*x^5+1/3*A*b^2*x^3
```

maxima [A] time = 1.32, size = 51, normalized size = 0.93

$$\frac{1}{9}\,Bc^2x^9 + \frac{1}{7}\left(2\,Bbc + Ac^2\right)x^7 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{5}\left(Bb^2 + 2\,Abc\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

mupad [B] time = 0.05, size = 51, normalized size = 0.93

$$x^5\left(\frac{B\,b^2}{5} + \frac{2\,A\,c\,b}{5}\right) + x^7\left(\frac{A\,c^2}{7} + \frac{2\,B\,b\,c}{7}\right) + \frac{A\,b^2\,x^3}{3} + \frac{B\,c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x)
```

```
[Out] x^5*((B*b^2)/5 + (2*A*b*c)/5) + x^7*((A*c^2)/7 + (2*B*b*c)/7) + (A*b^2*x^3)
/3 + (B*c^2*x^9)/9
```

sympy [A] time = 0.08, size = 56, normalized size = 1.02

$$\frac{Ab^2x^3}{3} + \frac{Bc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Bbc}{7}\right) + x^5\left(\frac{2Abc}{5} + \frac{Bb^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] A*b**2*x**3/3 + B*c**2*x**9/9 + x**7*(A*c**2/7 + 2*B*b*c/7) + x**5*(2*A*b*c
/5 + B*b**2/5)
```

3.15 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} \, dx$

Optimal. Leaf size=42

$$\frac{B\left(b+cx^2\right)^4}{8c^2}-\frac{\left(b+cx^2\right)^3(bB-Ac)}{6c^2}$$

```
[Out] -1/6*(-A*c+B*b)*(c*x^2+b)^3/c^2+1/8*B*(c*x^2+b)^4/c^2
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{B\left(b+cx^2\right)^4}{8c^2}-\frac{\left(b+cx^2\right)^3(bB-Ac)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -((b*B - A*c)*(b + c*x^2)^3)/(6*c^2) + (B*(b + c*x^2)^4)/(8*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^3}\,dx &= \int x\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^2}{c}+\frac{B(b+cx)^3}{c}\right)dx,x,x^2\right)\\
&= -\frac{(bB-Ac)\left(b+cx^2\right)^3}{6c^2}+\frac{B\left(b+cx^2\right)^4}{8c^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12Ab^2+4cx^4(Ac+2bB)+6bx^2(2Ac+bB)+3Bc^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] (x^2*(12*A*b^2 + 6*b*(b*B + 2*A*c)*x^2 + 4*c*(2*b*B + A*c)*x^4 + 3*B*c^2*x^
6))/24
```

fricas [A] time = 0.86, size = 51, normalized size = 1.21

$$\frac{1}{8}\,Bc^2x^8+\frac{1}{6}\left(2\,Bbc+Ac^2\right)x^6+\frac{1}{2}\,Ab^2x^2+\frac{1}{4}\left(Bb^2+2\,Abc\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

giac [A] time = 0.16, size = 53, normalized size = 1.26

$$\frac{1}{8}\,Bc^2x^8+\frac{1}{3}\,Bbcx^6+\frac{1}{6}\,Ac^2x^6+\frac{1}{4}\,Bb^2x^4+\frac{1}{2}\,Abcx^4+\frac{1}{2}\,Ab^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/6*A*c^2*x^6 + 1/4*B*b^2*x^4 + 1/2*A*b*c*x
^4 + 1/2*A*b^2*x^2
```

maple [A] time = 0.04, size = 52, normalized size = 1.24

$$\frac{B\,c^2x^8}{8}+\frac{\left(A\,c^2+2bBc\right)x^6}{6}+\frac{A\,b^2x^2}{2}+\frac{\left(2Abc+B\,b^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x)
```

```
[Out] 1/8*B*c^2*x^8+1/6*(A*c^2+2*B*b*c)*x^6+1/4*(2*A*b*c+B*b^2)*x^4+1/2*b^2*A*x^2
```

maxima [A] time = 1.36, size = 51, normalized size = 1.21

$$\frac{1}{8}\,Bc^2x^8+\frac{1}{6}\left(2\,Bbc+Ac^2\right)x^6+\frac{1}{2}\,Ab^2x^2+\frac{1}{4}\left(Bb^2+2\,Abc\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

mupad [B] time = 0.05, size = 51, normalized size = 1.21

$$x^4\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+x^6\left(\frac{A\,c^2}{6}+\frac{B\,b\,c}{3}\right)+\frac{A\,b^2\,x^2}{2}+\frac{B\,c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x)
```

```
[Out] x^4*((B*b^2)/4 + (A*b*c)/2) + x^6*((A*c^2)/6 + (B*b*c)/3) + (A*b^2*x^2)/2 +
 (B*c^2*x^8)/8
```

sympy [A] time = 0.08, size = 53, normalized size = 1.26

$$\frac{Ab^2x^2}{2} + \frac{Bc^2x^8}{8} + x^6\left(\frac{Ac^2}{6} + \frac{Bbc}{3}\right) + x^4\left(\frac{Abc}{2} + \frac{Bb^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] A*b**2*x**2/2 + B*c**2*x**8/8 + x**6*(A*c**2/6 + B*b*c/3) + x**4*(A*b*c/2 +
 B*b**2/4)
```

3.16 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx$

Optimal. Leaf size=50

$$Ab^2x + \frac{1}{5}cx^5(Ac + 2bB) + \frac{1}{3}bx^3(2Ac + bB) + \frac{1}{7}Bc^2x^7$$

```
[Out] A*b^2*x+1/3*b*(2*A*c+B*b)*x^3+1/5*c*(A*c+2*B*b)*x^5+1/7*B*c^2*x^7
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1584, 373}

$$Ab^2x + \frac{1}{5}cx^5(Ac + 2bB) + \frac{1}{3}bx^3(2Ac + bB) + \frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^2}{x^4}\,dx &= \int (A + Bx^2)(b + cx^2)^2\,dx \\
&= \int \left(Ab^2 + b(bB + 2Ac)x^2 + c(2bB + Ac)x^4 + Bc^2x^6\right)dx \\
&= Ab^2x + \frac{1}{3}b(bB + 2Ac)x^3 + \frac{1}{5}c(2bB + Ac)x^5 + \frac{1}{7}Bc^2x^7
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$Ab^2x + \frac{1}{5}cx^5(Ac + 2bB) + \frac{1}{3}bx^3(2Ac + bB) + \frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

fricas [A] time = 0.92, size = 48, normalized size = 0.96

$$\frac{1}{7}Bc^2x^7 + \frac{1}{5}\left(2\,Bbc + Ac^2\right)x^5 + Ab^2x + \frac{1}{3}\left(Bb^2 + 2\,Abc\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

giac [A] time = 0.17, size = 50, normalized size = 1.00

$$\frac{1}{7}Bc^2x^7 + \frac{2}{5}Bbcx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Abcx^3 + Ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/5*A*c^2*x^5 + 1/3*B*b^2*x^3 + 2/3*A*b*c*x
^3 + A*b^2*x
```

maple [A] time = 0.05, size = 49, normalized size = 0.98

$$\frac{B\,c^2x^7}{7} + \frac{\left(A\,c^2 + 2bBc\right)x^5}{5} + A\,b^2x + \frac{\left(2Abc + B\,b^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x)
```

```
[Out] 1/7*B*c^2*x^7+1/5*(A*c^2+2*B*b*c)*x^5+1/3*(2*A*b*c+B*b^2)*x^3+A*b^2*x
```

maxima [A] time = 1.24, size = 48, normalized size = 0.96

$$\frac{1}{7}Bc^2x^7 + \frac{1}{5}\left(2\,Bbc + Ac^2\right)x^5 + Ab^2x + \frac{1}{3}\left(Bb^2 + 2\,Abc\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^3\left(\frac{B\,b^2}{3} + \frac{2\,A\,c\,b}{3}\right) + x^5\left(\frac{A\,c^2}{5} + \frac{2\,B\,b\,c}{5}\right) + \frac{B\,c^2\,x^7}{7} + A\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x)
```

```
[Out] x^3*((B*b^2)/3 + (2*A*b*c)/3) + x^5*((A*c^2)/5 + (2*B*b*c)/5) + (B*c^2*x^7)
/7 + A*b^2*x
```

sympy [A] time = 0.08, size = 53, normalized size = 1.06

$$Ab^2x + \frac{Bc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Bbc}{5}\right) + x^3\left(\frac{2Abc}{3} + \frac{Bb^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] A*b**2*x + B*c**2*x**7/7 + x**5*(A*c**2/5 + 2*B*b*c/5) + x**3*(2*A*b*c/3 +
B*b**2/3)
```

3.17 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5} dx$

Optimal. Leaf size=43

$$Ab^2\log(x) + Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c}$$

```
[Out] A*b*c*x^2+1/4*A*c^2*x^4+1/6*B*(c*x^2+b)^3/c+A*b^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 80, 43}

$$Ab^2\log(x) + Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] A*b*c*x^2 + (A*c^2*x^4)/4 + (B*(b + c*x^2)^3)/(6*c) + A*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^5}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x}\,dx, x, x^2\right) \\
&= \frac{B\left(b+cx^2\right)^3}{6c} + \frac{1}{2}A\operatorname{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx, x, x^2\right) \\
&= \frac{B\left(b+cx^2\right)^3}{6c} + \frac{1}{2}A\operatorname{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)dx, x, x^2\right) \\
&= Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B\left(b+cx^2\right)^3}{6c} + Ab^2\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.19

$$Ab^2\log(x) + \frac{1}{4}cx^4(Ac+2bB) + \frac{1}{2}bx^2(2Ac+bB) + \frac{1}{6}Bc^2x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] (b*(b*B + 2*A*c)*x^2)/2 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2*x^6)/6 + A*b^2*L
og[x]
```

fricas [A] time = 0.83, size = 49, normalized size = 1.14

$$\frac{1}{6}Bc^2x^6 + \frac{1}{4}\left(2\,Bbc + Ac^2\right)x^4 + Ab^2\log(x) + \frac{1}{2}\left(Bb^2 + 2\,Abc\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + A*b^2*log(x) + 1/2*(B*b^2 + 2*A
*b*c)*x^2
```

giac [A] time = 0.15, size = 53, normalized size = 1.23

$$\frac{1}{6}Bc^2x^6 + \frac{1}{2}Bbcx^4 + \frac{1}{4}Ac^2x^4 + \frac{1}{2}Bb^2x^2 + Abcx^2 + \frac{1}{2}Ab^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/4*A*c^2*x^4 + 1/2*B*b^2*x^2 + A*b*c*x^2 +
 1/2*A*b^2*log(x^2)
```

maple [A] time = 0.05, size = 51, normalized size = 1.19

$$\frac{B\,c^2x^6}{6} + \frac{A\,c^2x^4}{4} + \frac{Bbc\,x^4}{2} + Abc\,x^2 + \frac{B\,b^2x^2}{2} + A\,b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x)
```

```
[Out] 1/6*B*c^2*x^6+1/4*A*c^2*x^4+1/2*B*x^4*b*c+A*b*c*x^2+1/2*B*b^2*x^2+A*b^2*ln(
x)
```

maxima [**A**] time = 1.34, size = 52, normalized size = 1.21

$$\frac{1}{6}\,Bc^2x^6 + \frac{1}{4}\left(2\,Bbc + Ac^2\right)x^4 + \frac{1}{2}\,Ab^2\log\left(x^2\right) + \frac{1}{2}\left(Bb^2 + 2\,Abc\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/2*A*b^2*log(x^2) + 1/2*(B*b^2
 + 2*A*b*c)*x^2
```

mupad [**B**] time = 0.04, size = 48, normalized size = 1.12

$$x^2\left(\frac{B\,b^2}{2} + A\,c\,b\right) + x^4\left(\frac{A\,c^2}{4} + \frac{B\,b\,c}{2}\right) + \frac{B\,c^2\,x^6}{6} + A\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x)
```

```
[Out] x^2*((B*b^2)/2 + A*b*c) + x^4*((A*c^2)/4 + (B*b*c)/2) + (B*c^2*x^6)/6 + A*b
^2*log(x)
```

sympy [**A**] time = 0.16, size = 49, normalized size = 1.14

$$Ab^2\log(x) + \frac{Bc^2x^6}{6} + x^4\left(\frac{Ac^2}{4} + \frac{Bbc}{2}\right) + x^2\left(Abc + \frac{Bb^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] A*b**2*log(x) + B*c**2*x**6/6 + x**4*(A*c**2/4 + B*b*c/2) + x**2*(A*b*c + B
*b**2/2)
```

3.18 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

```
[Out] -A*b^2/x+b*(2*A*c+B*b)*x+1/3*c*(A*c+2*B*b)*x^3+1/5*B*c^2*x^5
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^6}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x^2}\,dx\\
&= \int \left(b(bB+2Ac)+\frac{Ab^2}{x^2}+c(2bB+Ac)x^2+Bc^2x^4\right)dx\\
&= -\frac{Ab^2}{x}+b(bB+2Ac)x+\frac{1}{3}c(2bB+Ac)x^3+\frac{1}{5}Bc^2x^5
\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

fricas [A] time = 0.94, size = 53, normalized size = 1.10

$$\frac{3\,Bc^2x^6+5\left(2\,Bbc+Ac^2\right)x^4-15\,Ab^2+15\left(Bb^2+2\,Abc\right)x^2}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^6 + 5*(2*B*b*c + A*c^2)*x^4 - 15*A*b^2 + 15*(B*b^2 + 2*A*b*
c)*x^2)/x
```

giac [A] time = 0.15, size = 48, normalized size = 1.00

$$\frac{1}{5}\,Bc^2x^5+\frac{2}{3}\,Bbcx^3+\frac{1}{3}\,Ac^2x^3+Bb^2x+2\,Abcx-\frac{Ab^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/3*A*c^2*x^3 + B*b^2*x + 2*A*b*c*x - A*b^2
/x
```

maple [A] time = 0.05, size = 49, normalized size = 1.02

$$\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^3}{3}+\frac{2Bbc\,x^3}{3}+2Abcx+B\,b^2x-\frac{A\,b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x)
```

```
[Out] 1/5*B*c^2*x^5+1/3*A*x^3*c^2+2/3*B*x^3*b*c+2*A*b*c*x+B*b^2*x-A*b^2/x
```

maxima [A] time = 1.29, size = 48, normalized size = 1.00

$$\frac{1}{5}\,Bc^2x^5+\frac{1}{3}\left(2\,Bbc+Ac^2\right)x^3-\frac{Ab^2}{x}+\left(Bb^2+2\,Abc\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*B*c^2*x^5 + 1/3*(2*B*b*c + A*c^2)*x^3 - A*b^2/x + (B*b^2 + 2*A*b*c)*x
```

mupad [B] time = 0.05, size = 48, normalized size = 1.00

$$x^3\,\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+x\,\left(B\,b^2+2\,A\,c\,b\right)-\frac{A\,b^2}{x}+\frac{B\,c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x)
```

```
[Out] x^3*((A*c^2)/3 + (2*B*b*c)/3) + x*(B*b^2 + 2*A*b*c) - (A*b^2)/x + (B*c^2*x^
5)/5
```

sympy [A] time = 0.14, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{x}+\frac{Bc^2x^5}{5}+x^3\left(\frac{Ac^2}{3}+\frac{2Bbc}{3}\right)+x\left(2Abc+Bb^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -A*b**2/x + B*c**2*x**5/5 + x**3*(A*c**2/3 + 2*B*b*c/3) + x*(2*A*b*c + B*b*
*2)
```

3.19 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{2x^2}+\frac{1}{2}cx^2(Ac+2bB)+b\log(x)(2Ac+bB)+\frac{1}{4}Bc^2x^4$$

```
[Out] -1/2*A*b^2/x^2+1/2*c*(A*c+2*B*b)*x^2+1/4*B*c^2*x^4+b*(2*A*c+B*b)*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{2x^2}+\frac{1}{2}cx^2(Ac+2bB)+b\log(x)(2Ac+bB)+\frac{1}{4}Bc^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -(A*b^2)/(2*x^2) + (c*(2*b*B + A*c)*x^2)/2 + (B*c^2*x^4)/4 + b*(b*B + 2*A*c
)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(c(2bB+Ac)+\frac{Ab^2}{x^2}+\frac{b(bB+2Ac)}{x}+Bc^2x\right)dx,x,x^2\right) \\
&= -\frac{Ab^2}{2x^2}+\frac{1}{2}c(2bB+Ac)x^2+\frac{1}{4}Bc^2x^4+b(bB+2Ac)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2Ab^2}{x^2}+2cx^2(Ac+2bB)+4b\log(x)(2Ac+bB)+Bc^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] ((-2*A*b^2)/x^2 + 2*c*(2*b*B + A*c)*x^2 + B*c^2*x^4 + 4*b*(b*B + 2*A*c)*Log
[x])/4
```

fricas [A] time = 0.82, size = 54, normalized size = 1.06

$$\frac{Bc^2x^6+2\left(2\,Bbc+Ac^2\right)x^4+4\left(Bb^2+2\,Abc\right)x^2\log(x)-2\,Ab^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^2*x^6 + 2*(2*B*b*c + A*c^2)*x^4 + 4*(B*b^2 + 2*A*b*c)*x^2*log(x) -
 2*A*b^2)/x^2
```

giac [A] time = 0.19, size = 70, normalized size = 1.37

$$\frac{1}{4}\,Bc^2x^4+Bbcx^2+\frac{1}{2}\,Ac^2x^2+\frac{1}{2}\left(Bb^2+2\,Abc\right)\log\left(x^2\right)-\frac{Bb^2x^2+2\,Abcx^2+Ab^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/2*A*c^2*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*(B*b^2*x^2 + 2*A*b*c*x^2 + A*b^2)/x^2
```

maple [A] time = 0.05, size = 50, normalized size = 0.98

$$\frac{B\,c^2x^4}{4}+\frac{A\,c^2x^2}{2}+Bbc\,x^2+2Abc\ln(x)+B\,b^2\ln(x)-\frac{A\,b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x)
```

```
[Out] 1/4*B*c^2*x^4+1/2*A*x^2*c^2+B*x^2*b*c-1/2*A*b^2/x^2+2*A*ln(x)*b*c+B*ln(x)*b
^2
```

maxima [A] time = 1.35, size = 52, normalized size = 1.02

$$\frac{1}{4}\,Bc^2x^4+\frac{1}{2}\left(2\,Bbc+Ac^2\right)x^2+\frac{1}{2}\left(Bb^2+2\,Abc\right)\log\left(x^2\right)-\frac{Ab^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^2*x^4 + 1/2*(2*B*b*c + A*c^2)*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*A*b^2/x^2
```

mupad [B] time = 0.05, size = 48, normalized size = 0.94

$$x^2\left(\frac{A\,c^2}{2}+B\,b\,c\right)+\ln(x)\,\left(B\,b^2+2\,A\,c\,b\right)-\frac{A\,b^2}{2\,x^2}+\frac{B\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x)
```

```
[Out] x^2*((A*c^2)/2 + B*b*c) + log(x)*(B*b^2 + 2*A*b*c) - (A*b^2)/(2*x^2) + (B*c
^2*x^4)/4
```

sympy [A] time = 0.24, size = 48, normalized size = 0.94

$$-\frac{Ab^2}{2x^2} + \frac{Bc^2x^4}{4} + b\left(2Ac + Bb\right)\log\left(x\right) + x^2\left(\frac{Ac^2}{2} + Bbc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**7,x)
```

```
[Out] -A*b**2/(2*x**2) + B*c**2*x**4/4 + b*(2*A*c + B*b)*log(x) + x**2*(A*c**2/2
+ B*b*c)
```

3.20 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{3x^3} + cx(Ac+2bB) - \frac{b(2Ac+bB)}{x} + \frac{1}{3}Bc^2x^3$$

```
[Out] -1/3*A*b^2/x^3-b*(2*A*c+B*b)/x+c*(A*c+2*B*b)*x+1/3*B*c^2*x^3
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{Ab^2}{3x^3} + cx(Ac+2bB) - \frac{b(2Ac+bB)}{x} + \frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

```
[Out] -(A*b^2)/(3*x^3) - (b*(b*B + 2*A*c))/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^4} dx \\
&= \int \left(c(2bB+Ac) + \frac{Ab^2}{x^4} + \frac{b(bB+2Ac)}{x^2} + Bc^2x^2\right) dx \\
&= -\frac{Ab^2}{3x^3} - \frac{b(bB+2Ac)}{x} + c(2bB+Ac)x + \frac{1}{3}Bc^2x^3
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.04

$$\frac{b^2(-B)-2Abc}{x} - \frac{Ab^2}{3x^3} + cx(Ac+2bB) + \frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

```
[Out] -1/3*(A*b^2)/x^3 + (-(b^2*B) - 2*A*b*c)/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)
/3
```

fricas [A] time = 0.88, size = 52, normalized size = 1.08

$$\frac{Bc^2x^6 + 3\left(2\,Bbc + Ac^2\right)x^4 - Ab^2 - 3\left(Bb^2 + 2\,Abc\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/3*(B*c^2*x^6 + 3*(2*B*b*c + A*c^2)*x^4 - A*b^2 - 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

giac [A] time = 0.14, size = 50, normalized size = 1.04

$$\frac{1}{3}\,Bc^2x^3 + 2\,Bbcx + Ac^2x - \frac{3\,Bb^2x^2 + 6\,Abcx^2 + Ab^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] 1/3*B*c^2*x^3 + 2*B*b*c*x + A*c^2*x - 1/3*(3*B*b^2*x^2 + 6*A*b*c*x^2 + A*b^
2)/x^3
```

maple [A] time = 0.05, size = 46, normalized size = 0.96

$$\frac{B\,c^2x^3}{3} + A\,c^2x + 2Bbcx - \frac{A\,b^2}{3x^3} - \frac{(2Ac + bB)\,b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x)
```

```
[Out] 1/3*B*c^2*x^3+A*c^2*x+2*b*B*c*x-1/3*A*b^2/x^3-b*(2*A*c+B*b)/x
```

maxima [A] time = 1.32, size = 50, normalized size = 1.04

$$\frac{1}{3}\,Bc^2x^3 + \left(2\,Bbc + Ac^2\right)x - \frac{Ab^2 + 3\left(Bb^2 + 2\,Abc\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^2*x^3 + (2*B*b*c + A*c^2)*x - 1/3*(A*b^2 + 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

mupad [B] time = 0.05, size = 50, normalized size = 1.04

$$x\,\left(A\,c^2 + 2\,B\,b\,c\right) - \frac{x^2\,\left(B\,b^2 + 2\,A\,c\,b\right) + \frac{A\,b^2}{3}}{x^3} + \frac{B\,c^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x)
```

```
[Out] x*(A*c^2 + 2*B*b*c) - (x^2*(B*b^2 + 2*A*b*c) + (A*b^2)/3)/x^3 + (B*c^2*x^3)
/3
```

sympy [A] time = 0.26, size = 51, normalized size = 1.06

$$\frac{Bc^2x^3}{3} + x\left(Ac^2 + 2Bbc\right) + \frac{-Ab^2 + x^2\left(-6Abc - 3Bb^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] B*c**2*x**3/3 + x*(A*c**2 + 2*B*b*c) + (-A*b**2 + x**2*(-6*A*b*c - 3*B*b**2
))/(3*x**3)
```

3.21 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{4x^4} - \frac{b(2Ac + bB)}{2x^2} + c\log(x)(Ac + 2bB) + \frac{1}{2}Bc^2x^2$$

```
[Out] -1/4*A*b^2/x^4-1/2*b*(2*A*c+B*b)/x^2+1/2*B*c^2*x^2+c*(A*c+2*B*b)*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{4x^4} - \frac{b(2Ac + bB)}{2x^2} + c\log(x)(Ac + 2bB) + \frac{1}{2}Bc^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

```
[Out] -(A*b^2)/(4*x^4) - (b*(b*B + 2*A*c))/(2*x^2) + (B*c^2*x^2)/2 + c*(2*b*B + A
*c)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^5} dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^3} dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(Bc^2 + \frac{Ab^2}{x^3} + \frac{b(bB+2Ac)}{x^2} + \frac{c(2bB+Ac)}{x}\right) dx, x, x^2\right) \\
&= -\frac{Ab^2}{4x^4} - \frac{b(bB+2Ac)}{2x^2} + \frac{1}{2}Bc^2x^2 + c(2bB+Ac)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.98

$$c\log(x)(Ac+2bB)-\frac{Ab\left(b+4cx^2\right)+2Bx^2\left(b^2-c^2x^4\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

```
[Out] -1/4*(A*b*(b + 4*c*x^2) + 2*B*x^2*(b^2 - c^2*x^4))/x^4 + c*(2*b*B + A*c)*Lo
g[x]
```

fricas [A] time = 0.85, size = 55, normalized size = 1.08

$$\frac{2\,Bc^2x^6+4\left(2\,Bbc+Ac^2\right)x^4\log(x)-Ab^2-2\left(Bb^2+2\,Abc\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*c^2*x^6 + 4*(2*B*b*c + A*c^2)*x^4*log(x) - A*b^2 - 2*(B*b^2 + 2*A*
b*c)*x^2)/x^4
```

giac [A] time = 0.19, size = 72, normalized size = 1.41

$$\frac{1}{2}\,Bc^2x^2+\frac{1}{2}\left(2\,Bbc+Ac^2\right)\log\left(x^2\right)-\frac{6\,Bbcx^4+3\,Ac^2x^4+2\,Bb^2x^2+4\,Abcx^2+Ab^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(6*B*b*c*x^4 + 3*A*c^2
*x^4 + 2*B*b^2*x^2 + 4*A*b*c*x^2 + A*b^2)/x^4
```

maple [A] time = 0.06, size = 51, normalized size = 1.00

$$\frac{B\,c^2x^2}{2}+A\,c^2\ln(x)+2Bbc\ln(x)-\frac{Abc}{x^2}-\frac{B\,b^2}{2x^2}-\frac{A\,b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x)
```

```
[Out] 1/2*B*c^2*x^2-1/4*A*b^2/x^4-b/x^2*A*c-1/2*b^2/x^2*B+A*ln(x)*c^2+2*B*ln(x)*b
*c
```

maxima [A] time = 1.37, size = 54, normalized size = 1.06

$$\frac{1}{2}\,Bc^2x^2+\frac{1}{2}\left(2\,Bbc+Ac^2\right)\log\left(x^2\right)-\frac{Ab^2+2\left(Bb^2+2\,Abc\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(A*b^2 + 2*(B*b^2 + 2*
A*b*c)*x^2)/x^4
```

mupad [B] time = 0.08, size = 51, normalized size = 1.00

$$\ln(x)\,\left(A\,c^2+2\,B\,b\,c\right)-\frac{x^2\,\left(\frac{B\,b^2}{2}+A\,c\,b\right)+\frac{A\,b^2}{4}}{x^4}+\frac{B\,c^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x)
```

```
[Out] log(x)*(A*c^2 + 2*B*b*c) - (x^2*((B*b^2)/2 + A*b*c) + (A*b^2)/4)/x^4 + (B*c
^2*x^2)/2
```

sympy [A] time = 0.55, size = 51, normalized size = 1.00

$$\frac{Bc^2x^2}{2} + c\left(Ac + 2Bb\right)\log\left(x\right) + \frac{-Ab^2 + x^2\left(-4Abc - 2Bb^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] B*c**2*x**2/2 + c*(A*c + 2*B*b)*log(x) + (-A*b**2 + x**2*(-4*A*b*c - 2*B*b*
*2))/(4*x**4)
```

3.22 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} \, dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{3x^3}-\frac{c(Ac+2bB)}{x}+Bc^2x$$

```
[Out] -1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{3x^3}-\frac{c(Ac+2bB)}{x}+Bc^2x$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

```
[Out] -(A*b^2)/(5*x^5) - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^6} \, dx \\
&= \int \left(Bc^2+\frac{Ab^2}{x^6}+\frac{b(bB+2Ac)}{x^4}+\frac{c(2bB+Ac)}{x^2}\right) dx \\
&= -\frac{Ab^2}{5x^5}-\frac{b(bB+2Ac)}{3x^3}-\frac{c(2bB+Ac)}{x}+Bc^2x
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{3x^3}-\frac{c(Ac+2bB)}{x}+Bc^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

```
[Out] -1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

fricas [**A**] time = 1.01, size = 53, normalized size = 1.10

$$\frac{15\,Bc^2x^6-15\,\left(2\,Bbc+Ac^2\right)x^4-3\,Ab^2-5\,\left(Bb^2+2\,Abc\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*c^2*x^6 - 15*(2*B*b*c + A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 + 2*A*b*
c)*x^2)/x^5
```

giac [**A**] time = 0.15, size = 53, normalized size = 1.10

$$Bc^2x-\frac{30\,Bbcx^4+15\,Ac^2x^4+5\,Bb^2x^2+10\,Abcx^2+3\,Ab^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

```
[Out] B*c^2*x - 1/15*(30*B*b*c*x^4 + 15*A*c^2*x^4 + 5*B*b^2*x^2 + 10*A*b*c*x^2 +
3*A*b^2)/x^5
```

maple [**A**] time = 0.06, size = 45, normalized size = 0.94

$$B\,c^2x-\frac{(Ac+2bB)\,c}{x}-\frac{A\,b^2}{5x^5}-\frac{(2Ac+bB)\,b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x)
```

```
[Out] -1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

maxima [**A**] time = 1.39, size = 51, normalized size = 1.06

$$Bc^2x-\frac{15\,\left(2\,Bbc+Ac^2\right)x^4+3\,Ab^2+5\,\left(Bb^2+2\,Abc\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] B*c^2*x - 1/15*(15*(2*B*b*c + A*c^2)*x^4 + 3*A*b^2 + 5*(B*b^2 + 2*A*b*c)*x^
2)/x^5
```

mupad [**B**] time = 0.07, size = 50, normalized size = 1.04

$$B\,c^2\,x-\frac{x^2\,\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}\right)+x^4\,\left(A\,c^2+2\,B\,b\,c\right)+\frac{A\,b^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x)
```

```
[Out] B*c^2*x - (x^2*((B*b^2)/3 + (2*A*b*c)/3) + x^4*(A*c^2 + 2*B*b*c) + (A*b^2)/
5)/x^5
```

sympy [**A**] time = 0.61, size = 54, normalized size = 1.12

$$Bc^2x+\frac{-3Ab^2+x^4\left(-15Ac^2-30Bbc\right)+x^2\left(-10Abc-5Bb^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] B*c**2*x + (-3*A*b**2 + x**4*(-15*A*c**2 - 30*B*b*c) + x**2*(-10*A*b*c - 5*
B*b**2))/(15*x**5)
```

3.23 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{6x^6}-\frac{b(2Ac+bB)}{4x^4}-\frac{c(Ac+2bB)}{2x^2}+Bc^2\log(x)$$

```
[Out] -1/6*A*b^2/x^6-1/4*b*(2*A*c+B*b)/x^4-1/2*c*(A*c+2*B*b)/x^2+B*c^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{Ab^2}{6x^6}-\frac{b(2Ac+bB)}{4x^4}-\frac{c(Ac+2bB)}{2x^2}+Bc^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -(A*b^2)/(6*x^6) - (b*(b*B + 2*A*c))/(4*x^4) - (c*(2*b*B + A*c))/(2*x^2) +
B*c^2*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x^7}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab^2}{x^4}+\frac{b(bB+2Ac)}{x^3}+\frac{c(2bB+Ac)}{x^2}+\frac{Bc^2}{x}\right)dx,x,x^2\right)\\ &= -\frac{Ab^2}{6x^6}-\frac{b(bB+2Ac)}{4x^4}-\frac{c(2bB+Ac)}{2x^2}+Bc^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.04

$$Bc^2\log(x) - \frac{2A\left(b^2+3bcx^2+3c^2x^4\right)+3bBx^2\left(b+4cx^2\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -1/12*(3*b*B*x^2*(b + 4*c*x^2) + 2*A*(b^2 + 3*b*c*x^2 + 3*c^2*x^4))/x^6 + B
*c^2*Log[x]
```

fricas [A] time = 0.86, size = 55, normalized size = 1.08

$$\frac{12\,Bc^2x^6\log(x) - 6\left(2\,Bbc + Ac^2\right)x^4 - 2\,Ab^2 - 3\left(Bb^2 + 2\,Abc\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*c^2*x^6*log(x) - 6*(2*B*b*c + A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 +
2*A*b*c)*x^2)/x^6
```

giac [A] time = 0.19, size = 66, normalized size = 1.29

$$\frac{1}{2}\,Bc^2\log\left(x^2\right) - \frac{11\,Bc^2x^6 + 12\,Bbcx^4 + 6\,Ac^2x^4 + 3\,Bb^2x^2 + 6\,Abcx^2 + 2\,Ab^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(11*B*c^2*x^6 + 12*B*b*c*x^4 + 6*A*c^2*x^4 + 3*B*
b^2*x^2 + 6*A*b*c*x^2 + 2*A*b^2)/x^6
```

maple [A] time = 0.05, size = 52, normalized size = 1.02

$$B\,c^2\ln(x) - \frac{A\,c^2}{2x^2} - \frac{Bbc}{x^2} - \frac{Abc}{2x^4} - \frac{B\,b^2}{4x^4} - \frac{A\,b^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x)
```

```
[Out] -1/2*b/x^4*A*c-1/4*b^2/x^4*B-1/2*c^2/x^2*A-c/x^2*b*B-1/6*A*b^2/x^6+B*c^2*ln
(x)
```

maxima [A] time = 1.27, size = 55, normalized size = 1.08

$$\frac{1}{2}\,Bc^2\log\left(x^2\right) - \frac{6\left(2\,Bbc + Ac^2\right)x^4 + 2\,Ab^2 + 3\left(Bb^2 + 2\,Abc\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(6*(2*B*b*c + A*c^2)*x^4 + 2*A*b^2 + 3*(B*b^2 + 2
*A*b*c)*x^2)/x^6
```

mupad [B] time = 0.09, size = 51, normalized size = 1.00

$$B\,c^2\,\ln(x) - \frac{x^2\,\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+x^4\,\left(\frac{A\,c^2}{2}+B\,b\,c\right)+\frac{A\,b^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x)
```

```
[Out] B*c^2*log(x) - (x^2*((B*b^2)/4 + (A*b*c)/2) + x^4*((A*c^2)/2 + B*b*c) + (A*
b^2)/6)/x^6
```

sympy [A] time = 0.99, size = 56, normalized size = 1.10

$$Bc^2\log(x) + \frac{-2Ab^2 + x^4\left(-6Ac^2 - 12Bbc\right) + x^2\left(-6Abc - 3Bb^2\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] B*c**2*log(x) + (-2*A*b**2 + x**4*(-6*A*c**2 - 12*B*b*c) + x**2*(-6*A*b*c -
 3*B*b**2))/(12*x**6)
```

3.24 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=53

$$-\frac{Ab^2}{7x^7}-\frac{b(2Ac+bB)}{5x^5}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{x}$$

```
[Out] -1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1584, 448}

$$-\frac{Ab^2}{7x^7}-\frac{b(2Ac+bB)}{5x^5}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

```
[Out] -(A*b^2)/(7*x^7) - (b*(b*B + 2*A*c))/(5*x^5) - (c*(2*b*B + A*c))/(3*x^3) -
(B*c^2)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{12}}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^2}{x^8}\,dx\\
&= \int \left(\frac{Ab^2}{x^8}+\frac{b(bB+2Ac)}{x^6}+\frac{c(2bB+Ac)}{x^4}+\frac{Bc^2}{x^2}\right)dx\\
&= -\frac{Ab^2}{7x^7}-\frac{b(bB+2Ac)}{5x^5}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.11

$$\frac{b^2(-B)-2Abc}{5x^5}-\frac{Ab^2}{7x^7}+\frac{-Ac^2-2bBc}{3x^3}-\frac{Bc^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

```
[Out] -1/7*(A*b^2)/x^7 + (-(b^2*B) - 2*A*b*c)/(5*x^5) + (-2*b*B*c - A*c^2)/(3*x^3
) - (B*c^2)/x
```

fricas [A] time = 1.24, size = 53, normalized size = 1.00

$$-\frac{105\,Bc^2x^6 + 35\left(2\,Bbc + Ac^2\right)x^4 + 15\,Ab^2 + 21\left(Bb^2 + 2\,Abc\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

giac [A] time = 0.15, size = 55, normalized size = 1.04

$$-\frac{105\,Bc^2x^6 + 70\,Bbcx^4 + 35\,Ac^2x^4 + 21\,Bb^2x^2 + 42\,Abcx^2 + 15\,Ab^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 70*B*b*c*x^4 + 35*A*c^2*x^4 + 21*B*b^2*x^2 + 42*A*b
*c*x^2 + 15*A*b^2)/x^7
```

maple [A] time = 0.05, size = 48, normalized size = 0.91

$$-\frac{B\,c^2}{x} - \frac{(Ac + 2bB)\,c}{3x^3} - \frac{A\,b^2}{7x^7} - \frac{(2Ac + bB)\,b}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x)
```

```
[Out] -1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

maxima [A] time = 1.37, size = 53, normalized size = 1.00

$$-\frac{105\,Bc^2x^6 + 35\left(2\,Bbc + Ac^2\right)x^4 + 15\,Ab^2 + 21\left(Bb^2 + 2\,Abc\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

mupad [B] time = 0.04, size = 52, normalized size = 0.98

$$-\frac{x^2\left(\frac{B\,b^2}{5} + \frac{2\,A\,c\,b}{5}\right) + x^4\left(\frac{A\,c^2}{3} + \frac{2\,B\,b\,c}{3}\right) + \frac{A\,b^2}{7} + B\,c^2\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x)
```

```
[Out] -(x^2*((B*b^2)/5 + (2*A*b*c)/5) + x^4*((A*c^2)/3 + (2*B*b*c)/3) + (A*b^2)/7
 + B*c^2*x^6)/x^7
```

sympy [A] time = 1.07, size = 58, normalized size = 1.09

$$\frac{-15Ab^2 - 105Bc^2x^6 + x^4\left(-35Ac^2 - 70Bbc\right) + x^2\left(-42Abc - 21Bb^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] (-15*A*b**2 - 105*B*c**2*x**6 + x**4*(-35*A*c**2 - 70*B*b*c) + x**2*(-42*A*
b*c - 21*B*b**2))/(105*x**7)
```

3.25 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx$

Optimal. Leaf size=75

$$\frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2x^7(3Ac + bB) + \frac{1}{11}c^2x^{11}(Ac + 3bB) + \frac{1}{3}bcx^9(Ac + bB) + \frac{1}{13}Bc^3x^{13}$$

```
[Out] 1/5*A*b^3*x^5+1/7*b^2*(3*A*c+B*b)*x^7+1/3*b*c*(A*c+B*b)*x^9+1/11*c^2*(A*c+3
*B*b)*x^11+1/13*B*c^3*x^13
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{1}{7}b^2x^7(3Ac + bB) + \frac{1}{5}Ab^3x^5 + \frac{1}{11}c^2x^{11}(Ac + 3bB) + \frac{1}{3}bcx^9(Ac + bB) + \frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^2} dx &= \int x^4 (A + Bx^2)(b + cx^2)^3 dx \\
&= \int (Ab^3x^4 + b^2(bB + 3Ac)x^6 + 3bc(bB + Ac)x^8 + c^2(3bB + Ac)x^{10} + Bc^3x^{12}) \\
&= \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB + 3Ac)x^7 + \frac{1}{3}bc(bB + Ac)x^9 + \frac{1}{11}c^2(3bB + Ac)x^{11} + \frac{1}{13}Bc
\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.00

$$\frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2x^7(3Ac + bB) + \frac{1}{11}c^2x^{11}(Ac + 3bB) + \frac{1}{3}bcx^9(Ac + bB) + \frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

fricas [A] time = 0.77, size = 73, normalized size = 0.97

$$\frac{1}{13}\,Bc^3x^{13} + \frac{1}{11}\left(3\,Bbc^2 + Ac^3\right)x^{11} + \frac{1}{3}\left(Bb^2c + Abc^2\right)x^9 + \frac{1}{5}\,Ab^3x^5 + \frac{1}{7}\left(Bb^3 + 3\,Ab^2c\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

giac [A] time = 0.19, size = 77, normalized size = 1.03

$$\frac{1}{13}\,Bc^3x^{13} + \frac{3}{11}\,Bbc^2x^{11} + \frac{1}{11}\,Ac^3x^{11} + \frac{1}{3}\,Bb^2cx^9 + \frac{1}{3}\,Abc^2x^9 + \frac{1}{7}\,Bb^3x^7 + \frac{3}{7}\,Ab^2cx^7 + \frac{1}{5}\,Ab^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 3/7*A*b^2*c*x^7 + 1/5*A*b^3*x^5
```

maple [A] time = 0.04, size = 76, normalized size = 1.01

$$\frac{B\,c^3x^{13}}{13} + \frac{\left(A\,c^3 + 3Bb\,c^2\right)x^{11}}{11} + \frac{A\,b^3x^5}{5} + \frac{\left(3Ab\,c^2 + 3Bc\,b^2\right)x^9}{9} + \frac{\left(3Ac\,b^2 + B\,b^3\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x)
```

```
[Out] 1/13*B*c^3*x^13+1/11*(A*c^3+3*B*b*c^2)*x^11+1/9*(3*A*b*c^2+3*B*b^2*c)*x^9+1
/7*(3*A*b^2*c+B*b^3)*x^7+1/5*A*b^3*x^5
```

maxima [A] time = 1.34, size = 73, normalized size = 0.97

$$\frac{1}{13}\,Bc^3x^{13} + \frac{1}{11}\left(3\,Bbc^2 + Ac^3\right)x^{11} + \frac{1}{3}\left(Bb^2c + Abc^2\right)x^9 + \frac{1}{5}\,Ab^3x^5 + \frac{1}{7}\left(Bb^3 + 3\,Ab^2c\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

mupad [B] time = 0.06, size = 69, normalized size = 0.92

$$x^7\left(\frac{B\,b^3}{7} + \frac{3\,A\,c\,b^2}{7}\right) + x^{11}\left(\frac{A\,c^3}{11} + \frac{3\,B\,b\,c^2}{11}\right) + \frac{A\,b^3\,x^5}{5} + \frac{B\,c^3\,x^{13}}{13} + \frac{b\,c\,x^9\,\left(A\,c + B\,b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x)
```

```
[Out] x^7*((B*b^3)/7 + (3*A*b^2*c)/7) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + (A*b
^3*x^5)/5 + (B*c^3*x^13)/13 + (b*c*x^9*(A*c + B*b))/3
```

sympy [A] time = 0.08, size = 80, normalized size = 1.07

$$\frac{Ab^3x^5}{5} + \frac{Bc^3x^{13}}{13} + x^{11}\left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11}\right) + x^9\left(\frac{Abc^2}{3} + \frac{Bb^2c}{3}\right) + x^7\left(\frac{3Ab^2c}{7} + \frac{Bb^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] A*b**3*x**5/5 + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*
(A*b*c**2/3 + B*b**2*c/3) + x**7*(3*A*b**2*c/7 + B*b**3/7)
```

3.26 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=68

$$-\frac{(b+cx^2)^5(2bB-Ac)}{10c^3}+\frac{b(b+cx^2)^4(bB-Ac)}{8c^3}+\frac{B(b+cx^2)^6}{12c^3}$$

```
[Out] 1/8*b*(-A*c+B*b)*(c*x^2+b)^4/c^3-1/10*(-A*c+2*B*b)*(c*x^2+b)^5/c^3+1/12*B*(
c*x^2+b)^6/c^3
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{(b+cx^2)^5(2bB-Ac)}{10c^3}+\frac{b(b+cx^2)^4(bB-Ac)}{8c^3}+\frac{B(b+cx^2)^6}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] (b*(b*B - A*c)*(b + c*x^2)^4)/(8*c^3) - ((2*b*B - A*c)*(b + c*x^2)^5)/(10*c
^3) + (B*(b + c*x^2)^6)/(12*c^3)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}dx &= \int x^3(A+Bx^2)(b+cx^2)^3 dx\\ &= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)(b+cx)^3 dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB-Ac)(b+cx)^3}{c^2}+\frac{(-2bB+Ac)(b+cx)^4}{c^2}+\frac{B(b+cx)^5}{c^2}\right) dx, x,\right.\\ &= \frac{b(bB-Ac)(b+cx^2)^4}{8c^3}-\frac{(2bB-Ac)(b+cx^2)^5}{10c^3}+\frac{B(b+cx^2)^6}{12c^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 1.01

$$\frac{1}{120}x^4\left(30Ab^3+20b^2x^2(3Ac+bB)+12c^2x^6(Ac+3bB)+45bcx^4(Ac+bB)+10Bc^3x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] (x^4*(30*A*b^3 + 20*b^2*(b*B + 3*A*c)*x^2 + 45*b*c*(b*B + A*c)*x^4 + 12*c^2
*(3*b*B + A*c)*x^6 + 10*B*c^3*x^8))/120
```

fricas [A] time = 0.93, size = 73, normalized size = 1.07

$$\frac{1}{12}Bc^3x^{12}+\frac{1}{10}\left(3Bbc^2+Ac^3\right)x^{10}+\frac{3}{8}\left(Bb^2c+Abc^2\right)x^8+\frac{1}{4}Ab^3x^4+\frac{1}{6}\left(Bb^3+3Ab^2c\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

giac [A] time = 0.21, size = 77, normalized size = 1.13

$$\frac{1}{12}Bc^3x^{12}+\frac{3}{10}Bbc^2x^{10}+\frac{1}{10}Ac^3x^{10}+\frac{3}{8}Bb^2cx^8+\frac{3}{8}Abc^2x^8+\frac{1}{6}Bb^3x^6+\frac{1}{2}Ab^2cx^6+\frac{1}{4}Ab^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + 1/2*A*b^2*c*x^6 + 1/4*A*b^3*x^4
```

maple [A] time = 0.04, size = 76, normalized size = 1.12

$$\frac{Bc^3x^{12}}{12}+\frac{\left(Ac^3+3Bbc^2\right)x^{10}}{10}+\frac{Ab^3x^4}{4}+\frac{\left(3Abc^2+3Bcb^2\right)x^8}{8}+\frac{\left(3Acb^2+Bb^3\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x)
```

```
[Out] 1/12*B*c^3*x^12+1/10*(A*c^3+3*B*b*c^2)*x^10+1/8*(3*A*b*c^2+3*B*b^2*c)*x^8+1
/6*(3*A*b^2*c+B*b^3)*x^6+1/4*A*b^3*x^4
```

maxima [A] time = 1.32, size = 73, normalized size = 1.07

$$\frac{1}{12}Bc^3x^{12}+\frac{1}{10}\left(3Bbc^2+Ac^3\right)x^{10}+\frac{3}{8}\left(Bb^2c+Abc^2\right)x^8+\frac{1}{4}Ab^3x^4+\frac{1}{6}\left(Bb^3+3Ab^2c\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

mupad [B] time = 0.03, size = 69, normalized size = 1.01

$$x^6\left(\frac{Bb^3}{6}+\frac{Acb^2}{2}\right)+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+\frac{Ab^3x^4}{4}+\frac{Bc^3x^{12}}{12}+\frac{3bcx^8\,(Ac+Bb)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x)
```

```
[Out] x^6*((B*b^3)/6 + (A*b^2*c)/2) + x^10*((A*c^3)/10 + (3*B*b*c^2)/10) + (A*b^3
*x^4)/4 + (B*c^3*x^12)/12 + (3*b*c*x^8*(A*c + B*b))/8
```

sympy [A] time = 0.09, size = 82, normalized size = 1.21

$$\frac{Ab^3x^4}{4} + \frac{Bc^3x^{12}}{12} + x^{10}\left(\frac{Ac^3}{10} + \frac{3Bbc^2}{10}\right) + x^8\left(\frac{3Abc^2}{8} + \frac{3Bb^2c}{8}\right) + x^6\left(\frac{Ab^2c}{2} + \frac{Bb^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] A*b**3*x**4/4 + B*c**3*x**12/12 + x**10*(A*c**3/10 + 3*B*b*c**2/10) + x**8*
(3*A*b*c**2/8 + 3*B*b**2*c/8) + x**6*(A*b**2*c/2 + B*b**3/6)
```

3.27 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4} \, dx$

Optimal. Leaf size=75

$$\frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2x^5(3Ac + bB) + \frac{1}{9}c^2x^9(Ac + 3bB) + \frac{3}{7}bcx^7(Ac + bB) + \frac{1}{11}Bc^3x^{11}$$

```
[Out] 1/3*A*b^3*x^3+1/5*b^2*(3*A*c+B*b)*x^5+3/7*b*c*(A*c+B*b)*x^7+1/9*c^2*(A*c+3*
B*b)*x^9+1/11*B*c^3*x^11
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{1}{5}b^2x^5(3Ac + bB) + \frac{1}{3}Ab^3x^3 + \frac{1}{9}c^2x^9(Ac + 3bB) + \frac{3}{7}bcx^7(Ac + bB) + \frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^4} \, dx &= \int x^2 (A + Bx^2)(b + cx^2)^3 \, dx \\ &= \int (Ab^3x^2 + b^2(bB + 3Ac)x^4 + 3bc(bB + Ac)x^6 + c^2(3bB + Ac)x^8 + Bc^3x^{10}) \, d \\ &= \frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2(bB + 3Ac)x^5 + \frac{3}{7}bc(bB + Ac)x^7 + \frac{1}{9}c^2(3bB + Ac)x^9 + \frac{1}{11}Bc^3x \end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.00

$$\frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2x^5(3Ac + bB) + \frac{1}{9}c^2x^9(Ac + 3bB) + \frac{3}{7}bcx^7(Ac + bB) + \frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

fricas [A] time = 0.83, size = 73, normalized size = 0.97

$$\frac{1}{11}\,Bc^{3}x^{11}+\frac{1}{9}\left(3\,Bbc^{2}+Ac^{3}\right)x^{9}+\frac{3}{7}\left(Bb^{2}c+Abc^{2}\right)x^{7}+\frac{1}{3}\,Ab^{3}x^{3}+\frac{1}{5}\left(Bb^{3}+3\,Ab^{2}c\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

giac [A] time = 0.15, size = 77, normalized size = 1.03

$$\frac{1}{11}\,Bc^{3}x^{11}+\frac{1}{3}\,Bbc^{2}x^{9}+\frac{1}{9}\,Ac^{3}x^{9}+\frac{3}{7}\,Bb^{2}cx^{7}+\frac{3}{7}\,Abc^{2}x^{7}+\frac{1}{5}\,Bb^{3}x^{5}+\frac{3}{5}\,Ab^{2}cx^{5}+\frac{1}{3}\,Ab^{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/11*B*c^3*x^11 + 1/3*B*b*c^2*x^9 + 1/9*A*c^3*x^9 + 3/7*B*b^2*c*x^7 + 3/7*A
*b*c^2*x^7 + 1/5*B*b^3*x^5 + 3/5*A*b^2*c*x^5 + 1/3*A*b^3*x^3
```

maple [A] time = 0.04, size = 76, normalized size = 1.01

$$\frac{B\,c^{3}x^{11}}{11}+\frac{\left(A\,c^{3}+3Bb\,c^{2}\right)x^{9}}{9}+\frac{A\,b^{3}x^{3}}{3}+\frac{\left(3Ab\,c^{2}+3Bc\,b^{2}\right)x^{7}}{7}+\frac{\left(3Ac\,b^{2}+B\,b^{3}\right)x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x)
```

```
[Out] 1/11*B*c^3*x^11+1/9*(A*c^3+3*B*b*c^2)*x^9+1/7*(3*A*b*c^2+3*B*b^2*c)*x^7+1/5
*(3*A*b^2*c+B*b^3)*x^5+1/3*A*b^3*x^3
```

maxima [A] time = 1.37, size = 73, normalized size = 0.97

$$\frac{1}{11}\,Bc^{3}x^{11}+\frac{1}{9}\left(3\,Bbc^{2}+Ac^{3}\right)x^{9}+\frac{3}{7}\left(Bb^{2}c+Abc^{2}\right)x^{7}+\frac{1}{3}\,Ab^{3}x^{3}+\frac{1}{5}\left(Bb^{3}+3\,Ab^{2}c\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

mupad [B] time = 0.03, size = 69, normalized size = 0.92

$$x^{5}\left(\frac{B\,b^{3}}{5}+\frac{3\,A\,c\,b^{2}}{5}\right)+x^{9}\left(\frac{A\,c^{3}}{9}+\frac{B\,b\,c^{2}}{3}\right)+\frac{A\,b^{3}\,x^{3}}{3}+\frac{B\,c^{3}\,x^{11}}{11}+\frac{3\,b\,c\,x^{7}\,\left(A\,c+B\,b\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x)
```

```
[Out] x^5*((B*b^3)/5 + (3*A*b^2*c)/5) + x^9*((A*c^3)/9 + (B*b*c^2)/3) + (A*b^3*x^
3)/3 + (B*c^3*x^11)/11 + (3*b*c*x^7*(A*c + B*b))/7
```

sympy [A] time = 0.09, size = 82, normalized size = 1.09

$$\frac{Ab^{3}x^{3}}{3}+\frac{Bc^{3}x^{11}}{11}+x^{9}\left(\frac{Ac^{3}}{9}+\frac{Bbc^{2}}{3}\right)+x^{7}\left(\frac{3Abc^{2}}{7}+\frac{3Bb^{2}c}{7}\right)+x^{5}\left(\frac{3Ab^{2}c}{5}+\frac{Bb^{3}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] A*b**3*x**3/3 + B*c**3*x**11/11 + x**9*(A*c**3/9 + B*b*c**2/3) + x**7*(3*A*
b*c**2/7 + 3*B*b**2*c/7) + x**5*(3*A*b**2*c/5 + B*b**3/5)
```

3.28 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx$

Optimal. Leaf size=42

$$\frac{B(b+cx^2)^5}{10c^2}-\frac{(b+cx^2)^4(bB-Ac)}{8c^2}$$

```
[Out] -1/8*(-A*c+B*b)*(c*x^2+b)^4/c^2+1/10*B*(c*x^2+b)^5/c^2
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{B(b+cx^2)^5}{10c^2}-\frac{(b+cx^2)^4(bB-Ac)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

```
[Out] -((b*B - A*c)*(b + c*x^2)^4)/(8*c^2) + (B*(b + c*x^2)^5)/(10*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx &= \int x(A+Bx^2)(b+cx^2)^3\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)^3\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^3}{c}+\frac{B(b+cx)^4}{c}\right)dx,x,x^2\right)\\
&= -\frac{(bB-Ac)(b+cx^2)^4}{8c^2}+\frac{B(b+cx^2)^5}{10c^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.64

$$\frac{1}{40}x^2\left(20Ab^3+10b^2x^2(3Ac+bB)+5c^2x^6(Ac+3bB)+20bcx^4(Ac+bB)+4Bc^3x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

```
[Out] (x^2*(20*A*b^3 + 10*b^2*(b*B + 3*A*c)*x^2 + 20*b*c*(b*B + A*c)*x^4 + 5*c^2*
(3*b*B + A*c)*x^6 + 4*B*c^3*x^8))/40
```

fricas [A] time = 0.88, size = 73, normalized size = 1.74

$$\frac{1}{10}Bc^3x^{10}+\frac{1}{8}\left(3Bbc^2+Ac^3\right)x^8+\frac{1}{2}\left(Bb^2c+Abc^2\right)x^6+\frac{1}{2}Ab^3x^2+\frac{1}{4}\left(Bb^3+3Ab^2c\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

giac [B] time = 0.15, size = 77, normalized size = 1.83

$$\frac{1}{10}Bc^3x^{10}+\frac{3}{8}Bbc^2x^8+\frac{1}{8}Ac^3x^8+\frac{1}{2}Bb^2cx^6+\frac{1}{2}Abc^2x^6+\frac{1}{4}Bb^3x^4+\frac{3}{4}Ab^2cx^4+\frac{1}{2}Ab^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/10*B*c^3*x^10 + 3/8*B*b*c^2*x^8 + 1/8*A*c^3*x^8 + 1/2*B*b^2*c*x^6 + 1/2*A
*b*c^2*x^6 + 1/4*B*b^3*x^4 + 3/4*A*b^2*c*x^4 + 1/2*A*b^3*x^2
```

maple [A] time = 0.04, size = 76, normalized size = 1.81

$$\frac{Bc^3x^{10}}{10}+\frac{\left(Ac^3+3Bbc^2\right)x^8}{8}+\frac{Ab^3x^2}{2}+\frac{\left(3Abc^2+3Bcb^2\right)x^6}{6}+\frac{\left(3Acb^2+Bb^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x)
```

```
[Out] 1/10*B*c^3*x^10+1/8*(A*c^3+3*B*b*c^2)*x^8+1/6*(3*A*b*c^2+3*B*b^2*c)*x^6+1/4
*(3*A*b^2*c+B*b^3)*x^4+1/2*A*b^3*x^2
```

maxima [A] time = 1.44, size = 73, normalized size = 1.74

$$\frac{1}{10}Bc^3x^{10}+\frac{1}{8}\left(3Bbc^2+Ac^3\right)x^8+\frac{1}{2}\left(Bb^2c+Abc^2\right)x^6+\frac{1}{2}Ab^3x^2+\frac{1}{4}\left(Bb^3+3Ab^2c\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

mupad [B] time = 0.03, size = 69, normalized size = 1.64

$$x^4\left(\frac{Bb^3}{4}+\frac{3Acb^2}{4}\right)+x^8\left(\frac{Ac^3}{8}+\frac{3Bbc^2}{8}\right)+\frac{Ab^3x^2}{2}+\frac{Bc^3x^{10}}{10}+\frac{bcx^6\,(Ac+Bb)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x)
```

```
[Out] x^4*((B*b^3)/4 + (3*A*b^2*c)/4) + x^8*((A*c^3)/8 + (3*B*b*c^2)/8) + (A*b^3*
x^2)/2 + (B*c^3*x^10)/10 + (b*c*x^6*(A*c + B*b))/2
```

sympy [B] time = 0.09, size = 80, normalized size = 1.90

$$\frac{Ab^3x^2}{2} + \frac{Bc^3x^{10}}{10} + x^8\left(\frac{Ac^3}{8} + \frac{3Bbc^2}{8}\right) + x^6\left(\frac{Abc^2}{2} + \frac{Bb^2c}{2}\right) + x^4\left(\frac{3Ab^2c}{4} + \frac{Bb^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] A*b**3*x**2/2 + B*c**3*x**10/10 + x**8*(A*c**3/8 + 3*B*b*c**2/8) + x**6*(A*
b*c**2/2 + B*b**2*c/2) + x**4*(3*A*b**2*c/4 + B*b**3/4)
```

3.29 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} \, dx$

Optimal. Leaf size=70

$$Ab^3x + \frac{1}{3}b^2x^3(3Ac + bB) + \frac{1}{7}c^2x^7(Ac + 3bB) + \frac{3}{5}bcx^5(Ac + bB) + \frac{1}{9}Bc^3x^9$$

```
[Out] A*b^3*x+1/3*b^2*(3*A*c+B*b)*x^3+3/5*b*c*(A*c+B*b)*x^5+1/7*c^2*(A*c+3*B*b)*x
^7+1/9*B*c^3*x^9
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 373}

$$\frac{1}{3}b^2x^3(3Ac + bB) + Ab^3x + \frac{1}{7}c^2x^7(Ac + 3bB) + \frac{3}{5}bcx^5(Ac + bB) + \frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx^2)(bx^2 + cx^4)^3}{x^6} \, dx &= \int (A + Bx^2)(b + cx^2)^3 \, dx \\
&= \int \left(Ab^3 + b^2(bB + 3Ac)x^2 + 3bc(bB + Ac)x^4 + c^2(3bB + Ac)x^6 + Bc^3x^8\right) dx \\
&= Ab^3x + \frac{1}{3}b^2(bB + 3Ac)x^3 + \frac{3}{5}bc(bB + Ac)x^5 + \frac{1}{7}c^2(3bB + Ac)x^7 + \frac{1}{9}Bc^3x^9
\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.00

$$Ab^3x + \frac{1}{3}b^2x^3(3Ac + bB) + \frac{1}{7}c^2x^7(Ac + 3bB) + \frac{3}{5}bcx^5(Ac + bB) + \frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

fricas [A] time = 0.83, size = 70, normalized size = 1.00

$$\frac{1}{9}\,Bc^3x^9+\frac{1}{7}\left(3\,Bbc^2+Ac^3\right)x^7+\frac{3}{5}\left(Bb^2c+Abc^2\right)x^5+Ab^3x+\frac{1}{3}\left(Bb^3+3\,Ab^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

giac [A] time = 0.17, size = 73, normalized size = 1.04

$$\frac{1}{9}\,Bc^3x^9+\frac{3}{7}\,Bbc^2x^7+\frac{1}{7}\,Ac^3x^7+\frac{3}{5}\,Bb^2cx^5+\frac{3}{5}\,Abc^2x^5+\frac{1}{3}\,Bb^3x^3+Ab^2cx^3+Ab^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/9*B*c^3*x^9 + 3/7*B*b*c^2*x^7 + 1/7*A*c^3*x^7 + 3/5*B*b^2*c*x^5 + 3/5*A*b
*c^2*x^5 + 1/3*B*b^3*x^3 + A*b^2*c*x^3 + A*b^3*x
```

maple [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{B\,c^3x^9}{9}+\frac{\left(A\,c^3+3Bb\,c^2\right)x^7}{7}+A\,b^3x+\frac{\left(3Ab\,c^2+3Bc\,b^2\right)x^5}{5}+\frac{\left(3Ac\,b^2+B\,b^3\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x)
```

```
[Out] 1/9*B*c^3*x^9+1/7*(A*c^3+3*B*b*c^2)*x^7+1/5*(3*A*b*c^2+3*B*b^2*c)*x^5+1/3*(
3*A*b^2*c+B*b^3)*x^3+A*b^3*x
```

maxima [A] time = 1.28, size = 70, normalized size = 1.00

$$\frac{1}{9}\,Bc^3x^9+\frac{1}{7}\left(3\,Bbc^2+Ac^3\right)x^7+\frac{3}{5}\left(Bb^2c+Abc^2\right)x^5+Ab^3x+\frac{1}{3}\left(Bb^3+3\,Ab^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

mupad [B] time = 0.03, size = 65, normalized size = 0.93

$$x^3\,\left(\frac{B\,b^3}{3}+A\,c\,b^2\right)+x^7\,\left(\frac{A\,c^3}{7}+\frac{3\,B\,b\,c^2}{7}\right)+\frac{B\,c^3\,x^9}{9}+A\,b^3\,x+\frac{3\,b\,c\,x^5\,\left(A\,c+B\,b\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x)
```

```
[Out] x^3*((B*b^3)/3 + A*b^2*c) + x^7*((A*c^3)/7 + (3*B*b*c^2)/7) + (B*c^3*x^9)/9
 + A*b^3*x + (3*b*c*x^5*(A*c + B*b))/5
```

sympy [A] time = 0.09, size = 76, normalized size = 1.09

$$Ab^3x+\frac{Bc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Bbc^2}{7}\right)+x^5\left(\frac{3Abc^2}{5}+\frac{3Bb^2c}{5}\right)+x^3\left(Ab^2c+\frac{Bb^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] A*b**3*x + B*c**3*x**9/9 + x**7*(A*c**3/7 + 3*B*b*c**2/7) + x**5*(3*A*b*c**
2/5 + 3*B*b**2*c/5) + x**3*(A*b**2*c + B*b**3/3)
```

3.30 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx$

Optimal. Leaf size=60

$$Ab^3\log(x)+\frac{3}{2}Ab^2cx^2+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B(b+cx^2)^4}{8c}$$

```
[Out] 3/2*A*b^2*c*x^2+3/4*A*b*c^2*x^4+1/6*A*c^3*x^6+1/8*B*(c*x^2+b)^4/c+A*b^3*ln(
x)
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 80, 43}

$$\frac{3}{2}Ab^2cx^2+Ab^3\log(x)+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

```
[Out] (3*A*b^2*c*x^2)/2 + (3*A*b*c^2*x^4)/4 + (A*c^3*x^6)/6 + (B*(b + c*x^2)^4)/(
8*c) + A*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x}\,dx, x, x^2\right)$$
$$= \frac{B(b+cx^2)^4}{8c} + \frac{1}{2}A\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx, x, x^2\right)$$
$$= \frac{B(b+cx^2)^4}{8c} + \frac{1}{2}A\operatorname{Subst}\left(\int \left(3b^2c + \frac{b^3}{x} + 3bc^2x + c^3x^2\right)dx, x, x^2\right)$$
$$= \frac{3}{2}Ab^2cx^2 + \frac{3}{4}Abc^2x^4 + \frac{1}{6}Ac^3x^6 + \frac{B(b+cx^2)^4}{8c} + Ab^3\log(x)$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.18

$$Ab^3\log(x) + \frac{1}{2}b^2x^2(3Ac+bB) + \frac{1}{6}c^2x^6(Ac+3bB) + \frac{3}{4}bcx^4(Ac+bB) + \frac{1}{8}Bc^3x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

```
[Out] (b^2*(b*B + 3*A*c)*x^2)/2 + (3*b*c*(b*B + A*c)*x^4)/4 + (c^2*(3*b*B + A*c)*
x^6)/6 + (B*c^3*x^8)/8 + A*b^3*Log[x]
```

fricas [A] time = 0.91, size = 71, normalized size = 1.18

$$\frac{1}{8}Bc^3x^8 + \frac{1}{6}\left(3Bbc^2 + Ac^3\right)x^6 + \frac{3}{4}\left(Bb^2c + Abc^2\right)x^4 + Ab^3\log(x) + \frac{1}{2}\left(Bb^3 + 3Ab^2c\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 A*b^3*log(x) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

giac [A] time = 0.15, size = 78, normalized size = 1.30

$$\frac{1}{8}Bc^3x^8 + \frac{1}{2}Bbc^2x^6 + \frac{1}{6}Ac^3x^6 + \frac{3}{4}Bb^2cx^4 + \frac{3}{4}Abc^2x^4 + \frac{1}{2}Bb^3x^2 + \frac{3}{2}Ab^2cx^2 + \frac{1}{2}Ab^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/8*B*c^3*x^8 + 1/2*B*b*c^2*x^6 + 1/6*A*c^3*x^6 + 3/4*B*b^2*c*x^4 + 3/4*A*b
*c^2*x^4 + 1/2*B*b^3*x^2 + 3/2*A*b^2*c*x^2 + 1/2*A*b^3*log(x^2)
```

maple [A] time = 0.04, size = 76, normalized size = 1.27

$$\frac{Bc^3x^8}{8} + \frac{Ac^3x^6}{6} + \frac{Bbc^2x^6}{2} + \frac{3Abc^2x^4}{4} + \frac{3Bb^2cx^4}{4} + \frac{3Ab^2cx^2}{2} + \frac{Bb^3x^2}{2} + Ab^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x)
```

```
[Out] 1/8*B*c^3*x^8+1/6*A*c^3*x^6+1/2*B*x^6*b*c^2+3/4*A*b*c^2*x^4+3/4*B*x^4*b^2*c
+3/2*A*b^2*c*x^2+1/2*B*x^2*b^3+A*b^3*ln(x)
```

maxima [A] time = 1.31, size = 74, normalized size = 1.23

$$\frac{1}{8}Bc^3x^8 + \frac{1}{6}\left(3\,Bbc^2 + Ac^3\right)x^6 + \frac{3}{4}\left(Bb^2c + Abc^2\right)x^4 + \frac{1}{2}Ab^3\log\left(x^2\right) + \frac{1}{2}\left(Bb^3 + 3\,Ab^2c\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 1/2*A*b^3*log(x^2) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

mupad [B] time = 0.03, size = 67, normalized size = 1.12

$$x^2\left(\frac{B\,b^3}{2} + \frac{3\,A\,c\,b^2}{2}\right) + x^6\left(\frac{A\,c^3}{6} + \frac{B\,b\,c^2}{2}\right) + \frac{B\,c^3\,x^8}{8} + A\,b^3\,\ln(x) + \frac{3\,b\,c\,x^4\,\left(A\,c + B\,b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x)
```

```
[Out] x^2*((B*b^3)/2 + (3*A*b^2*c)/2) + x^6*((A*c^3)/6 + (B*b*c^2)/2) + (B*c^3*x^
8)/8 + A*b^3*log(x) + (3*b*c*x^4*(A*c + B*b))/4
```

sympy [A] time = 0.18, size = 80, normalized size = 1.33

$$Ab^3\log\left(x\right) + \frac{Bc^3x^8}{8} + x^6\left(\frac{Ac^3}{6} + \frac{Bbc^2}{2}\right) + x^4\left(\frac{3Abc^2}{4} + \frac{3Bb^2c}{4}\right) + x^2\left(\frac{3Ab^2c}{2} + \frac{Bb^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] A*b**3*log(x) + B*c**3*x**8/8 + x**6*(A*c**3/6 + B*b*c**2/2) + x**4*(3*A*b*
c**2/4 + 3*B*b**2*c/4) + x**2*(3*A*b**2*c/2 + B*b**3/2)
```

3.31 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx$

Optimal. Leaf size=65

$$-\frac{Ab^3}{x}+b^2x(3Ac+bB)+\frac{1}{5}c^2x^5(Ac+3bB)+bcx^3(Ac+bB)+\frac{1}{7}Bc^3x^7$$

```
[Out] -A*b^3/x+b^2*(3*A*c+B*b)*x+b*c*(A*c+B*b)*x^3+1/5*c^2*(A*c+3*B*b)*x^5+1/7*B*
c^3*x^7
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$b^2x(3Ac+bB)-\frac{Ab^3}{x}+\frac{1}{5}c^2x^5(Ac+3bB)+bcx^3(Ac+bB)+\frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^2}\,dx\\ &= \int \left(b^2(bB+3Ac)+\frac{Ab^3}{x^2}+3bc(bB+Ac)x^2+c^2(3bB+Ac)x^4+Bc^3x^6\right)dx\\ &= -\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3+\frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.00

$$-\frac{Ab^3}{x}+b^2x(3Ac+bB)+\frac{1}{5}c^2x^5(Ac+3bB)+bcx^3(Ac+bB)+\frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

fricas [A] time = 0.97, size = 75, normalized size = 1.15

$$\frac{5\,Bc^3x^8 + 7\left(3\,Bbc^2 + Ac^3\right)x^6 + 35\left(Bb^2c + Abc^2\right)x^4 - 35\,Ab^3 + 35\left(Bb^3 + 3\,Ab^2c\right)x^2}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*c^3*x^8 + 7*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4
 - 35*A*b^3 + 35*(B*b^3 + 3*A*b^2*c)*x^2)/x
```

giac [A] time = 0.16, size = 70, normalized size = 1.08

$$\frac{1}{7}\,Bc^3x^7 + \frac{3}{5}\,Bbc^2x^5 + \frac{1}{5}\,Ac^3x^5 + Bb^2cx^3 + Abc^2x^3 + Bb^3x + 3\,Ab^2cx - \frac{Ab^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/7*B*c^3*x^7 + 3/5*B*b*c^2*x^5 + 1/5*A*c^3*x^5 + B*b^2*c*x^3 + A*b*c^2*x^3
 + B*b^3*x + 3*A*b^2*c*x - A*b^3/x
```

maple [A] time = 0.05, size = 71, normalized size = 1.09

$$\frac{B\,c^3x^7}{7} + \frac{A\,c^3x^5}{5} + \frac{3Bb\,c^2x^5}{5} + Ab\,c^2x^3 + B\,b^2c\,x^3 + 3A\,b^2cx + B\,b^3x - \frac{A\,b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x)
```

```
[Out] 1/7*B*c^3*x^7+1/5*A*x^5*c^3+3/5*B*x^5*b*c^2+A*x^3*b*c^2+B*x^3*b^2*c+3*A*c*b
^2*x+B*b^3*x-A*b^3/x
```

maxima [A] time = 1.34, size = 69, normalized size = 1.06

$$\frac{1}{7}\,Bc^3x^7 + \frac{1}{5}\left(3\,Bbc^2 + Ac^3\right)x^5 + \left(Bb^2c + Abc^2\right)x^3 - \frac{Ab^3}{x} + \left(Bb^3 + 3\,Ab^2c\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/7*B*c^3*x^7 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + (B*b^2*c + A*b*c^2)*x^3 - A*b
^3/x + (B*b^3 + 3*A*b^2*c)*x
```

mupad [B] time = 0.03, size = 65, normalized size = 1.00

$$x\,\left(B\,b^3 + 3\,A\,c\,b^2\right) + x^5\,\left(\frac{A\,c^3}{5} + \frac{3\,B\,b\,c^2}{5}\right) - \frac{A\,b^3}{x} + \frac{B\,c^3\,x^7}{7} + b\,c\,x^3\,\left(A\,c + B\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x)
```

```
[Out] x*(B*b^3 + 3*A*b^2*c) + x^5*((A*c^3)/5 + (3*B*b*c^2)/5) - (A*b^3)/x + (B*c^
3*x^7)/7 + b*c*x^3*(A*c + B*b)
```

sympy [A] time = 0.17, size = 68, normalized size = 1.05

$$-\frac{Ab^3}{x} + \frac{Bc^3x^7}{7} + x^5\left(\frac{Ac^3}{5} + \frac{3Bbc^2}{5}\right) + x^3\left(Abc^2 + Bb^2c\right) + x\left(3Ab^2c + Bb^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -A*b**3/x + B*c**3*x**7/7 + x**5*(A*c**3/5 + 3*B*b*c**2/5) + x**3*(A*b*c**2
 + B*b**2*c) + x*(3*A*b**2*c + B*b**3)
```

3.32 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} \, dx$

Optimal. Leaf size=71

$$-\frac{Ab^3}{2x^2}+b^2\log(x)(3Ac+bB)+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{3}{2}bcx^2(Ac+bB)+\frac{1}{6}Bc^3x^6$$

```
[Out] -1/2*A*b^3/x^2+3/2*b*c*(A*c+B*b)*x^2+1/4*c^2*(A*c+3*B*b)*x^4+1/6*B*c^3*x^6+
b^2*(3*A*c+B*b)*ln(x)
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$b^2\log(x)(3Ac+bB)-\frac{Ab^3}{2x^2}+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{3}{2}bcx^2(Ac+bB)+\frac{1}{6}Bc^3x^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -(A*b^3)/(2*x^2) + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + b^2*(b*B + 3*A*c)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^3} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(3bc(bB+Ac)+\frac{Ab^3}{x^2}+\frac{b^2(bB+3Ac)}{x}+c^2(3bB+Ac)x+Bc^3x^2\right) d \\
&= -\frac{Ab^3}{2x^2}+\frac{3}{2}bc(bB+Ac)x^2+\frac{1}{4}c^2(3bB+Ac)x^4+\frac{1}{6}Bc^3x^6+b^2(bB+3Ac)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.03

$$-\frac{Ab^3}{2x^2}+\log(x)\left(3Ab^2c+b^3B\right)+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{3}{2}bcx^2(Ac+bB)+\frac{1}{6}Bc^3x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -1/2*(A*b^3)/x^2 + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + (b^3*B + 3*A*b^2*c)*Log[x]
```

fricas [A] time = 0.56, size = 77, normalized size = 1.08

$$\frac{2\,Bc^3x^8+3\left(3\,Bbc^2+Ac^3\right)x^6+18\left(Bb^2c+Abc^2\right)x^4-6\,Ab^3+12\left(Bb^3+3\,Ab^2c\right)x^2\log(x)}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^3*x^8 + 3*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c^2)*x^4
- 6*A*b^3 + 12*(B*b^3 + 3*A*b^2*c)*x^2*log(x))/x^2
```

giac [A] time = 0.17, size = 97, normalized size = 1.37

$$\frac{1}{6}\,Bc^3x^6+\frac{3}{4}\,Bbc^2x^4+\frac{1}{4}\,Ac^3x^4+\frac{3}{2}\,Bb^2cx^2+\frac{3}{2}\,Abc^2x^2+\frac{1}{2}\left(Bb^3+3\,Ab^2c\right)\log\left(x^2\right)-\frac{Bb^3x^2+3\,Ab^2cx^2+Ab^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/6*B*c^3*x^6 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + 3/2*B*b^2*c*x^2 + 3/2*A*b
*c^2*x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2) - 1/2*(B*b^3*x^2 + 3*A*b^2*c*x^
2 + A*b^3)/x^2
```

maple [A] time = 0.05, size = 75, normalized size = 1.06

$$\frac{B\,c^3x^6}{6}+\frac{A\,c^3x^4}{4}+\frac{3Bb\,c^2x^4}{4}+\frac{3Ab\,c^2x^2}{2}+\frac{3B\,b^2c\,x^2}{2}+3A\,b^2c\ln(x)+B\,b^3\ln(x)-\frac{A\,b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x)
```

```
[Out] 1/6*B*c^3*x^6+1/4*A*x^4*c^3+3/4*B*x^4*b*c^2+3/2*A*x^2*b*c^2+3/2*B*x^2*b^2*c
-1/2*A*b^3/x^2+3*A*ln(x)*b^2*c+B*ln(x)*b^3
```

maxima [A] time = 1.30, size = 74, normalized size = 1.04

$$\frac{1}{6}\,Bc^3x^6+\frac{1}{4}\left(3\,Bbc^2+Ac^3\right)x^4+\frac{3}{2}\left(Bb^2c+Abc^2\right)x^2-\frac{Ab^3}{2\,x^2}+\frac{1}{2}\left(Bb^3+3\,Ab^2c\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^3*x^6 + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + 3/2*(B*b^2*c + A*b*c^2)*x^2 -
 1/2*A*b^3/x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2)
```

mupad [B] time = 0.04, size = 67, normalized size = 0.94

$$x^4\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right)+\ln(x)\,\left(B\,b^3+3\,A\,c\,b^2\right)-\frac{A\,b^3}{2\,x^2}+\frac{B\,c^3\,x^6}{6}+\frac{3\,b\,c\,x^2\,\left(A\,c+B\,b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x)
```

```
[Out] x^4*((A*c^3)/4 + (3*B*b*c^2)/4) + log(x)*(B*b^3 + 3*A*b^2*c) - (A*b^3)/(2*x
^2) + (B*c^3*x^6)/6 + (3*b*c*x^2*(A*c + B*b))/2
```

sympy [A] time = 0.27, size = 78, normalized size = 1.10

$$-\frac{Ab^3}{2x^2} + \frac{Bc^3x^6}{6} + b^2(3Ac + Bb)\log(x) + x^4\left(\frac{Ac^3}{4} + \frac{3Bbc^2}{4}\right) + x^2\left(\frac{3Abc^2}{2} + \frac{3Bb^2c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**9,x)
```

```
[Out] -A*b**3/(2*x**2) + B*c**3*x**6/6 + b**2*(3*A*c + B*b)*log(x) + x**4*(A*c**3
/4 + 3*B*b*c**2/4) + x**2*(3*A*b*c**2/2 + 3*B*b**2*c/2)
```

3.33 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=69

$$-\frac{Ab^3}{3x^3}-\frac{b^2(3Ac+bB)}{x}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

```
[Out] -1/3*A*b^3/x^3-b^2*(3*A*c+B*b)/x+3*b*c*(A*c+B*b)*x+1/3*c^2*(A*c+3*B*b)*x^3+
1/5*B*c^3*x^5
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{x}-\frac{Ab^3}{3x^3}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

```
[Out] -(A*b^3)/(3*x^3) - (b^2*(b*B + 3*A*c))/x + 3*b*c*(b*B + A*c)*x + (c^2*(3*b*
B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{10}}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^3}{x^4}\,dx\\ &= \int \left(3bc(bB+Ac)+\frac{Ab^3}{x^4}+\frac{b^2(bB+3Ac)}{x^2}+c^2(3bB+Ac)x^2+Bc^3x^4\right)dx\\ &= -\frac{Ab^3}{3x^3}-\frac{b^2(bB+3Ac)}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.03

$$-\frac{Ab^3}{3x^3}+\frac{b^3(-B)-3Ab^2c}{x}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

```
[Out] -1/3*(A*b^3)/x^3 + (-(b^3*B) - 3*A*b^2*c)/x + 3*b*c*(b*B + A*c)*x + (c^2*(3
*b*B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

fricas [A] time = 0.86, size = 75, normalized size = 1.09

$$\frac{3\,Bc^3x^8 + 5\,\left(3\,Bbc^2 + Ac^3\right)x^6 + 45\,\left(Bb^2c + Abc^2\right)x^4 - 5\,Ab^3 - 15\,\left(Bb^3 + 3\,Ab^2c\right)x^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^3*x^8 + 5*(3*B*b*c^2 + A*c^3)*x^6 + 45*(B*b^2*c + A*b*c^2)*x^4
 - 5*A*b^3 - 15*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

giac [A] time = 0.15, size = 74, normalized size = 1.07

$$\frac{1}{5}\,Bc^3x^5 + Bbc^2x^3 + \frac{1}{3}\,Ac^3x^3 + 3\,Bb^2cx + 3\,Abc^2x - \frac{3\,Bb^3x^2 + 9\,Ab^2cx^2 + Ab^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/5*B*c^3*x^5 + B*b*c^2*x^3 + 1/3*A*c^3*x^3 + 3*B*b^2*c*x + 3*A*b*c^2*x - 1
/3*(3*B*b^3*x^2 + 9*A*b^2*c*x^2 + A*b^3)/x^3
```

maple [A] time = 0.05, size = 70, normalized size = 1.01

$$\frac{B\,c^3x^5}{5} + \frac{A\,c^3x^3}{3} + Bb\,c^2x^3 + 3Ab\,c^2x + 3B\,b^2cx - \frac{A\,b^3}{3x^3} - \frac{(3Ac + bB)\,b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x)
```

```
[Out] 1/5*B*c^3*x^5+1/3*A*x^3*c^3+B*x^3*b*c^2+3*A*b*c^2*x+3*B*c*b^2*x-1/3*A*b^3/x
^3-b^2*(3*A*c+B*b)/x
```

maxima [A] time = 1.33, size = 73, normalized size = 1.06

$$\frac{1}{5}\,Bc^3x^5 + \frac{1}{3}\,\left(3\,Bbc^2 + Ac^3\right)x^3 + 3\,\left(Bb^2c + Abc^2\right)x - \frac{Ab^3 + 3\,\left(Bb^3 + 3\,Ab^2c\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/5*B*c^3*x^5 + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3*(B*b^2*c + A*b*c^2)*x - 1/3
*(A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

mupad [B] time = 0.06, size = 68, normalized size = 0.99

$$x^3\,\left(\frac{A\,c^3}{3} + B\,b\,c^2\right) - \frac{\frac{A\,b^3}{3} + x^2\,\left(B\,b^3 + 3\,A\,c\,b^2\right)}{x^3} + \frac{B\,c^3\,x^5}{5} + 3\,b\,c\,x\,\left(A\,c + B\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x)
```

```
[Out] x^3*((A*c^3)/3 + B*b*c^2) - ((A*b^3)/3 + x^2*(B*b^3 + 3*A*b^2*c))/x^3 + (B*
c^3*x^5)/5 + 3*b*c*x*(A*c + B*b)
```

sympy [A] time = 0.29, size = 75, normalized size = 1.09

$$\frac{Bc^3x^5}{5} + x^3\left(\frac{Ac^3}{3} + Bbc^2\right) + x\left(3Abc^2 + 3Bb^2c\right) + \frac{-Ab^3 + x^2\left(-9Ab^2c - 3Bb^3\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**10,x)
```

```
[Out] B*c**3*x**5/5 + x**3*(A*c**3/3 + B*b*c**2) + x*(3*A*b*c**2 + 3*B*b**2*c) +
(-A*b**3 + x**2*(-9*A*b**2*c - 3*B*b**3))/(3*x**3)
```

3.34 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} \, dx$

Optimal. Leaf size=72

$$-\frac{Ab^3}{4x^4}-\frac{b^2(3Ac+bB)}{2x^2}+\frac{1}{2}c^2x^2(Ac+3bB)+3bc\log(x)(Ac+bB)+\frac{1}{4}Bc^3x^4$$

```
[Out] -1/4*A*b^3/x^4-1/2*b^2*(3*A*c+B*b)/x^2+1/2*c^2*(A*c+3*B*b)*x^2+1/4*B*c^3*x^
4+3*b*c*(A*c+B*b)*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{b^2(3Ac+bB)}{2x^2}-\frac{Ab^3}{4x^4}+\frac{1}{2}c^2x^2(Ac+3bB)+3bc\log(x)(Ac+bB)+\frac{1}{4}Bc^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

```
[Out] -(A*b^3)/(4*x^4) - (b^2*(b*B + 3*A*c))/(2*x^2) + (c^2*(3*b*B + A*c)*x^2)/2
+ (B*c^3*x^4)/4 + 3*b*c*(b*B + A*c)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^5} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^3} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^3}+\frac{b^2(bB+3Ac)}{x^2}+\frac{3bc(bB+Ac)}{x}+Bc^3x\right) dx, \right. \\
&= -\frac{Ab^3}{4x^4}-\frac{b^2(bB+3Ac)}{2x^2}+\frac{1}{2}c^2(3bB+Ac)x^2+\frac{1}{4}Bc^3x^4+3bc(bB+Ac)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.01

$$\frac{Bx^2\left(-2b^3+6bc^2x^4+c^3x^6\right)-A\left(b^3+6b^2cx^2-2c^3x^6\right)}{4x^4}+3bc\log(x)(Ac+bB)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

```
[Out] (-(A*(b^3 + 6*b^2*c*x^2 - 2*c^3*x^6)) + B*x^2*(-2*b^3 + 6*b*c^2*x^4 + c^3*x
^6))/(4*x^4) + 3*b*c*(b*B + A*c)*Log[x]
```

fricas [A] time = 0.69, size = 76, normalized size = 1.06

$$\frac{Bc^3x^8+2\left(3\,Bbc^2+Ac^3\right)x^6+12\left(Bb^2c+Abc^2\right)x^4\log(x)-Ab^3-2\left(Bb^3+3\,Ab^2c\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^8 + 2*(3*B*b*c^2 + A*c^3)*x^6 + 12*(B*b^2*c + A*b*c^2)*x^4*log
(x) - A*b^3 - 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

giac [A] time = 0.15, size = 98, normalized size = 1.36

$$\frac{1}{4}\,Bc^3x^4+\frac{3}{2}\,Bbc^2x^2+\frac{1}{2}\,Ac^3x^2+\frac{3}{2}\left(Bb^2c+Abc^2\right)\log\left(x^2\right)-\frac{9\,Bb^2cx^4+9\,Abc^2x^4+2\,Bb^3x^2+6\,Ab^2cx^2+Ab^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/4*B*c^3*x^4 + 3/2*B*b*c^2*x^2 + 1/2*A*c^3*x^2 + 3/2*(B*b^2*c + A*b*c^2)*l
og(x^2) - 1/4*(9*B*b^2*c*x^4 + 9*A*b*c^2*x^4 + 2*B*b^3*x^2 + 6*A*b^2*c*x^2
+ A*b^3)/x^4
```

maple [A] time = 0.05, size = 76, normalized size = 1.06

$$\frac{B\,c^3x^4}{4}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+3Ab\,c^2\ln(x)+3B\,b^2c\ln(x)-\frac{3A\,b^2c}{2x^2}-\frac{B\,b^3}{2x^2}-\frac{A\,b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x)
```

```
[Out] 1/4*B*c^3*x^4+1/2*A*x^2*c^3+3/2*B*x^2*b*c^2-1/4*A*b^3/x^4-3/2*b^2/x^2*A*c-1
/2*b^3/x^2*B+3*A*ln(x)*b*c^2+3*B*ln(x)*b^2*c
```

maxima [A] time = 1.28, size = 76, normalized size = 1.06

$$\frac{1}{4}\,Bc^3x^4+\frac{1}{2}\left(3\,Bbc^2+Ac^3\right)x^2+\frac{3}{2}\left(Bb^2c+Abc^2\right)\log\left(x^2\right)-\frac{Ab^3+2\left(Bb^3+3\,Ab^2c\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^3*x^4 + 1/2*(3*B*b*c^2 + A*c^3)*x^2 + 3/2*(B*b^2*c + A*b*c^2)*log(x
^2) - 1/4*(A*b^3 + 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

mupad [B] time = 0.07, size = 76, normalized size = 1.06

$$\ln(x)\,\left(3\,B\,b^2\,c+3\,A\,b\,c^2\right)-\frac{\frac{A\,b^3}{4}+x^2\,\left(\frac{B\,b^3}{2}+\frac{3\,A\,c\,b^2}{2}\right)}{x^4}+x^2\,\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right)+\frac{B\,c^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x)
```

```
[Out] log(x)*(3*A*b*c^2 + 3*B*b^2*c) - ((A*b^3)/4 + x^2*((B*b^3)/2 + (3*A*b^2*c)/
2))/x^4 + x^2*((A*c^3)/2 + (3*B*b*c^2)/2) + (B*c^3*x^4)/4
```

sympy [A] time = 0.59, size = 75, normalized size = 1.04

$$\frac{Bc^3x^4}{4} + 3bc\,(Ac + Bb)\log{(x)} + x^2\left(\frac{Ac^3}{2} + \frac{3Bbc^2}{2}\right) + \frac{-Ab^3 + x^2\left(-6Ab^2c - 2Bb^3\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] B*c**3*x**4/4 + 3*b*c*(A*c + B*b)*log(x) + x**2*(A*c**3/2 + 3*B*b*c**2/2) +
 (-A*b**3 + x**2*(-6*A*b**2*c - 2*B*b**3))/(4*x**4)
```

3.35 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} \, dx$

Optimal. Leaf size=68

$$-\frac{Ab^3}{5x^5}-\frac{b^2(3Ac+bB)}{3x^3}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3$$

```
[Out] -1/5*A*b^3/x^5-1/3*b^2*(3*A*c+B*b)/x^3-3*b*c*(A*c+B*b)/x+c^2*(A*c+3*B*b)*x+
1/3*B*c^3*x^3
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{3x^3}-\frac{Ab^3}{5x^5}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

```
[Out] -(A*b^3)/(5*x^5) - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{12}}\,dx &= \int \frac{\left(A+Bx^2\right)\left(b+cx^2\right)^3}{x^6}\,dx\\
&= \int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^6}+\frac{b^2(bB+3Ac)}{x^4}+\frac{3bc(bB+Ac)}{x^2}+Bc^3x^2\right)dx\\
&= -\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3
\end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 1.00

$$-\frac{Ab^3}{5x^5}-\frac{b^2(3Ac+bB)}{3x^3}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

```
[Out] -1/5*(A*b^3)/x^5 - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

fricas [A] time = 0.93, size = 75, normalized size = 1.10

$$\frac{5\,Bc^3x^8 + 15\left(3\,Bbc^2 + Ac^3\right)x^6 - 45\left(Bb^2c + Abc^2\right)x^4 - 3\,Ab^3 - 5\left(Bb^3 + 3\,Ab^2c\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/15*(5*B*c^3*x^8 + 15*(3*B*b*c^2 + A*c^3)*x^6 - 45*(B*b^2*c + A*b*c^2)*x^4
 - 3*A*b^3 - 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

giac [A] time = 0.16, size = 75, normalized size = 1.10

$$\frac{1}{3}\,Bc^3x^3 + 3\,Bbc^2x + Ac^3x - \frac{45\,Bb^2cx^4 + 45\,Abc^2x^4 + 5\,Bb^3x^2 + 15\,Ab^2cx^2 + 3\,Ab^3}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
```

```
[Out] 1/3*B*c^3*x^3 + 3*B*b*c^2*x + A*c^3*x - 1/15*(45*B*b^2*c*x^4 + 45*A*b*c^2*x
^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 3*A*b^3)/x^5
```

maple [A] time = 0.05, size = 64, normalized size = 0.94

$$\frac{B\,c^3x^3}{3} + A\,c^3x + 3Bb\,c^2x - \frac{3\,(Ac + bB)\,bc}{x} - \frac{A\,b^3}{5x^5} - \frac{(3Ac + bB)\,b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x)
```

```
[Out] 1/3*B*c^3*x^3+A*c^3*x+3*B*b*c^2*x-1/5*A*b^3/x^5-1/3*b^2*(3*A*c+B*b)/x^3-3*b
*c*(A*c+B*b)/x
```

maxima [A] time = 1.33, size = 73, normalized size = 1.07

$$\frac{1}{3}\,Bc^3x^3 + \left(3\,Bbc^2 + Ac^3\right)x - \frac{45\left(Bb^2c + Abc^2\right)x^4 + 3\,Ab^3 + 5\left(Bb^3 + 3\,Ab^2c\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^3*x^3 + (3*B*b*c^2 + A*c^3)*x - 1/15*(45*(B*b^2*c + A*b*c^2)*x^4 +
3*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

mupad [B] time = 0.06, size = 73, normalized size = 1.07

$$x\,\left(A\,c^3 + 3\,B\,b\,c^2\right) - \frac{x^4\,\left(3\,B\,b^2\,c + 3\,A\,b\,c^2\right) + \frac{A\,b^3}{5} + x^2\,\left(\frac{B\,b^3}{3} + A\,c\,b^2\right)}{x^5} + \frac{B\,c^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x)
```

```
[Out] x*(A*c^3 + 3*B*b*c^2) - (x^4*(3*A*b*c^2 + 3*B*b^2*c) + (A*b^3)/5 + x^2*((B*
b^3)/3 + A*b^2*c))/x^5 + (B*c^3*x^3)/3
```

sympy [A] time = 0.68, size = 78, normalized size = 1.15

$$\frac{Bc^3x^3}{3} + x\left(Ac^3 + 3Bbc^2\right) + \frac{-3Ab^3 + x^4\left(-45Abc^2 - 45Bb^2c\right) + x^2\left(-15Ab^2c - 5Bb^3\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] B*c**3*x**3/3 + x*(A*c**3 + 3*B*b*c**2) + (-3*A*b**3 + x**4*(-45*A*b*c**2 -
 45*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(15*x**5)
```

3.36 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}} dx$

Optimal. Leaf size=71

$$-\frac{Ab^3}{6x^6}-\frac{b^2(3Ac+bB)}{4x^4}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

```
[Out] -1/6*A*b^3/x^6-1/4*b^2*(3*A*c+B*b)/x^4-3/2*b*c*(A*c+B*b)/x^2+1/2*B*c^3*x^2+
c^2*(A*c+3*B*b)*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 76}

$$-\frac{b^2(3Ac+bB)}{4x^4}-\frac{Ab^3}{6x^6}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -(A*b^3)/(6*x^6) - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^7}dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^4}dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(Bc^3+\frac{Ab^3}{x^4}+\frac{b^2(bB+3Ac)}{x^3}+\frac{3bc(bB+Ac)}{x^2}+\frac{c^2(3bB+Ac)}{x}\right)dx,\right.\\
&= -\frac{Ab^3}{6x^6}-\frac{b^2(bB+3Ac)}{4x^4}-\frac{3bc(bB+Ac)}{2x^2}+\frac{1}{2}Bc^3x^2+c^2(3bB+Ac)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 1.00

$$-\frac{Ab^3}{6x^6}-\frac{b^2(3Ac+bB)}{4x^4}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

fricas [A] time = 0.91, size = 77, normalized size = 1.08

$$\frac{6\,Bc^3x^8+12\left(3\,Bbc^2+Ac^3\right)x^6\log(x)-18\left(Bb^2c+Abc^2\right)x^4-2\,Ab^3-3\left(Bb^3+3\,Ab^2c\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/12*(6*B*c^3*x^8 + 12*(3*B*b*c^2 + A*c^3)*x^6*log(x) - 18*(B*b^2*c + A*b*c
^2)*x^4 - 2*A*b^3 - 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

giac [A] time = 0.15, size = 99, normalized size = 1.39

$$\frac{1}{2}\,Bc^3x^2+\frac{1}{2}\left(3\,Bbc^2+Ac^3\right)\log\left(x^2\right)-\frac{33\,Bbc^2x^6+11\,Ac^3x^6+18\,Bb^2cx^4+18\,Abc^2x^4+3\,Bb^3x^2+9\,Ab^2cx^2+2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(33*B*b*c^2*x^6 + 1
1*A*c^3*x^6 + 18*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 3*B*b^3*x^2 + 9*A*b^2*c*x^2
 + 2*A*b^3)/x^6
```

maple [A] time = 0.05, size = 75, normalized size = 1.06

$$\frac{B\,c^3x^2}{2}+A\,c^3\ln(x)+3Bb\,c^2\ln(x)-\frac{3Ab\,c^2}{2x^2}-\frac{3B\,b^2c}{2x^2}-\frac{3A\,b^2c}{4x^4}-\frac{B\,b^3}{4x^4}-\frac{A\,b^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x)
```

```
[Out] 1/2*B*c^3*x^2-3/4*b^2/x^4*A*c-1/4*b^3/x^4*B-3/2*b*c^2/x^2*A-3/2*b^2*c/x^2*B
-1/6*A*b^3/x^6+A*ln(x)*c^3+3*B*ln(x)*b*c^2
```

maxima [A] time = 1.44, size = 77, normalized size = 1.08

$$\frac{1}{2}\,Bc^3x^2+\frac{1}{2}\left(3\,Bbc^2+Ac^3\right)\log\left(x^2\right)-\frac{18\left(Bb^2c+Abc^2\right)x^4+2\,Ab^3+3\left(Bb^3+3\,Ab^2c\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(18*(B*b^2*c + A*b*
c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

mupad [B] time = 0.09, size = 75, normalized size = 1.06

$$\ln(x)\,\left(A\,c^3+3\,B\,b\,c^2\right)-\frac{x^4\,\left(\frac{3\,B\,b^2\,c}{2}+\frac{3\,A\,b\,c^2}{2}\right)+\frac{A\,b^3}{6}+x^2\,\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)}{x^6}+\frac{B\,c^3\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x)
```

```
[Out] log(x)*(A*c^3 + 3*B*b*c^2) - (x^4*((3*A*b*c^2)/2 + (3*B*b^2*c)/2) + (A*b^3)
/6 + x^2*((B*b^3)/4 + (3*A*b^2*c)/4))/x^6 + (B*c^3*x^2)/2
```

sympy [A] time = 1.29, size = 78, normalized size = 1.10

$$\frac{Bc^3x^2}{2} + c^2\left(Ac + 3Bb\right)\log\left(x\right) + \frac{-2Ab^3 + x^4\left(-18Abc^2 - 18Bb^2c\right) + x^2\left(-9Ab^2c - 3Bb^3\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**13,x)
```

```
[Out] B*c**3*x**2/2 + c**2*(A*c + 3*B*b)*log(x) + (-2*A*b**3 + x**4*(-18*A*b*c**2
 - 18*B*b**2*c) + x**2*(-9*A*b**2*c - 3*B*b**3))/(12*x**6)
```

3.37 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx$

Optimal. Leaf size=66

$$-\frac{Ab^3}{7x^7}-\frac{b^2(3Ac+bB)}{5x^5}-\frac{c^2(Ac+3bB)}{x}-\frac{bc(Ac+bB)}{x^3}+Bc^3x$$

```
[Out] -1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x+
B*c^3*x
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{5x^5}-\frac{Ab^3}{7x^7}-\frac{c^2(Ac+3bB)}{x}-\frac{bc(Ac+bB)}{x^3}+Bc^3x$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

```
[Out] -(A*b^3)/(7*x^7) - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^8}\,dx\\
&= \int \left(Bc^3+\frac{Ab^3}{x^8}+\frac{b^2(bB+3Ac)}{x^6}+\frac{3bc(bB+Ac)}{x^4}+\frac{c^2(3bB+Ac)}{x^2}\right)dx\\
&= -\frac{Ab^3}{7x^7}-\frac{b^2(bB+3Ac)}{5x^5}-\frac{bc(bB+Ac)}{x^3}-\frac{c^2(3bB+Ac)}{x}+Bc^3x
\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 1.00

$$-\frac{Ab^3}{7x^7}-\frac{b^2(3Ac+bB)}{5x^5}-\frac{c^2(Ac+3bB)}{x}-\frac{bc(Ac+bB)}{x^3}+Bc^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

```
[Out] -1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

fricas [A] time = 0.70, size = 75, normalized size = 1.14

$$\frac{35\,Bc^3x^8-35\,\left(3\,Bbc^2+Ac^3\right)x^6-35\,\left(Bb^2c+Abc^2\right)x^4-5\,Ab^3-7\,\left(Bb^3+3\,Ab^2c\right)x^2}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] 1/35*(35*B*c^3*x^8 - 35*(3*B*b*c^2 + A*c^3)*x^6 - 35*(B*b^2*c + A*b*c^2)*x^
4 - 5*A*b^3 - 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

giac [A] time = 0.21, size = 77, normalized size = 1.17

$$Bc^3x-\frac{105\,Bbc^2x^6+35\,Ac^3x^6+35\,Bb^2cx^4+35\,Abc^2x^4+7\,Bb^3x^2+21\,Ab^2cx^2+5\,Ab^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
```

```
[Out] B*c^3*x - 1/35*(105*B*b*c^2*x^6 + 35*A*c^3*x^6 + 35*B*b^2*c*x^4 + 35*A*b*c^
2*x^4 + 7*B*b^3*x^2 + 21*A*b^2*c*x^2 + 5*A*b^3)/x^7
```

maple [A] time = 0.05, size = 63, normalized size = 0.95

$$B\,c^3x-\frac{(Ac+3bB)\,c^2}{x}-\frac{(Ac+bB)\,bc}{x^3}-\frac{A\,b^3}{7x^7}-\frac{(3Ac+bB)\,b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x)
```

```
[Out] -1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x+
B*c^3*x
```

maxima [A] time = 1.30, size = 73, normalized size = 1.11

$$Bc^3x-\frac{35\,\left(3\,Bbc^2+Ac^3\right)x^6+35\,\left(Bb^2c+Abc^2\right)x^4+5\,Ab^3+7\,\left(Bb^3+3\,Ab^2c\right)x^2}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] B*c^3*x - 1/35*(35*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4 + 5
*A*b^3 + 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

mupad [B] time = 0.08, size = 71, normalized size = 1.08

$$B\,c^3\,x-\frac{x^4\,\left(B\,b^2\,c+A\,b\,c^2\right)+\frac{A\,b^3}{7}+x^2\,\left(\frac{B\,b^3}{5}+\frac{3\,A\,c\,b^2}{5}\right)+x^6\,\left(A\,c^3+3\,B\,b\,c^2\right)}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x)
```

```
[Out] B*c^3*x - (x^4*(A*b*c^2 + B*b^2*c) + (A*b^3)/7 + x^2*((B*b^3)/5 + (3*A*b^2*
c)/5) + x^6*(A*c^3 + 3*B*b*c^2))/x^7
```

sympy [A] time = 1.50, size = 80, normalized size = 1.21

$$Bc^3x+\frac{-5Ab^3+x^6\left(-35Ac^3-105Bbc^2\right)+x^4\left(-35Abc^2-35Bb^2c\right)+x^2\left(-21Ab^2c-7Bb^3\right)}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**14,x)
```

```
[Out] B*c**3*x + (-5*A*b**3 + x**6*(-35*A*c**3 - 105*B*b*c**2) + x**4*(-35*A*b*c*
*2 - 35*B*b**2*c) + x**2*(-21*A*b**2*c - 7*B*b**3))/(35*x**7)
```

3.38 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx$

Optimal. Leaf size=63

$$-\frac{A(b+cx^2)^4}{8bx^8}-\frac{b^3B}{6x^6}-\frac{3b^2Bc}{4x^4}-\frac{3bBc^2}{2x^2}+Bc^3\log(x)$$

```
[Out] -1/6*b^3*B/x^6-3/4*b^2*B*c/x^4-3/2*b*B*c^2/x^2-1/8*A*(c*x^2+b)^4/b/x^8+B*c^
3*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 78, 43}

$$-\frac{A(b+cx^2)^4}{8bx^8}-\frac{3b^2Bc}{4x^4}-\frac{b^3B}{6x^6}-\frac{3bBc^2}{2x^2}+Bc^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

```
[Out] -(b^3*B)/(6*x^6) - (3*b^2*B*c)/(4*x^4) - (3*b*B*c^2)/(2*x^2) - (A*(b + c*x^
2)^4)/(8*b*x^8) + B*c^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x^9}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{A(b+cx^2)^4}{8bx^8} + \frac{1}{2}B\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{A(b+cx^2)^4}{8bx^8} + \frac{1}{2}B\operatorname{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx, x, x^2\right)$$
$$= -\frac{b^3B}{6x^6} - \frac{3b^2Bc}{4x^4} - \frac{3bBc^2}{2x^2} - \frac{A(b+cx^2)^4}{8bx^8} + Bc^3\log(x)$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.22

$$Bc^3\log(x) - \frac{3A(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)+2bBx^2(2b^2+9bcx^2+18c^2x^4)}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

```
[Out] -1/24*(2*b*B*x^2*(2*b^2 + 9*b*c*x^2 + 18*c^2*x^4) + 3*A*(b^3 + 4*b^2*c*x^2
+ 6*b*c^2*x^4 + 4*c^3*x^6))/x^8 + B*c^3*Log[x]
```

fricas [A] time = 0.83, size = 77, normalized size = 1.22

$$\frac{24\,Bc^3x^8\log(x) - 12\,(3\,Bbc^2 + Ac^3)x^6 - 18\,(Bb^2c + Abc^2)x^4 - 3\,Ab^3 - 4\,(Bb^3 + 3\,Ab^2c)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] 1/24*(24*B*c^3*x^8*log(x) - 12*(3*B*b*c^2 + A*c^3)*x^6 - 18*(B*b^2*c + A*b*
c^2)*x^4 - 3*A*b^3 - 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

giac [A] time = 0.17, size = 90, normalized size = 1.43

$$\frac{1}{2}Bc^3\log(x^2) - \frac{25\,Bc^3x^8 + 36\,Bbc^2x^6 + 12\,Ac^3x^6 + 18\,Bb^2cx^4 + 18\,Abc^2x^4 + 4\,Bb^3x^2 + 12\,Ab^2cx^2 + 3\,Ab^3}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
```

```
[Out] 1/2*B*c^3*log(x^2) - 1/24*(25*B*c^3*x^8 + 36*B*b*c^2*x^6 + 12*A*c^3*x^6 + 1
8*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 4*B*b^3*x^2 + 12*A*b^2*c*x^2 + 3*A*b^3)/x^
8
```

maple [A] time = 0.05, size = 76, normalized size = 1.21

$$B\,c^3\ln(x) - \frac{A\,c^3}{2x^2} - \frac{3Bb\,c^2}{2x^2} - \frac{3Ab\,c^2}{4x^4} - \frac{3B\,b^2c}{4x^4} - \frac{A\,b^2c}{2x^6} - \frac{B\,b^3}{6x^6} - \frac{A\,b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x)
```

```
[Out] -3/4*b*c^2/x^4*A-3/4*b^2*B*c/x^4-1/8*A*b^3/x^8-1/2*c^3/x^2*A-3/2*b*B*c^2/x^
2-1/2*b^2/x^6*A*c-1/6*b^3*B/x^6+B*c^3*ln(x)
```

maxima [A] time = 1.32, size = 77, normalized size = 1.22

$$\frac{1}{2}Bc^3\log\left(x^2\right) - \frac{12\left(3Bbc^2 + Ac^3\right)x^6 + 18\left(Bb^2c + Abc^2\right)x^4 + 3Ab^3 + 4\left(Bb^3 + 3Ab^2c\right)x^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^3*log(x^2) - 1/24*(12*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c
^2)*x^4 + 3*A*b^3 + 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

mupad [B] time = 0.10, size = 75, normalized size = 1.19

$$B\,c^3\,\ln(x) - \frac{x^4\left(\frac{3\,B\,b^2\,c}{4} + \frac{3\,A\,b\,c^2}{4}\right) + \frac{A\,b^3}{8} + x^2\left(\frac{B\,b^3}{6} + \frac{A\,c\,b^2}{2}\right) + x^6\left(\frac{A\,c^3}{2} + \frac{3\,B\,b\,c^2}{2}\right)}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x)
```

```
[Out] B*c^3*log(x) - (x^4*((3*A*b*c^2)/4 + (3*B*b^2*c)/4) + (A*b^3)/8 + x^2*((B*b
^3)/6 + (A*b^2*c)/2) + x^6*((A*c^3)/2 + (3*B*b*c^2)/2))/x^8
```

sympy [A] time = 2.37, size = 82, normalized size = 1.30

$$Bc^3\log(x) + \frac{-3Ab^3 + x^6\left(-12Ac^3 - 36Bbc^2\right) + x^4\left(-18Abc^2 - 18Bb^2c\right) + x^2\left(-12Ab^2c - 4Bb^3\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**15,x)
```

```
[Out] B*c**3*log(x) + (-3*A*b**3 + x**6*(-12*A*c**3 - 36*B*b*c**2) + x**4*(-18*A*
b*c**2 - 18*B*b**2*c) + x**2*(-12*A*b**2*c - 4*B*b**3))/(24*x**8)
```

3.39 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}} \, dx$

Optimal. Leaf size=73

$$-\frac{Ab^3}{9x^9}-\frac{b^2(3Ac+bB)}{7x^7}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

```
[Out] -1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+3
*B*b)/x^3-B*c^3/x
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$-\frac{b^2(3Ac+bB)}{7x^7}-\frac{Ab^3}{9x^9}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

```
[Out] -(A*b^3)/(9*x^9) - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^{10}} \, dx\\ &= \int \left(\frac{Ab^3}{x^{10}}+\frac{b^2(bB+3Ac)}{x^8}+\frac{3bc(bB+Ac)}{x^6}+\frac{c^2(3bB+Ac)}{x^4}+\frac{Bc^3}{x^2}\right) dx\\ &= -\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.00

$$-\frac{Ab^3}{9x^9}-\frac{b^2(3Ac+bB)}{7x^7}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

```
[Out] -1/9*(A*b^3)/x^9 - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

fricas [A] time = 0.82, size = 75, normalized size = 1.03

$$-\frac{315 \, B c^{3} x^{8} + 105 \, {\left(3 \, B b c^{2} + A c^{3}\right)} x^{6} + 189 \, {\left(B b^{2} c + A b c^{2}\right)} x^{4} + 35 \, A b^{3} + 45 \, {\left(B b^{3} + 3 \, A b^{2} c\right)} x^{2}}{315 \, x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

giac [A] time = 0.15, size = 79, normalized size = 1.08

$$-\frac{315 \, B c^{3} x^{8} + 315 \, B b c^{2} x^{6} + 105 \, A c^{3} x^{6} + 189 \, B b^{2} c x^{4} + 189 \, A b c^{2} x^{4} + 45 \, B b^{3} x^{2} + 135 \, A b^{2} c x^{2} + 35 \, A b^{3}}{315 \, x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 315*B*b*c^2*x^6 + 105*A*c^3*x^6 + 189*B*b^2*c*x^4 +
 189*A*b*c^2*x^4 + 45*B*b^3*x^2 + 135*A*b^2*c*x^2 + 35*A*b^3)/x^9
```

maple [A] time = 0.05, size = 66, normalized size = 0.90

$$-\frac{B\,c^{3}}{x}-\frac{\left(Ac+3bB\right)c^{2}}{3x^{3}}-\frac{3\left(Ac+bB\right)bc}{5x^{5}}-\frac{A\,b^{3}}{9x^{9}}-\frac{\left(3Ac+bB\right)b^{2}}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x)
```

```
[Out] -1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+3
*B*b)/x^3-B*c^3/x
```

maxima [A] time = 1.38, size = 75, normalized size = 1.03

$$-\frac{315 \, B c^{3} x^{8} + 105 \, {\left(3 \, B b c^{2} + A c^{3}\right)} x^{6} + 189 \, {\left(B b^{2} c + A b c^{2}\right)} x^{4} + 35 \, A b^{3} + 45 \, {\left(B b^{3} + 3 \, A b^{2} c\right)} x^{2}}{315 \, x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

mupad [B] time = 0.07, size = 74, normalized size = 1.01

$$-\frac{x^4\,\left(\frac{3\,B\,b^2\,c}{5}+\frac{3\,A\,b\,c^2}{5}\right)+\frac{A\,b^3}{9}+x^2\,\left(\frac{B\,b^3}{7}+\frac{3\,A\,c\,b^2}{7}\right)+x^6\,\left(\frac{A\,c^3}{3}+B\,b\,c^2\right)+B\,c^3\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x)
```

```
[Out] -(x^4*((3*A*b*c^2)/5 + (3*B*b^2*c)/5) + (A*b^3)/9 + x^2*((B*b^3)/7 + (3*A*b
^2*c)/7) + x^6*((A*c^3)/3 + B*b*c^2) + B*c^3*x^8)/x^9
```

sympy [A] time = 2.45, size = 83, normalized size = 1.14

$$\frac{-35Ab^{3} - 315Bc^{3}x^{8} + x^{6}\left(-105Ac^{3} - 315Bbc^{2}\right) + x^{4}\left(-189Abc^{2} - 189Bb^{2}c\right) + x^{2}\left(-135Ab^{2}c - 45Bb^{3}\right)}{315x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] (-35*A*b**3 - 315*B*c**3*x**8 + x**6*(-105*A*c**3 - 315*B*b*c**2) + x**4*(-
189*A*b*c**2 - 189*B*b**2*c) + x**2*(-135*A*b**2*c - 45*B*b**3))/(315*x**9)
```

3.40 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx$

Optimal. Leaf size=49

$$-\frac{(b+cx^2)^4(5bB-Ac)}{40b^2x^8}-\frac{A(b+cx^2)^4}{10bx^{10}}$$

```
[Out] -1/10*A*(c*x^2+b)^4/b/x^10-1/40*(-A*c+5*B*b)*(c*x^2+b)^4/b^2/x^8
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 446, 78, 37}

$$-\frac{(b+cx^2)^4(5bB-Ac)}{40b^2x^8}-\frac{A(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

```
[Out] -(A*(b + c*x^2)^4)/(10*b*x^10) - ((5*b*B - A*c)*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x^{11}}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^6}\,dx, x, x^2\right)$$
$$= -\frac{A(b+cx^2)^4}{10bx^{10}} + \frac{(5bB-Ac)\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx, x, x^2\right)}{10b}$$
$$= -\frac{A(b+cx^2)^4}{10bx^{10}} - \frac{(5bB-Ac)(b+cx^2)^4}{40b^2x^8}$$

Mathematica [**A**] time = 0.02, size = 78, normalized size = 1.59

$$-\frac{A(4b^3+15b^2cx^2+20bc^2x^4+10c^3x^6)+5Bx^2(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)}{40x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

```
[Out] -1/40*(5*B*x^2*(b^3 + 4*b^2*c*x^2 + 6*b*c^2*x^4 + 4*c^3*x^6) + A*(4*b^3 + 1
5*b^2*c*x^2 + 20*b*c^2*x^4 + 10*c^3*x^6))/x^10
```

fricas [**A**] time = 0.55, size = 75, normalized size = 1.53

$$-\frac{20\,Bc^3x^8+10\,(3\,Bbc^2+Ac^3)x^6+20\,(Bb^2c+Abc^2)x^4+4\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

giac [**A**] time = 0.16, size = 79, normalized size = 1.61

$$-\frac{20\,Bc^3x^8+30\,Bbc^2x^6+10\,Ac^3x^6+20\,Bb^2cx^4+20\,Abc^2x^4+5\,Bb^3x^2+15\,Ab^2cx^2+4\,Ab^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 30*B*b*c^2*x^6 + 10*A*c^3*x^6 + 20*B*b^2*c*x^4 + 20*A
*b*c^2*x^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 4*A*b^3)/x^10
```

maple [**A**] time = 0.05, size = 66, normalized size = 1.35

$$-\frac{B\,c^3}{2x^2}-\frac{(Ac+3bB)\,c^2}{4x^4}-\frac{(Ac+bB)\,bc}{2x^6}-\frac{A\,b^3}{10x^{10}}-\frac{(3Ac+bB)\,b^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x)
```

```
[Out] -1/4*c^2*(A*c+3*B*b)/x^4-1/8*b^2*(3*A*c+B*b)/x^8-1/2*B*c^3/x^2-1/2*b*c*(A*c
+B*b)/x^6-1/10*A*b^3/x^10
```

maxima [A] time = 1.33, size = 75, normalized size = 1.53

$$-\frac{20\,Bc^{3}x^{8}+10\,\left(3\,Bbc^{2}+Ac^{3}\right)x^{6}+20\,\left(Bb^{2}c+Abc^{2}\right)x^{4}+4\,Ab^{3}+5\,\left(Bb^{3}+3\,Ab^{2}c\right)x^{2}}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

mupad [B] time = 0.07, size = 76, normalized size = 1.55

$$-\frac{x^{4}\,\left(\frac{B\,b^{2}\,c}{2}+\frac{A\,b\,c^{2}}{2}\right)+\frac{A\,b^{3}}{10}+x^{2}\,\left(\frac{B\,b^{3}}{8}+\frac{3\,A\,c\,b^{2}}{8}\right)+x^{6}\,\left(\frac{A\,c^{3}}{4}+\frac{3\,B\,b\,c^{2}}{4}\right)+\frac{B\,c^{3}\,x^{8}}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x)
```

```
[Out] -(x^4*((A*b*c^2)/2 + (B*b^2*c)/2) + (A*b^3)/10 + x^2*((B*b^3)/8 + (3*A*b^2*
c)/8) + x^6*((A*c^3)/4 + (3*B*b*c^2)/4) + (B*c^3*x^8)/2)/x^10
```

sympy [A] time = 3.43, size = 83, normalized size = 1.69

$$\frac{-4Ab^{3}-20Bc^{3}x^{8}+x^{6}\left(-10Ac^{3}-30Bbc^{2}\right)+x^{4}\left(-20Abc^{2}-20Bb^{2}c\right)+x^{2}\left(-15Ab^{2}c-5Bb^{3}\right)}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] (-4*A*b**3 - 20*B*c**3*x**8 + x**6*(-10*A*c**3 - 30*B*b*c**2) + x**4*(-20*A
*b*c**2 - 20*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(40*x**10)
```

3.41 $\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=119

$$-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{11/2}}+\frac{b^3x(bB-Ac)}{c^5}-\frac{b^2x^3(bB-Ac)}{3c^4}+\frac{bx^5(bB-Ac)}{5c^3}-\frac{x^7(bB-Ac)}{7c^2}+\frac{Bx^9}{9c}$$

```
[Out] b^3*(-A*c+B*b)*x/c^5-1/3*b^2*(-A*c+B*b)*x^3/c^4+1/5*b*(-A*c+B*b)*x^5/c^3-1/
7*(-A*c+B*b)*x^7/c^2+1/9*B*x^9/c-b^(7/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2
))/c^(11/2)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 302, 205}

$$-\frac{b^2x^3(bB-Ac)}{3c^4}+\frac{b^3x(bB-Ac)}{c^5}-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{11/2}}-\frac{x^7(bB-Ac)}{7c^2}+\frac{bx^5(bB-Ac)}{5c^3}+\frac{Bx^9}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) - ((b*B - A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B - A*
c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^8\left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$= \frac{Bx^9}{9c} - \frac{(9bB-9Ac)\int \frac{x^8}{b+cx^2}\,dx}{9c}$$
$$= \frac{Bx^9}{9c} - \frac{(9bB-9Ac)\int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4(b+cx^2)}\right)dx}{9c}$$
$$= \frac{b^3(bB-Ac)x}{c^5} - \frac{b^2(bB-Ac)x^3}{3c^4} + \frac{b(bB-Ac)x^5}{5c^3} - \frac{(bB-Ac)x^7}{7c^2} + \frac{Bx^9}{9c} - \frac{\left(b^4(bB-Ac)\right)\int}{c^5}$$
$$= \frac{b^3(bB-Ac)x}{c^5} - \frac{b^2(bB-Ac)x^3}{3c^4} + \frac{b(bB-Ac)x^5}{5c^3} - \frac{(bB-Ac)x^7}{7c^2} + \frac{Bx^9}{9c} - \frac{b^{7/2}(bB-Ac)\tan}{c^{11/2}}$$

Mathematica [A] time = 0.09, size = 119, normalized size = 1.00

$$-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{11/2}}+\frac{b^3x(bB-Ac)}{c^5}-\frac{b^2x^3(bB-Ac)}{3c^4}+\frac{bx^5(bB-Ac)}{5c^3}+\frac{x^7(Ac-bB)}{7c^2}+\frac{Bx^9}{9c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) + ((-(b*B) + A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B -
 A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

fricas [A] time = 0.83, size = 274, normalized size = 2.30

$$\left[\frac{70\,Bc^4x^9-90\left(Bbc^3-Ac^4\right)x^7+126\left(Bb^2c^2-Abc^3\right)x^5-210\left(Bb^3c-Ab^2c^2\right)x^3-315\left(Bb^4-Ab^3c\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2}{\cdots}\right.}{630\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/630*(70*B*c^4*x^9 - 90*(B*b*c^3 - A*c^4)*x^7 + 126*(B*b^2*c^2 - A*b*c^3)
*x^5 - 210*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(-b/c)*log
((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 630*(B*b^4 - A*b^3*c)*x)/c^5
, 1/315*(35*B*c^4*x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)
*x^5 - 105*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(b/c)*arct
an(c*x*sqrt(b/c)/b) + 315*(B*b^4 - A*b^3*c)*x)/c^5]
```

giac [A] time = 0.17, size = 133, normalized size = 1.12

$$-\frac{\left(Bb^5-Ab^4c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^5}+\frac{35\,Bc^8x^9-45\,Bbc^7x^7+45\,Ac^8x^7+63\,Bb^2c^6x^5-63\,Abc^7x^5-105\,Bb^3c^5x^3+105\,\cdots}{315\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^8*
x^9 - 45*B*b*c^7*x^7 + 45*A*c^8*x^7 + 63*B*b^2*c^6*x^5 - 63*A*b*c^7*x^5 - 1
```

```
05*B*b^3*c^5*x^3 + 105*A*b^2*c^6*x^3 + 315*B*b^4*c^4*x - 315*A*b^3*c^5*x)/c
^9
```

maple [A] time = 0.05, size = 140, normalized size = 1.18

$$\frac{B\,x^9}{9c}+\frac{A\,x^7}{7c}-\frac{Bb\,x^7}{7c^2}-\frac{Ab\,x^5}{5c^2}+\frac{B\,b^2x^5}{5c^3}+\frac{A\,b^2x^3}{3c^3}-\frac{B\,b^3x^3}{3c^4}+\frac{A\,b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^4}-\frac{B\,b^5\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^5}-\frac{A\,b^3x}{c^4}+\frac{B\,b^4x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/9*B*x^9/c+1/7/c*A*x^7-1/7/c^2*B*x^7*b-1/5/c^2*A*x^5*b+1/5/c^3*B*x^5*b^2+1
/3/c^3*A*x^3*b^2-1/3/c^4*B*x^3*b^3-1/c^4*A*b^3*x+1/c^5*B*b^4*x+b^4/c^4/(b*c
)^(1/2)*arctan(c*x/(b*c)^(1/2))*A-b^5/c^5/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2
))*B
```

maxima [A] time = 3.00, size = 124, normalized size = 1.04

$$-\frac{\left(Bb^5-Ab^4c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^5}+\frac{35\,Bc^4x^9-45\left(Bbc^3-Ac^4\right)x^7+63\left(Bb^2c^2-Abc^3\right)x^5-105\left(Bb^3c-Ab^2c^2\right)x^3+}{315\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^4*
x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)*x^5 - 105*(B*b^3*
c - A*b^2*c^2)*x^3 + 315*(B*b^4 - A*b^3*c)*x)/c^5
```

mupad [B] time = 0.17, size = 144, normalized size = 1.21

$$x^7\left(\frac{A}{7\,c}-\frac{B\,b}{7\,c^2}\right)+\frac{B\,x^9}{9\,c}+\frac{b^2\,x^3\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{3\,c^2}-\frac{b^{7/2}\,\mathrm{atan}\left(\frac{b^{7/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^5-A\,b^4\,c}\right)\,(A\,c-B\,b)}{c^{11/2}}-\frac{b\,x^5\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{5\,c}-\frac{b^3\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^7*(A/(7*c) - (B*b)/(7*c^2)) + (B*x^9)/(9*c) + (b^2*x^3*(A/c - (B*b)/c^2))
/(3*c^2) - (b^(7/2)*atan((b^(7/2)*c^(1/2)*x*(A*c - B*b))/(B*b^5 - A*b^4*c))
*(A*c - B*b))/c^(11/2) - (b*x^5*(A/c - (B*b)/c^2))/(5*c) - (b^3*x*(A/c - (B
*b)/c^2))/c^3
```

sympy [A] time = 0.44, size = 204, normalized size = 1.71

$$\frac{Bx^9}{9c}+x^7\left(\frac{A}{7c}-\frac{Bb}{7c^2}\right)+x^5\left(-\frac{Ab}{5c^2}+\frac{Bb^2}{5c^3}\right)+x^3\left(\frac{Ab^2}{3c^3}-\frac{Bb^3}{3c^4}\right)+x\left(-\frac{Ab^3}{c^4}+\frac{Bb^4}{c^5}\right)+\frac{\sqrt{-\frac{b^7}{c^{11}}}\,(-Ac+Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}}{-Ab^3}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**9/(9*c) + x**7*(A/(7*c) - B*b/(7*c**2)) + x**5*(-A*b/(5*c**2) + B*b**2
/(5*c**3)) + x**3*(A*b**2/(3*c**3) - B*b**3/(3*c**4)) + x*(-A*b**3/c**4 + B
*b**4/c**5) + sqrt(-b**7/c**11)*(-A*c + B*b)*log(-c**5*sqrt(-b**7/c**11)*(-
A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2 - sqrt(-b**7/c**11)*(-A*c + B*b)*log
(c**5*sqrt(-b**7/c**11)*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2
```

3.42 $\int \frac{x^9\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=96

$$\frac{b^3(bB-Ac)\log\left(b+cx^2\right)}{2c^5}-\frac{b^2x^2(bB-Ac)}{2c^4}+\frac{bx^4(bB-Ac)}{4c^3}-\frac{x^6(bB-Ac)}{6c^2}+\frac{Bx^8}{8c}$$

```
[Out] -1/2*b^2*(-A*c+B*b)*x^2/c^4+1/4*b*(-A*c+B*b)*x^4/c^3-1/6*(-A*c+B*b)*x^6/c^2
+1/8*B*x^8/c+1/2*b^3*(-A*c+B*b)*ln(c*x^2+b)/c^5
```

Rubi [A] time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b^2x^2(bB-Ac)}{2c^4}+\frac{b^3(bB-Ac)\log\left(b+cx^2\right)}{2c^5}-\frac{x^6(bB-Ac)}{6c^2}+\frac{bx^4(bB-Ac)}{4c^3}+\frac{Bx^8}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -(b^2*(b*B - A*c)*x^2)/(2*c^4) + (b*(b*B - A*c)*x^4)/(4*c^3) - ((b*B - A*c)
*x^6)/(6*c^2) + (B*x^8)/(8*c) + (b^3*(b*B - A*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^7\left(A+Bx^2\right)}{b+cx^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{b+cx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b^2(bB-Ac)}{c^4}+\frac{b(bB-Ac)x}{c^3}+\frac{(-bB+Ac)x^2}{c^2}+\frac{Bx^3}{c}+\frac{b^3(bB-Ac)}{c^4(b+cx)}\right)dx,x,\right.\\
&= -\frac{b^2(bB-Ac)x^2}{2c^4}+\frac{b(bB-Ac)x^4}{4c^3}-\frac{(bB-Ac)x^6}{6c^2}+\frac{Bx^8}{8c}+\frac{b^3(bB-Ac)\log\left(b+cx^2\right)}{2c^5}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.96

$$\frac{12b^3(bB-Ac)\log\left(b+cx^2\right)+cx^2\left(6b^2c\left(2A+Bx^2\right)-2bc^2x^2\left(3A+2Bx^2\right)+c^3x^4\left(4A+3Bx^2\right)-12b^3B\right)}{24c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-12*b^3*B + 6*b^2*c*(2*A + B*x^2) - 2*b*c^2*x^2*(3*A + 2*B*x^2) + c
^3*x^4*(4*A + 3*B*x^2)) + 12*b^3*(b*B - A*c)*Log[b + c*x^2])/(24*c^5)
```

fricas [A] time = 1.02, size = 98, normalized size = 1.02

$$\frac{3\,Bc^4x^8-4\left(Bbc^3-Ac^4\right)x^6+6\left(Bb^2c^2-Abc^3\right)x^4-12\left(Bb^3c-Ab^2c^2\right)x^2+12\left(Bb^4-Ab^3c\right)\log\left(cx^2+b\right)}{24\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*c^4*x^8 - 4*(B*b*c^3 - A*c^4)*x^6 + 6*(B*b^2*c^2 - A*b*c^3)*x^4 -
 12*(B*b^3*c - A*b^2*c^2)*x^2 + 12*(B*b^4 - A*b^3*c)*log(c*x^2 + b))/c^5
```

giac [A] time = 0.15, size = 101, normalized size = 1.05

$$\frac{3\,Bc^3x^8-4\,Bbc^2x^6+4\,Ac^3x^6+6\,Bb^2cx^4-6\,Abc^2x^4-12\,Bb^3x^2+12\,Ab^2cx^2}{24\,c^4}+\frac{\left(Bb^4-Ab^3c\right)\log\left(\left|cx^2+b\right|\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*B*b*c^2*x^6 + 4*A*c^3*x^6 + 6*B*b^2*c*x^4 - 6*A*b*c^2
*x^4 - 12*B*b^3*x^2 + 12*A*b^2*c*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(abs(c
*x^2 + b))/c^5
```

maple [A] time = 0.04, size = 110, normalized size = 1.15

$$\frac{B\,x^8}{8c}+\frac{A\,x^6}{6c}-\frac{Bb\,x^6}{6c^2}-\frac{Ab\,x^4}{4c^2}+\frac{B\,b^2x^4}{4c^3}+\frac{A\,b^2x^2}{2c^3}-\frac{B\,b^3x^2}{2c^4}-\frac{A\,b^3\ln\left(c\,x^2+b\right)}{2c^4}+\frac{B\,b^4\ln\left(c\,x^2+b\right)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/8*B*x^8/c+1/6/c*A*x^6-1/6/c^2*B*x^6*b-1/4/c^2*A*x^4*b+1/4/c^3*B*x^4*b^2+1
/2/c^3*A*x^2*b^2-1/2/c^4*B*x^2*b^3-1/2*b^3/c^4*ln(c*x^2+b)*A+1/2*b^4/c^5*ln
(c*x^2+b)*B
```

maxima [A] time = 1.38, size = 97, normalized size = 1.01

$$\frac{3\,Bc^3x^8-4\left(Bbc^2-Ac^3\right)x^6+6\left(Bb^2c-Abc^2\right)x^4-12\left(Bb^3-Ab^2c\right)x^2}{24\,c^4}+\frac{\left(Bb^4-Ab^3c\right)\log\left(cx^2+b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*(B*b*c^2 - A*c^3)*x^6 + 6*(B*b^2*c - A*b*c^2)*x^4 - 1
2*(B*b^3 - A*b^2*c)*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(c*x^2 + b)/c^5
```

mupad [B] time = 0.06, size = 100, normalized size = 1.04

$$x^6\left(\frac{A}{6\,c}-\frac{B\,b}{6\,c^2}\right)+\frac{B\,x^8}{8\,c}+\frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^4-A\,b^3\,c\right)}{2\,c^5}+\frac{b^2\,x^2\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{2\,c^2}-\frac{b\,x^4\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^6*(A/(6*c) - (B*b)/(6*c^2)) + (B*x^8)/(8*c) + (log(b + c*x^2)*(B*b^4 - A*
b^3*c))/(2*c^5) + (b^2*x^2*(A/c - (B*b)/c^2))/(2*c^2) - (b*x^4*(A/c - (B*b)
/c^2))/(4*c)
```

sympy [A] time = 0.35, size = 94, normalized size = 0.98

$$\frac{Bx^8}{8c}+\frac{b^3\left(-Ac+Bb\right)\log\left(b+cx^2\right)}{2c^5}+x^6\left(\frac{A}{6c}-\frac{Bb}{6c^2}\right)+x^4\left(-\frac{Ab}{4c^2}+\frac{Bb^2}{4c^3}\right)+x^2\left(\frac{Ab^2}{2c^3}-\frac{Bb^3}{2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**8/(8*c) + b**3*(-A*c + B*b)*log(b + c*x**2)/(2*c**5) + x**6*(A/(6*c) -
 B*b/(6*c**2)) + x**4*(-A*b/(4*c**2) + B*b**2/(4*c**3)) + x**2*(A*b**2/(2*c
**3) - B*b**3/(2*c**4))
```

3.43 $\int \frac{x^8\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=98

$$\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^2x(bB-Ac)}{c^4}+\frac{bx^3(bB-Ac)}{3c^3}-\frac{x^5(bB-Ac)}{5c^2}+\frac{Bx^7}{7c}$$

```
[Out] -b^2*(-A*c+B*b)*x/c^4+1/3*b*(-A*c+B*b)*x^3/c^3-1/5*(-A*c+B*b)*x^5/c^2+1/7*B
*x^7/c+b^(5/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 302, 205}

$$-\frac{b^2x(bB-Ac)}{c^4}+\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{x^5(bB-Ac)}{5c^2}+\frac{bx^3(bB-Ac)}{3c^3}+\frac{Bx^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) - ((b*B - A*c)*x^5
)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/c^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^8 \left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^6 \left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$= \frac{Bx^7}{7c} - \frac{(7bB-7Ac)\int \frac{x^6}{b+cx^2}\,dx}{7c}$$
$$= \frac{Bx^7}{7c} - \frac{(7bB-7Ac)\int \left(\frac{b^2}{c^3} - \frac{bx^2}{c^2} + \frac{x^4}{c} - \frac{b^3}{c^3\left(b+cx^2\right)}\right)dx}{7c}$$
$$= -\frac{b^2(bB-Ac)x}{c^4} + \frac{b(bB-Ac)x^3}{3c^3} - \frac{(bB-Ac)x^5}{5c^2} + \frac{Bx^7}{7c} + \frac{\left(b^3(bB-Ac)\right)\int \frac{1}{b+cx^2}\,dx}{c^4}$$
$$= -\frac{b^2(bB-Ac)x}{c^4} + \frac{b(bB-Ac)x^3}{3c^3} - \frac{(bB-Ac)x^5}{5c^2} + \frac{Bx^7}{7c} + \frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}$$

Mathematica [A] time = 0.07, size = 98, normalized size = 1.00

$$\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}} - \frac{b^2x(bB-Ac)}{c^4} + \frac{bx^3(bB-Ac)}{3c^3} + \frac{x^5(Ac-bB)}{5c^2} + \frac{Bx^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) + ((-(b*B) + A*c)*
x^5)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/c^(9/2)
```

fricas [A] time = 0.99, size = 228, normalized size = 2.33

$$\left[\frac{30\,Bc^3x^7 - 42\left(Bbc^2 - Ac^3\right)x^5 + 70\left(Bb^2c - Abc^2\right)x^3 - 105\left(Bb^3 - Ab^2c\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2 - 2\,cx\sqrt{-\frac{b}{c}} - b}{cx^2+b}\right) - 210\left(Bb^3 - \right.}{210\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*c^3*x^7 - 42*(B*b*c^2 - A*c^3)*x^5 + 70*(B*b^2*c - A*b*c^2)*x^
3 - 105*(B*b^3 - A*b^2*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*
x^2 + b)) - 210*(B*b^3 - A*b^2*c)*x)/c^4, 1/105*(15*B*c^3*x^7 - 21*(B*b*c^2
 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 + 105*(B*b^3 - A*b^2*c)*sqrt(b/c
)*arctan(c*x*sqrt(b/c)/b) - 105*(B*b^3 - A*b^2*c)*x)/c^4]
```

giac [A] time = 0.18, size = 108, normalized size = 1.10

$$\frac{\left(Bb^4 - Ab^3c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^4} + \frac{15\,Bc^6x^7 - 21\,Bbc^5x^5 + 21\,Ac^6x^5 + 35\,Bb^2c^4x^3 - 35\,Abc^5x^3 - 105\,Bb^3c^3x + 105\,Ab^2}{105\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^6*x
^7 - 21*B*b*c^5*x^5 + 21*A*c^6*x^5 + 35*B*b^2*c^4*x^3 - 35*A*b*c^5*x^3 - 10
5*B*b^3*c^3*x + 105*A*b^2*c^4*x)/c^7
```

maple [A] time = 0.05, size = 116, normalized size = 1.18

$$\frac{B\,x^{7}}{7c}+\frac{A\,x^{5}}{5c}-\frac{Bb\,x^{5}}{5c^{2}}-\frac{Ab\,x^{3}}{3c^{2}}+\frac{B\,b^{2}x^{3}}{3c^{3}}-\frac{A\,b^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^{3}}+\frac{B\,b^{4}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^{4}}+\frac{A\,b^{2}x}{c^{3}}-\frac{B\,b^{3}x}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/7*B*x^7/c+1/5/c*A*x^5-1/5/c^2*B*x^5*b-1/3/c^2*A*x^3*b+1/3/c^3*B*x^3*b^2+1
/c^3*A*b^2*x-1/c^4*B*b^3*x-b^3/c^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A+
b^4/c^4/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 2.94, size = 100, normalized size = 1.02

$$\frac{\left(Bb^{4}-Ab^{3}c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^{4}}+\frac{15\,Bc^{3}x^{7}-21\left(Bbc^{2}-Ac^{3}\right)x^{5}+35\left(Bb^{2}c-Abc^{2}\right)x^{3}-105\left(Bb^{3}-Ab^{2}c\right)x}{105\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^3*x
^7 - 21*(B*b*c^2 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 - 105*(B*b^3 - A
*b^2*c)*x)/c^4
```

mupad [B] time = 0.04, size = 118, normalized size = 1.20

$$x^{5}\left(\frac{A}{5\,c}-\frac{B\,b}{5\,c^{2}}\right)+\frac{B\,x^{7}}{7\,c}+\frac{b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^{4}-A\,b^{3}\,c}\right)\,(A\,c-B\,b)}{c^{9/2}}-\frac{b\,x^{3}\,\left(\frac{A}{c}-\frac{B\,b}{c^{2}}\right)}{3\,c}+\frac{b^{2}\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^{2}}\right)}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^5*(A/(5*c) - (B*b)/(5*c^2)) + (B*x^7)/(7*c) + (b^(5/2)*atan((b^(5/2)*c^(1
/2)*x*(A*c - B*b))/(B*b^4 - A*b^3*c))*(A*c - B*b))/c^(9/2) - (b*x^3*(A/c -
(B*b)/c^2))/(3*c) + (b^2*x*(A/c - (B*b)/c^2))/c^2
```

sympy [B] time = 0.41, size = 180, normalized size = 1.84

$$\frac{Bx^{7}}{7c}+x^{5}\left(\frac{A}{5c}-\frac{Bb}{5c^{2}}\right)+x^{3}\left(-\frac{Ab}{3c^{2}}+\frac{Bb^{2}}{3c^{3}}\right)+x\left(\frac{Ab^{2}}{c^{3}}-\frac{Bb^{3}}{c^{4}}\right)-\frac{\sqrt{-\frac{b^{5}}{c^{9}}}\,(-Ac+Bb)\log\left(-\frac{c^{4}\sqrt{-\frac{b^{5}}{c^{9}}}\,(-Ac+Bb)}{-Ab^{2}c+Bb^{3}}+x\right)}{2}+\frac{\sqrt{-\frac{b^{5}}{c^{9}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**7/(7*c) + x**5*(A/(5*c) - B*b/(5*c**2)) + x**3*(-A*b/(3*c**2) + B*b**2
/(3*c**3)) + x*(A*b**2/c**3 - B*b**3/c**4) - sqrt(-b**5/c**9)*(-A*c + B*b)*
log(-c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c + B*b**3) + x)/2 + sqrt(
-b**5/c**9)*(-A*c + B*b)*log(c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c
+ B*b**3) + x)/2
```

3.44 $\int \frac{x^7\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=75

$$-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}+\frac{bx^2(bB-Ac)}{2c^3}-\frac{x^4(bB-Ac)}{4c^2}+\frac{Bx^6}{6c}$$

```
[Out] 1/2*b*(-A*c+B*b)*x^2/c^3-1/4*(-A*c+B*b)*x^4/c^2+1/6*B*x^6/c-1/2*b^2*(-A*c+B
*b)*ln(c*x^2+b)/c^4
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}-\frac{x^4(bB-Ac)}{4c^2}+\frac{bx^2(bB-Ac)}{2c^3}+\frac{Bx^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x^2)/(2*c^3) - ((b*B - A*c)*x^4)/(4*c^2) + (B*x^6)/(6*c) - (
b^2*(b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^5\left(A+Bx^2\right)}{b+cx^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{b+cx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^3}+\frac{(-bB+Ac)x}{c^2}+\frac{Bx^2}{c}-\frac{b^2(bB-Ac)}{c^3(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{b(bB-Ac)x^2}{2c^3}-\frac{(bB-Ac)x^4}{4c^2}+\frac{Bx^6}{6c}-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 71, normalized size = 0.95

$$\frac{cx^2\left(-3bc\left(2A+Bx^2\right)+c^2x^2\left(3A+2Bx^2\right)+6b^2B\right)+6b^2(Ac-bB)\log\left(b+cx^2\right)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(6*b^2*B - 3*b*c*(2*A + B*x^2) + c^2*x^2*(3*A + 2*B*x^2)) + 6*b^2*(-
(b*B) + A*c)*Log[b + c*x^2])/(12*c^4)
```

fricas [**A**] time = 0.84, size = 75, normalized size = 1.00

$$\frac{2\,Bc^3x^6-3\left(Bbc^2-Ac^3\right)x^4+6\left(Bb^2c-Abc^2\right)x^2-6\left(Bb^3-Ab^2c\right)\log\left(cx^2+b\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^3*x^6 - 3*(B*b*c^2 - A*c^3)*x^4 + 6*(B*b^2*c - A*b*c^2)*x^2 - 6
*(B*b^3 - A*b^2*c)*log(c*x^2 + b))/c^4
```

giac [**A**] time = 0.15, size = 77, normalized size = 1.03

$$\frac{2\,Bc^2x^6-3\,Bbcx^4+3\,Ac^2x^4+6\,Bb^2x^2-6\,Abcx^2}{12\,c^3}-\frac{\left(Bb^3-Ab^2c\right)\log\left(\left|cx^2+b\right|\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*B*b*c*x^4 + 3*A*c^2*x^4 + 6*B*b^2*x^2 - 6*A*b*c*x^2)/
c^3 - 1/2*(B*b^3 - A*b^2*c)*log(abs(c*x^2 + b))/c^4
```

maple [**A**] time = 0.04, size = 86, normalized size = 1.15

$$\frac{B\,x^6}{6c}+\frac{A\,x^4}{4c}-\frac{Bb\,x^4}{4c^2}-\frac{Ab\,x^2}{2c^2}+\frac{B\,b^2x^2}{2c^3}+\frac{A\,b^2\ln\left(c\,x^2+b\right)}{2c^3}-\frac{B\,b^3\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/6*B*x^6/c+1/4/c*A*x^4-1/4/c^2*B*x^4*b-1/2/c^2*A*x^2*b+1/2/c^3*B*x^2*b^2+1
/2*b^2/c^3*ln(c*x^2+b)*A-1/2*b^3/c^4*ln(c*x^2+b)*B
```

maxima [**A**] time = 1.34, size = 74, normalized size = 0.99

$$\frac{2\,Bc^2x^6-3\left(Bbc-Ac^2\right)x^4+6\left(Bb^2-Abc\right)x^2}{12\,c^3}-\frac{\left(Bb^3-Ab^2c\right)\log\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*(B*b*c - A*c^2)*x^4 + 6*(B*b^2 - A*b*c)*x^2)/c^3 - 1/
2*(B*b^3 - A*b^2*c)*log(c*x^2 + b)/c^4
```

mupad [**B**] time = 0.09, size = 76, normalized size = 1.01

$$x^4\left(\frac{A}{4\,c}-\frac{B\,b}{4\,c^2}\right)+\frac{B\,x^6}{6\,c}-\frac{\ln\left(c\,x^2+b\right)\left(B\,b^3-A\,b^2\,c\right)}{2\,c^4}-\frac{b\,x^2\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^4*(A/(4*c) - (B*b)/(4*c^2)) + (B*x^6)/(6*c) - (log(b + c*x^2)*(B*b^3 - A*
b^2*c))/(2*c^4) - (b*x^2*(A/c - (B*b)/c^2))/(2*c)
```

sympy [**A**] time = 0.32, size = 70, normalized size = 0.93

$$\frac{Bx^6}{6c} - \frac{b^2\left(-Ac + Bb\right)\log\left(b + cx^2\right)}{2c^4} + x^4\left(\frac{A}{4c} - \frac{Bb}{4c^2}\right) + x^2\left(-\frac{Ab}{2c^2} + \frac{Bb^2}{2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**6/(6*c) - b**2*(-A*c + B*b)*log(b + c*x**2)/(2*c**4) + x**4*(A/(4*c) -
 B*b/(4*c**2)) + x**2*(-A*b/(2*c**2) + B*b**2/(2*c**3))
```

3.45 $\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=77

$$-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{bx(bB-Ac)}{c^3}-\frac{x^3(bB-Ac)}{3c^2}+\frac{Bx^5}{5c}$$

```
[Out] b*(-A*c+B*b)*x/c^3-1/3*(-A*c+B*b)*x^3/c^2+1/5*B*x^5/c-b^(3/2)*(-A*c+B*b)*ar
ctan(x*c^(1/2)/b^(1/2))/c^(7/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 302, 205}

$$-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}-\frac{x^3(bB-Ac)}{3c^2}+\frac{bx(bB-Ac)}{c^3}+\frac{Bx^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x)/c^3 - ((b*B - A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(3/2
)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6 \left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^4 \left(A+Bx^2\right)}{b+cx^2}\,dx \\
&= \frac{Bx^5}{5c} - \frac{(5bB-5Ac)\int \frac{x^4}{b+cx^2}\,dx}{5c} \\
&= \frac{Bx^5}{5c} - \frac{(5bB-5Ac)\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right) dx}{5c} \\
&= \frac{b(bB-Ac)x}{c^3} - \frac{(bB-Ac)x^3}{3c^2} + \frac{Bx^5}{5c} - \frac{\left(b^2(bB-Ac)\right)\int \frac{1}{b+cx^2}\,dx}{c^3} \\
&= \frac{b(bB-Ac)x}{c^3} - \frac{(bB-Ac)x^3}{3c^2} + \frac{Bx^5}{5c} - \frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.00

$$
-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}} + \frac{bx(bB-Ac)}{c^3} + \frac{x^3(Ac-bB)}{3c^2} + \frac{Bx^5}{5c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x)/c^3 + ((-(b*B) + A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(
3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

fricas [A] time = 0.96, size = 178, normalized size = 2.31

$$
\left[\frac{6\,Bc^2x^5 - 10\left(Bbc-Ac^2\right)x^3 - 15\left(Bb^2-Abc\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right) + 30\left(Bb^2-Abc\right)x}{30\,c^3}, \frac{3\,Bc^2x^5 - 5\left(Bbc-A\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*c^2*x^5 - 10*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(B*b^2 - A*b*c)*x)/c^
3, 1/15*(3*B*c^2*x^5 - 5*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(b/c)
*arctan(c*x*sqrt(b/c)/b) + 15*(B*b^2 - A*b*c)*x)/c^3]
```

giac [A] time = 0.18, size = 85, normalized size = 1.10

$$
-\frac{\left(Bb^3-Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3} + \frac{3\,Bc^4x^5 - 5\,Bbc^3x^3 + 5\,Ac^4x^3 + 15\,Bb^2c^2x - 15\,Abc^3x}{15\,c^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^4*x^
5 - 5*B*b*c^3*x^3 + 5*A*c^4*x^3 + 15*B*b^2*c^2*x - 15*A*b*c^3*x)/c^5
```

maple [A] time = 0.05, size = 92, normalized size = 1.19

$$\frac{B\,x^5}{5c}+\frac{A\,x^3}{3c}-\frac{Bb\,x^3}{3c^2}+\frac{A\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^2}-\frac{B\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^3}-\frac{Abx}{c^2}+\frac{B\,b^2x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/5*B*x^5/c+1/3/c*A*x^3-1/3/c^2*B*x^3*b-1/c^2*A*b*x+1/c^3*B*b^2*x+b^2/c^2/(
b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A-b^3/c^3/(b*c)^(1/2)*arctan(1/(b*c)^(
1/2)*c*x)*B
```

maxima [A] time = 2.86, size = 78, normalized size = 1.01

$$-\frac{\left(Bb^3-Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3}+\frac{3\,Bc^2x^5-5\left(Bbc-Ac^2\right)x^3+15\left(Bb^2-Abc\right)x}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^2*x^
5 - 5*(B*b*c - A*c^2)*x^3 + 15*(B*b^2 - A*b*c)*x)/c^3
```

mupad [B] time = 0.07, size = 96, normalized size = 1.25

$$x^3\left(\frac{A}{3\,c}-\frac{B\,b}{3\,c^2}\right)+\frac{B\,x^5}{5\,c}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^3-A\,b^2\,c}\right)\,(A\,c-B\,b)}{c^{7/2}}-\frac{b\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^3*(A/(3*c) - (B*b)/(3*c^2)) + (B*x^5)/(5*c) - (b^(3/2)*atan((b^(3/2)*c^(1
/2)*x*(A*c - B*b))/(B*b^3 - A*b^2*c))*(A*c - B*b))/c^(7/2) - (b*x*(A/c - (B
*b)/c^2))/c
```

sympy [B] time = 0.37, size = 153, normalized size = 1.99

$$\frac{Bx^5}{5c}+x^3\left(\frac{A}{3c}-\frac{Bb}{3c^2}\right)+x\left(-\frac{Ab}{c^2}+\frac{Bb^2}{c^3}\right)+\frac{\sqrt{-\frac{b^3}{c^7}}\,(-Ac+Bb)\log\left(-\frac{c^3\sqrt{-\frac{b^3}{c^7}}\,(-Ac+Bb)}{-Abc+Bb^2}+x\right)}{2}-\frac{\sqrt{-\frac{b^3}{c^7}}\,(-Ac+Bb)\log\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**5/(5*c) + x**3*(A/(3*c) - B*b/(3*c**2)) + x*(-A*b/c**2 + B*b**2/c**3)
+ sqrt(-b**3/c**7)*(-A*c + B*b)*log(-c**3*sqrt(-b**3/c**7)*(-A*c + B*b)/(-A
*b*c + B*b**2) + x)/2 - sqrt(-b**3/c**7)*(-A*c + B*b)*log(c**3*sqrt(-b**3/c
**7)*(-A*c + B*b)/(-A*b*c + B*b**2) + x)/2
```

3.46 $\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=54

$$\frac{b(bB-Ac)\log\left(b+cx^2\right)}{2c^3}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

```
[Out] -1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/2*b*(-A*c+B*b)*ln(c*x^2+b)/c^3
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{x^2(bB-Ac)}{2c^2}+\frac{b(bB-Ac)\log\left(b+cx^2\right)}{2c^3}+\frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + (b*(b*B - A*c)*Log[b + c*x^2])
/(2*c^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^3\left(A+Bx^2\right)}{b+cx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{b+cx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-bB+Ac}{c^2}+\frac{Bx}{c}+\frac{b(bB-Ac)}{c^2(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{b(bB-Ac)\log\left(b+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.87

$$\frac{cx^2\left(2Ac-2bB+Bcx^2\right)+2b(bB-Ac)\log\left(b+cx^2\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b*B + 2*A*c + B*c*x^2) + 2*b*(b*B - A*c)*Log[b + c*x^2])/(4*c^3)
```

fricas [A] time = 0.85, size = 51, normalized size = 0.94

$$\frac{Bc^2x^4-2\left(Bbc-Ac^2\right)x^2+2\left(Bb^2-Abc\right)\log\left(cx^2+b\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^2*x^4 - 2*(B*b*c - A*c^2)*x^2 + 2*(B*b^2 - A*b*c)*log(c*x^2 + b))/
c^3
```

giac [A] time = 0.15, size = 52, normalized size = 0.96

$$\frac{Bcx^4-2\,Bbx^2+2\,Acx^2}{4\,c^2}+\frac{\left(Bb^2-Abc\right)\log\left(\left|cx^2+b\right|\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(abs(c*x
^2 + b))/c^3
```

maple [A] time = 0.05, size = 62, normalized size = 1.15

$$\frac{B\,x^4}{4c}+\frac{A\,x^2}{2c}-\frac{Bb\,x^2}{2c^2}-\frac{Ab\ln\left(c\,x^2+b\right)}{2c^2}+\frac{B\,b^2\ln\left(c\,x^2+b\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*B*x^4/c+1/2/c*A*x^2-1/2/c^2*B*x^2*b-1/2*b/c^2*ln(c*x^2+b)*A+1/2*b^2/c^3
*ln(c*x^2+b)*B
```

maxima [A] time = 1.33, size = 50, normalized size = 0.93

$$\frac{Bcx^4-2\,(Bb-Ac)x^2}{4\,c^2}+\frac{\left(Bb^2-Abc\right)\log\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*c*x^4 - 2*(B*b - A*c)*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(c*x^2 + b)/
c^3
```

mupad [B] time = 0.07, size = 52, normalized size = 0.96

$$x^2\left(\frac{A}{2\,c}-\frac{B\,b}{2\,c^2}\right)+\frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^2-A\,b\,c\right)}{2\,c^3}+\frac{B\,x^4}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^2*(A/(2*c) - (B*b)/(2*c^2)) + (log(b + c*x^2)*(B*b^2 - A*b*c))/(2*c^3) +
(B*x^4)/(4*c)
```

sympy [A] time = 0.29, size = 46, normalized size = 0.85

$$\frac{Bx^4}{4c} + \frac{b\left(-Ac + Bb\right)\log\left(b + cx^2\right)}{2c^3} + x^2\left(\frac{A}{2c} - \frac{Bb}{2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**4/(4*c) + b*(-A*c + B*b)*log(b + c*x**2)/(2*c**3) + x**2*(A/(2*c) - B*
b/(2*c**2))
```

3.47 $\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

```
[Out] -(-A*c+B*b)*x/c^2+1/3*B*x^3/c+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/
c^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 459, 321, 205}

$$-\frac{x(bB-Ac)}{c^2}+\frac{\sqrt{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^2\left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$= \frac{Bx^3}{3c} - \frac{(3bB-3Ac)\int \frac{x^2}{b+cx^2}\,dx}{3c}$$
$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{(b(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\sqrt{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.98

$$\frac{\sqrt{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}} + \frac{x(Ac-bB)}{c^2} + \frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

fricas [A] time = 0.93, size = 129, normalized size = 2.22

$$\left[\frac{2\,Bcx^3 - 3\,(Bb-Ac)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right) - 6\,(Bb-Ac)x}{6\,c^2},\ \frac{Bcx^3 + 3\,(Bb-Ac)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right) - 3\,(Bb-}{3\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*c*x^3 - 3*(B*b - A*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
b)/(c*x^2 + b)) - 6*(B*b - A*c)*x)/c^2, 1/3*(B*c*x^3 + 3*(B*b - A*c)*sqrt(b
/c)*arctan(c*x*sqrt(b/c)/b) - 3*(B*b - A*c)*x)/c^2]
```

giac [A] time = 0.16, size = 57, normalized size = 0.98

$$\frac{\left(Bb^2 - Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2} + \frac{Bc^2x^3 - 3\,Bbcx + 3\,Ac^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c^2*x^3 - 3*
B*b*c*x + 3*A*c^2*x)/c^3
```

maple [A] time = 0.05, size = 68, normalized size = 1.17

$$\frac{B\,x^3}{3c} - \frac{Ab\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c} + \frac{B\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^2} + \frac{Ax}{c} - \frac{Bbx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 1/3*B*x^3/c+1/c*A*x-1/c^2*b*B*x-b/c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A
+b^2/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 2.86, size = 53, normalized size = 0.91

$$\frac{\left(Bb^2 - Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2} + \frac{Bcx^3 - 3\,(Bb - Ac)x}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c*x^3 - 3*(B
*b - A*c)*x)/c^2
```

mupad [B] time = 0.11, size = 70, normalized size = 1.21

$$x\left(\frac{A}{c} - \frac{B\,b}{c^2}\right) + \frac{B\,x^3}{3\,c} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^2-A\,b\,c}\right)(A\,c - B\,b)}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (B*b)/c^2) + (B*x^3)/(3*c) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(A*c
 - B*b))/(B*b^2 - A*b*c))*(A*c - B*b))/c^(5/2)
```

sympy [A] time = 0.34, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3c} + x\left(\frac{A}{c} - \frac{Bb}{c^2}\right) - \frac{\sqrt{-\frac{b}{c^5}}\,(-Ac + Bb)\log\left(-c^2\sqrt{-\frac{b}{c^5}} + x\right)}{2} + \frac{\sqrt{-\frac{b}{c^5}}\,(-Ac + Bb)\log\left(c^2\sqrt{-\frac{b}{c^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**3/(3*c) + x*(A/c - B*b/c**2) - sqrt(-b/c**5)*(-A*c + B*b)*log(-c**2*sq
rt(-b/c**5) + x)/2 + sqrt(-b/c**5)*(-A*c + B*b)*log(c**2*sqrt(-b/c**5) + x)
/2
```

3.48 $\int \frac{x^3\left(A+Bx^2\right)}{bx^2+cx^4} \, dx$

Optimal. Leaf size=35

$$\frac{Bx^2}{2c} - \frac{(bB - Ac)\log\left(b + cx^2\right)}{2c^2}$$

```
[Out] 1/2*B*x^2/c-1/2*(-A*c+B*b)*ln(c*x^2+b)/c^2
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{Bx^2}{2c} - \frac{(bB - Ac)\log\left(b + cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b*B - A*c)*Log[b + c*x^2])/(2*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{bx^2+cx^4} \, dx &= \int \frac{x\left(A+Bx^2\right)}{b+cx^2} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{b+cx} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c} + \frac{-bB+Ac}{c(b+cx)}\right) dx, x, x^2\right) \\
&= \frac{Bx^2}{2c} - \frac{(bB-Ac)\log\left(b+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.89

$$\frac{(Ac - bB)\log\left(b + cx^2\right) + Bcx^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (B*c*x^2 + (-(b*B) + A*c)*Log[b + c*x^2])/(2*c^2)
```

fricas [A] time = 0.76, size = 30, normalized size = 0.86

$$\frac{Bcx^2 - (Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(B*c*x^2 - (B*b - A*c)*log(c*x^2 + b))/c^2
```

giac [A] time = 0.23, size = 32, normalized size = 0.91

$$\frac{Bx^2}{2\,c} - \frac{(Bb - Ac)\log\left(\left|cx^2 + b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/c^2
```

maple [A] time = 0.04, size = 40, normalized size = 1.14

$$\frac{B\,x^2}{2c} + \frac{A\ln\left(c\,x^2 + b\right)}{2c} - \frac{Bb\ln\left(c\,x^2 + b\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2),x)
[Out] 1/2*B*x^2/c+1/2/c*ln(c*x^2+b)*A-1/2/c^2*ln(c*x^2+b)*b*B
```

maxima [A] time = 1.30, size = 31, normalized size = 0.89

$$\frac{Bx^2}{2\,c} - \frac{(Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(c*x^2 + b)/c^2
```

mupad [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{B\,x^2}{2\,c} + \frac{\ln\left(c\,x^2 + b\right)\,(A\,c - B\,b)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] (B*x^2)/(2*c) + (log(b + c*x^2)*(A*c - B*b))/(2*c^2)
```

sympy [A] time = 0.25, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2c} - \frac{(-Ac + Bb)\log\left(b + cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**2/(2*c) - (-A*c + B*b)*log(b + c*x**2)/(2*c**2)
```

3.49 $\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=40

$$\frac{Bx}{c}-\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}$$

```
[Out] B*x/c-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(3/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 388, 205}

$$\frac{Bx}{c}-\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{A+Bx^2}{b+cx^2}\,dx\\ &= \frac{Bx}{c}-\frac{(bB-Ac)\int \frac{1}{b+cx^2}\,dx}{c}\\ &= \frac{Bx}{c}-\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{Bx}{c}-\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

fricas [A] time = 0.90, size = 99, normalized size = 2.48

$$\left[\frac{2\,Bbcx + (Bb - Ac)\sqrt{-bc}\,\log\left(\frac{cx^2 - 2\,\sqrt{-bc}\,x - b}{cx^2 + b}\right)}{2\,bc^2}, \frac{Bbcx - (Bb - Ac)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{bc^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] [1/2*(2*B*b*c*x + (B*b - A*c)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(
c*x^2 + b)))/(b*c^2), (B*b*c*x - (B*b - A*c)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b*c^2)]
```

giac [A] time = 0.16, size = 34, normalized size = 0.85

$$\frac{Bx}{c} - \frac{(Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

maple [A] time = 0.04, size = 45, normalized size = 1.12

$$\frac{A\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}} - \frac{Bb\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c} + \frac{Bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2),x)
[Out] B*x/c+1/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A-1/c/(b*c)^(1/2)*arctan(1/(b
*c)^(1/2)*c*x)*b*B
```

maxima [A] time = 2.92, size = 34, normalized size = 0.85

$$\frac{Bx}{c} - \frac{(Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

mupad [B] time = 0.05, size = 31, normalized size = 0.78

$$\frac{B\,x}{c} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c - B\,b)}{\sqrt{b}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (B*x)/c + (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(1/2)*c^(3/2))
```

sympy [B] time = 0.29, size = 82, normalized size = 2.05

$$\frac{Bx}{c} + \frac{\sqrt{-\frac{1}{bc^3}}\,(-Ac+Bb)\log\left(-bc\sqrt{-\frac{1}{bc^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{bc^3}}\,(-Ac+Bb)\log\left(bc\sqrt{-\frac{1}{bc^3}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x/c + sqrt(-1/(b*c**3))*(-A*c + B*b)*log(-b*c*sqrt(-1/(b*c**3)) + x)/2 -
sqrt(-1/(b*c**3))*(-A*c + B*b)*log(b*c*sqrt(-1/(b*c**3)) + x)/2
```

3.50 $\int \frac{x(A+Bx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=34

$$\frac{(bB-Ac)\log(b+cx^2)}{2bc}+\frac{A\log(x)}{b}$$

```
[Out] A*ln(x)/b+1/2*(-A*c+B*b)*ln(c*x^2+b)/b/c
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 72}

$$\frac{(bB-Ac)\log(b+cx^2)}{2bc}+\frac{A\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{A+Bx^2}{x(b+cx^2)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(b+cx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{bx}+\frac{bB-Ac}{b(b+cx)}\right)dx, x, x^2\right)\\
&= \frac{A\log(x)}{b}+\frac{(bB-Ac)\log(b+cx^2)}{2bc}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{(bB-Ac)\log(b+cx^2)}{2bc}+\frac{A\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

fricas [A] time = 0.99, size = 32, normalized size = 0.94

$$\frac{2\,Ac\log(x) + (Bb - Ac)\log\left(cx^2 + b\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(2*A*c*log(x) + (B*b - A*c)*log(c*x^2 + b))/(b*c)
```

giac [A] time = 0.20, size = 34, normalized size = 1.00

$$\frac{A\log(|x|)}{b} + \frac{(Bb - Ac)\log\left(\left|cx^2 + b\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] A*log(abs(x))/b + 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/(b*c)
```

maple [A] time = 0.05, size = 37, normalized size = 1.09

$$\frac{A\ln(x)}{b} - \frac{A\ln\left(c\,x^2 + b\right)}{2b} + \frac{B\ln\left(c\,x^2 + b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2),x)
[Out] -1/2/b*ln(c*x^2+b)*A+1/2/c*ln(c*x^2+b)*B+A*ln(x)/b
```

maxima [A] time = 1.34, size = 35, normalized size = 1.03

$$\frac{A\log\left(x^2\right)}{2\,b} + \frac{(Bb - Ac)\log\left(cx^2 + b\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*A*log(x^2)/b + 1/2*(B*b - A*c)*log(c*x^2 + b)/(b*c)
```

mupad [B] time = 0.08, size = 32, normalized size = 0.94

$$\frac{A\,\ln(x)}{b} - \frac{\ln\left(c\,x^2 + b\right)\,(A\,c - B\,b)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] (A*log(x))/b - (log(b + c*x^2)*(A*c - B*b))/(2*b*c)
```

sympy [A] time = 0.75, size = 26, normalized size = 0.76

$$\frac{A\log(x)}{b} + \frac{(-Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] A*log(x)/b + (-A*c + B*b)*log(b/c + x**2)/(2*b*c)
```

3.51 $\int \frac{A+Bx^2}{bx^2+cx^4} \, dx$

Optimal. Leaf size=42

$$\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

```
[Out] -A/b/x+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1593, 453, 205}

$$\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{bx^2+cx^4} \, dx &= \int \frac{A+Bx^2}{x^2\left(b+cx^2\right)} \, dx\\ &= -\frac{A}{bx}-\frac{(-bB+Ac)\int \frac{1}{b+cx^2} \, dx}{b}\\ &= -\frac{A}{bx}+\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 1.00

$$\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

fricas [A] time = 1.04, size = 105, normalized size = 2.50

$$\left[\frac{(Bb-Ac)\sqrt{-bc}\,x\log\left(\frac{cx^2+2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)-2\,Abc}{2\,b^2cx},\ \frac{(Bb-Ac)\sqrt{bc}\,x\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)-Abc}{b^2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b - A*c)*sqrt(-b*c)*x*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
) - 2*A*b*c)/(b^2*c*x), ((B*b - A*c)*sqrt(b*c)*x*arctan(sqrt(b*c)*x/b) - A*
b*c)/(b^2*c*x)]
```

giac [A] time = 0.16, size = 36, normalized size = 0.86

$$\frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

maple [A] time = 0.05, size = 48, normalized size = 1.14

$$-\frac{Ac\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;b}+\frac{B\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] -1/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A*c+1/(b*c)^(1/2)*arctan(1/(b*c)
^(1/2)*c*x)*B-A/b/x
```

maxima [A] time = 2.94, size = 36, normalized size = 0.86

$$\frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

mupad [B] time = 0.09, size = 35, normalized size = 0.83

$$-\frac{A}{b\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-B\,b\right)}{b^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4),x)
```

```
[Out] - A/(b*x) - (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(3/2)*c^(1/2))
```

sympy [B] time = 0.42, size = 82, normalized size = 1.95

$$-\frac{A}{bx}-\frac{\sqrt{-\frac{1}{b^3c}}\,(-Ac+Bb)\log\left(-b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{b^3c}}\,(-Ac+Bb)\log\left(b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(-1/(b**3*c))*(-A*c + B*b)*log(-b**2*sqrt(-1/(b**3*c)) + x)/
2 + sqrt(-1/(b**3*c))*(-A*c + B*b)*log(b**2*sqrt(-1/(b**3*c)) + x)/2
```

3.52 $\int \frac{A+Bx^2}{bx^2-cx^4} \, dx$

Optimal. Leaf size=41

$$\frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

```
[Out] -A/b/x+(A*c+B*b)*arctanh(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1593, 453, 208}

$$\frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{bx^2-cx^4}\,dx &= \int \frac{A+Bx^2}{x^2\left(b-cx^2\right)}\,dx\\ &= -\frac{A}{bx}+\frac{(bB+Ac)\int \frac{1}{b-cx^2}\,dx}{b}\\ &= -\frac{A}{bx}+\frac{(bB+Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

fricas [A] time = 0.82, size = 103, normalized size = 2.51

$$\left[\frac{(Bb+Ac)\sqrt{bc}\,x\log\left(\frac{cx^2+2\sqrt{bc}\,x+b}{cx^2-b}\right)-2\,Abc}{2\,b^2cx},-\frac{(Bb+Ac)\sqrt{-bc}\,x\arctan\left(\frac{\sqrt{-bc}\,x}{b}\right)+Abc}{b^2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b + A*c)*sqrt(b*c)*x*log((c*x^2 + 2*sqrt(b*c)*x + b)/(c*x^2 - b))
- 2*A*b*c)/(b^2*c*x), -((B*b + A*c)*sqrt(-b*c)*x*arctan(sqrt(-b*c)*x/b) + A
*b*c)/(b^2*c*x)]
```

giac [A] time = 0.15, size = 38, normalized size = 0.93

$$-\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{-bc}}\right)}{\sqrt{-bc}\,b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b + A*c)*arctan(c*x/sqrt(-b*c))/(sqrt(-b*c)*b) - A/(b*x)
```

maple [A] time = 0.05, size = 39, normalized size = 0.95

$$-\frac{(-Ac-bB)\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\ b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(-c*x^4+b*x^2),x)
```

```
[Out] -(-A*c-B*b)/b/(b*c)^(1/2)*arctanh(1/(b*c)^(1/2)*c*x)-A/b/x
```

maxima [A] time = 3.06, size = 51, normalized size = 1.24

$$-\frac{(Bb+Ac)\log\left(\frac{cx-\sqrt{bc}}{cx+\sqrt{bc}}\right)}{2\sqrt{bc}\,b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b + A*c)*log((c*x - sqrt(b*c))/(c*x + sqrt(b*c)))/(sqrt(b*c)*b) - A
/(b*x)
```

mupad [B] time = 0.15, size = 33, normalized size = 0.80

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)(A\,c+B\,b)}{b^{3/2}\sqrt{c}}-\frac{A}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 - c*x^4),x)
```

```
[Out] (atanh((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(b^(3/2)*c^(1/2)) - A/(b*x)
```

sympy [B] time = 0.44, size = 75, normalized size = 1.83

$$-\frac{A}{bx}-\frac{\sqrt{\frac{1}{b^3c}}\,(Ac+Bb)\log\left(-b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}+\frac{\sqrt{\frac{1}{b^3c}}\,(Ac+Bb)\log\left(b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(-c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(1/(b**3*c))*(A*c + B*b)*log(-b**2*sqrt(1/(b**3*c)) + x)/2 +
 sqrt(1/(b**3*c))*(A*c + B*b)*log(b**2*sqrt(1/(b**3*c)) + x)/2
```

3.53 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=49

$$-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

```
[Out] -1/2*A/b/x^2+(-A*c+B*b)*ln(x)/b^2-1/2*(-A*c+B*b)*ln(c*x^2+b)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*b*x^2) + ((b*B - A*c)*Log[x])/b^2 - ((b*B - A*c)*Log[b + c*x^2])/(2*b
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{A}{bx^2}+\frac{bB-Ac}{b^2x}-\frac{c(bB-Ac)}{b^2(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{(Ac-bB)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(b*x^2) + ((b*B - A*c)*Log[x])/b^2 + ((-(b*B) + A*c)*Log[b + c*x^2])
/(2*b^2)
```

fricas [A] time = 0.71, size = 47, normalized size = 0.96

$$-\frac{(Bb-Ac)x^2\log\left(cx^2+b\right)-2(Bb-Ac)x^2\log(x)+Ab}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*((B*b - A*c)*x^2*log(c*x^2 + b) - 2*(B*b - A*c)*x^2*log(x) + A*b)/(b^2
*x^2)
```

giac [A] time = 0.15, size = 71, normalized size = 1.45

$$\frac{(Bb-Ac)\log\left(x^2\right)}{2b^2}-\frac{\left(Bbc-Ac^2\right)\log\left(\left|cx^2+b\right|\right)}{2b^2c}-\frac{Bbx^2-Acx^2+Ab}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*(B*b*c - A*c^2)*log(abs(c*x^2 + b))/(b^2
*c) - 1/2*(B*b*x^2 - A*c*x^2 + A*b)/(b^2*x^2)
```

maple [A] time = 0.06, size = 56, normalized size = 1.14

$$-\frac{Ac\ln(x)}{b^2}+\frac{Ac\ln\left(cx^2+b\right)}{2b^2}+\frac{B\ln(x)}{b}-\frac{B\ln\left(cx^2+b\right)}{2b}-\frac{A}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2),x)
```

```
[Out] 1/2/b^2*ln(c*x^2+b)*A*c-1/2/b*ln(c*x^2+b)*B-1/2*A/b/x^2-1/b^2*ln(x)*A*c+1/b
*ln(x)*B
```

maxima [A] time = 1.32, size = 48, normalized size = 0.98

$$-\frac{(Bb-Ac)\log\left(cx^2+b\right)}{2b^2}+\frac{(Bb-Ac)\log\left(x^2\right)}{2b^2}-\frac{A}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*log(c*x^2 + b)/b^2 + 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*A/
(b*x^2)
```

mupad [B] time = 0.14, size = 46, normalized size = 0.94

$$\frac{\ln\left(cx^2+b\right)(Ac-Bb)}{2b^2}-\frac{A}{2bx^2}-\frac{\ln(x)(Ac-Bb)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(b + c*x^2)*(A*c - B*b))/(2*b^2) - A/(2*b*x^2) - (log(x)*(A*c - B*b))/b
^2
```

sympy [A] time = 0.76, size = 41, normalized size = 0.84

$$-\frac{A}{2bx^2} + \frac{(-Ac + Bb)\log(x)}{b^2} - \frac{(-Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2),x)
```

```
[Out] -A/(2*b*x**2) + (-A*c + B*b)*log(x)/b**2 - (-A*c + B*b)*log(b/c + x**2)/(2*
b**2)
```

3.54 $\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{bB-Ac}{b^2x}-\frac{A}{3bx^3}$$

```
[Out] -1/3*A/b/x^3+(A*c-B*b)/b^2/x-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b
^(5/2)
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 453, 325, 205}

$$-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{A}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(3*b*x^3) - (b*B - A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)}\,dx$$
$$= -\frac{A}{3bx^3} - \frac{(-3bB+3Ac)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{3b}$$
$$= -\frac{A}{3bx^3} - \frac{bB-Ac}{b^2x} - \frac{(c(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{A}{3bx^3} - \frac{bB-Ac}{b^2x} - \frac{\sqrt{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 0.98

$$-\frac{\sqrt{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}} + \frac{Ac-bB}{b^2x} - \frac{A}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*A/(b*x^3) + (-(b*B) + A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt
[c]*x)/Sqrt[b]])/b^(5/2)
```

fricas [A] time = 0.82, size = 135, normalized size = 2.21

$$\left[-\frac{3\,(Bb-Ac)x^3\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,(Bb-Ac)x^2+2\,Ab}{6\,b^2x^3}, -\frac{3\,(Bb-Ac)x^3\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,(Bb}{3\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*b - A*c)*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x
^2 + b)) + 6*(B*b - A*c)*x^2 + 2*A*b)/(b^2*x^3), -1/3*(3*(B*b - A*c)*x^3*sq
rt(c/b)*arctan(x*sqrt(c/b)) + 3*(B*b - A*c)*x^2 + A*b)/(b^2*x^3)]
```

giac [A] time = 0.16, size = 57, normalized size = 0.93

$$-\frac{\left(Bbc-Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2} - \frac{3\,Bbx^2-3\,Acx^2+Ab}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*B*b*x^2 - 3
*A*c*x^2 + A*b)/(b^2*x^3)
```

maple [A] time = 0.06, size = 72, normalized size = 1.18

$$\frac{A\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;b^2} - \frac{Bc\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;b} + \frac{Ac}{b^2x} - \frac{B}{bx} - \frac{A}{3b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2),x)
```

```
[Out] c^2/b^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A-c/b/(b*c)^(1/2)*arctan(1/(b
*c)^(1/2)*c*x)*B-1/3*A/b/x^3+1/b^2/x*A*c-1/b/x*B
```

maxima [A] time = 2.82, size = 56, normalized size = 0.92

$$-\frac{\left(Bbc - Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2} - \frac{3\,(Bb - Ac)x^2 + Ab}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*(B*b - A*c)
*x^2 + A*b)/(b^2*x^3)
```

mupad [B] time = 0.11, size = 53, normalized size = 0.87

$$\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)(A\,c - B\,b)}{b^{5/2}} - \frac{\frac{A}{3\,b} - \frac{x^2\,(A\,c - B\,b)}{b^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(5/2) - (A/(3*b) - (x^2*(
A*c - B*b))/b^2)/x^3
```

sympy [B] time = 0.44, size = 129, normalized size = 2.11

$$\frac{\sqrt{-\frac{c}{b^5}}\,(-Ac + Bb)\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)}{-Ac^2+Bbc} + x\right)}{2} - \frac{\sqrt{-\frac{c}{b^5}}\,(-Ac + Bb)\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)}{-Ac^2+Bbc} + x\right)}{2} + \frac{-Ab + x^2\,(3Ac - 3B}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**5)*(-A*c + B*b)*log(-b**3*sqrt(-c/b**5)*(-A*c + B*b)/(-A*c**2 +
B*b*c) + x)/2 - sqrt(-c/b**5)*(-A*c + B*b)*log(b**3*sqrt(-c/b**5)*(-A*c + B
*b)/(-A*c**2 + B*b*c) + x)/2 + (-A*b + x**2*(3*A*c - 3*B*b))/(3*b**2*x**3)
```

3.55 $\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=70

$$\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}-\frac{c\log(x)(bB-Ac)}{b^3}-\frac{bB-Ac}{2b^2x^2}-\frac{A}{4bx^4}$$

```
[Out] -1/4*A/b/x^4+1/2*(A*c-B*b)/b^2/x^2-c*(-A*c+B*b)*ln(x)/b^3+1/2*c*(-A*c+B*b)*
ln(c*x^2+b)/b^3
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{bB-Ac}{2b^2x^2}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}-\frac{c\log(x)(bB-Ac)}{b^3}-\frac{A}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(4*b*x^4) - (b*B - A*c)/(2*b^2*x^2) - (c*(b*B - A*c)*Log[x])/b^3 + (c*(b
*B - A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^3}+\frac{bB-Ac}{b^2x^2}-\frac{c(bB-Ac)}{b^3x}+\frac{c^2(bB-Ac)}{b^3(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{A}{4bx^4}-\frac{bB-Ac}{2b^2x^2}-\frac{c(bB-Ac)\log(x)}{b^3}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.00

$$\frac{4cx^4\log(x)(Ac-bB)-b\left(Ab-2Acx^2+2bBx^2\right)+2cx^4(bB-Ac)\log\left(b+cx^2\right)}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] (-(b*(A*b + 2*b*B*x^2 - 2*A*c*x^2)) + 4*c*(-(b*B) + A*c)*x^4*Log[x] + 2*c*(
b*B - A*c)*x^4*Log[b + c*x^2])/(4*b^3*x^4)
```

fricas [A] time = 0.74, size = 73, normalized size = 1.04

$$\frac{2\left(Bbc-Ac^2\right)x^4\log\left(cx^2+b\right)-4\left(Bbc-Ac^2\right)x^4\log(x)-Ab^2-2\left(Bb^2-Abc\right)x^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b*c - A*c^2)*x^4*log(c*x^2 + b) - 4*(B*b*c - A*c^2)*x^4*log(x) -
A*b^2 - 2*(B*b^2 - A*b*c)*x^2)/(b^3*x^4)
```

giac [A] time = 0.19, size = 100, normalized size = 1.43

$$-\frac{\left(Bbc-Ac^2\right)\log\left(x^2\right)}{2\,b^3}+\frac{\left(Bbc^2-Ac^3\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^3c}+\frac{3\,Bbcx^4-3\,Ac^2x^4-2\,Bb^2x^2+2\,Abcx^2-Ab^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c - A*c^2)*log(x^2)/b^3 + 1/2*(B*b*c^2 - A*c^3)*log(abs(c*x^2 + b
))/(b^3*c) + 1/4*(3*B*b*c*x^4 - 3*A*c^2*x^4 - 2*B*b^2*x^2 + 2*A*b*c*x^2 - A
*b^2)/(b^3*x^4)
```

maple [A] time = 0.05, size = 81, normalized size = 1.16

$$\frac{A\,c^2\ln(x)}{b^3}-\frac{A\,c^2\ln\left(c\,x^2+b\right)}{2b^3}-\frac{Bc\ln(x)}{b^2}+\frac{Bc\ln\left(c\,x^2+b\right)}{2b^2}+\frac{Ac}{2b^2x^2}-\frac{B}{2b\,x^2}-\frac{A}{4b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2),x)
```

```
[Out] -1/2*c^2/b^3*ln(c*x^2+b)*A+1/2*c/b^2*ln(c*x^2+b)*B-1/4*A/b/x^4+1/2/b^2/x^2*
A*c-1/2/b/x^2*B+c^2/b^3*ln(x)*A-c/b^2*ln(x)*B
```

maxima [A] time = 1.38, size = 70, normalized size = 1.00

$$\frac{\left(Bbc-Ac^2\right)\log\left(cx^2+b\right)}{2\,b^3}-\frac{\left(Bbc-Ac^2\right)\log\left(x^2\right)}{2\,b^3}-\frac{2\,(Bb-Ac)x^2+Ab}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c - A*c^2)*log(c*x^2 + b)/b^3 - 1/2*(B*b*c - A*c^2)*log(x^2)/b^3 -
 1/4*(2*(B*b - A*c)*x^2 + A*b)/(b^2*x^4)
```

mupad [B] time = 0.14, size = 70, normalized size = 1.00

$$\frac{\ln(x)\,\left(A\,c^2-B\,b\,c\right)}{b^3}-\frac{\ln\left(c\,x^2+b\right)\,\left(A\,c^2-B\,b\,c\right)}{2\,b^3}-\frac{\frac{A}{4\,b}-\frac{x^2\,(A\,c-B\,b)}{2\,b^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(A*c^2 - B*b*c))/b^3 - (log(b + c*x^2)*(A*c^2 - B*b*c))/(2*b^3) - (
A/(4*b) - (x^2*(A*c - B*b))/(2*b^2))/x^4
```

sympy [**A**] time = 0.92, size = 61, normalized size = 0.87

$$\frac{-Ab + x^2\left(2Ac - 2Bb\right)}{4b^2x^4} - \frac{c\left(-Ac + Bb\right)\log\left(x\right)}{b^3} + \frac{c\left(-Ac + Bb\right)\log\left(\frac{b}{c} + x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-A*b + x**2*(2*A*c - 2*B*b))/(4*b**2*x**4) - c*(-A*c + B*b)*log(x)/b**3 +
c*(-A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

3.56 $\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=78

$$\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c(bB-Ac)}{b^3x}-\frac{bB-Ac}{3b^2x^3}-\frac{A}{5bx^5}$$

```
[Out] -1/5*A/b/x^5+1/3*(A*c-B*b)/b^2/x^3+c*(-A*c+B*b)/b^3/x+c^(3/2)*(-A*c+B*b)*ar
ctan(x*c^(1/2)/b^(1/2))/b^(7/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 453, 325, 205}

$$\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}-\frac{A}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(5*b*x^5) - (b*B - A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^(3/2)
*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)}\,dx$$
$$= -\frac{A}{5bx^5} - \frac{(-5bB+5Ac)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{5b}$$
$$= -\frac{A}{5bx^5} - \frac{bB-Ac}{3b^2x^3} - \frac{(c(bB-Ac))\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{b^2}$$
$$= -\frac{A}{5bx^5} - \frac{bB-Ac}{3b^2x^3} + \frac{c(bB-Ac)}{b^3x} + \frac{\left(c^2(bB-Ac)\right)\int \frac{1}{b+cx^2}\,dx}{b^3}$$
$$= -\frac{A}{5bx^5} - \frac{bB-Ac}{3b^2x^3} + \frac{c(bB-Ac)}{b^3x} + \frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 1.00

$$\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}} + \frac{c(bB-Ac)}{b^3x} + \frac{Ac-bB}{3b^2x^3} - \frac{A}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*A/(b*x^5) + (-(b*B) + A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^
(3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

fricas [A] time = 1.13, size = 184, normalized size = 2.36

$$\left[-\frac{15\left(Bbc-Ac^2\right)x^5\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-30\left(Bbc-Ac^2\right)x^4+6\,Ab^2+10\left(Bb^2-Abc\right)x^2}{30\,b^3x^5}, \frac{15\left(Bbc-Ac^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*b*c - A*c^2)*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b
)/(c*x^2 + b)) - 30*(B*b*c - A*c^2)*x^4 + 6*A*b^2 + 10*(B*b^2 - A*b*c)*x^2)
/(b^3*x^5), 1/15*(15*(B*b*c - A*c^2)*x^5*sqrt(c/b)*arctan(x*sqrt(c/b)) + 15
*(B*b*c - A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)]
```

giac [A] time = 0.16, size = 81, normalized size = 1.04

$$\frac{\left(Bbc^2-Ac^3\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^3} + \frac{15\,Bbcx^4-15\,Ac^2x^4-5\,Bb^2x^2+5\,Abcx^2-3\,Ab^2}{15\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*B*b*c*x^
4 - 15*A*c^2*x^4 - 5*B*b^2*x^2 + 5*A*b*c*x^2 - 3*A*b^2)/(b^3*x^5)
```

maple [A] time = 0.05, size = 96, normalized size = 1.23

$$-\frac{A\,c^{3}\arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c}\;b^{3}}+\frac{B\,c^{2}\arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c}\;b^{2}}-\frac{A\,c^{2}}{b^{3} x}+\frac{B c}{b^{2} x}+\frac{A c}{3 b^{2} x^{3}}-\frac{B}{3 b\,x^{3}}-\frac{A}{5 b\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2),x)
```

```
[Out] -c^3/b^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A+c^2/b^2/(b*c)^(1/2)*arctan
(1/(b*c)^(1/2)*c*x)*B-1/5*A/b/x^5+1/3/b^2/x^3*A*c-1/3/b/x^3*B-c^2/b^3/x*A+c
/b^2/x*B
```

maxima [A] time = 3.05, size = 79, normalized size = 1.01

$$\frac{\left(B b c^{2}-A c^{3}\right)\arctan\left(\frac{c x}{\sqrt{b c}}\right)}{\sqrt{b c}\,b^{3}}+\frac{15\left(B b c-A c^{2}\right)x^{4}-3\,A b^{2}-5\left(B b^{2}-A b c\right)x^{2}}{15\,b^{3} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*(B*b*c -
 A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)
```

mupad [B] time = 0.11, size = 70, normalized size = 0.90

$$-\frac{\frac{A}{5\,b}-\frac{x^{2}\,(A\,c-B\,b)}{3\,b^{2}}+\frac{c\,x^{4}\,(A\,c-B\,b)}{b^{3}}}{x^{5}}-\frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x)
```

```
[Out] - (A/(5*b) - (x^2*(A*c - B*b))/(3*b^2) + (c*x^4*(A*c - B*b))/b^3)/x^5 - (c^
(3/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(7/2)
```

sympy [B] time = 0.58, size = 163, normalized size = 2.09

$$-\frac{\sqrt{-\frac{c^{3}}{b^{7}}}\,(-Ac+Bb)\log\left(-\frac{b^{4}\sqrt{-\frac{c^{3}}{b^{7}}}\,(-Ac+Bb)}{-Ac^{3}+Bbc^{2}}+x\right)}{2}+\frac{\sqrt{-\frac{c^{3}}{b^{7}}}\,(-Ac+Bb)\log\left(\frac{b^{4}\sqrt{-\frac{c^{3}}{b^{7}}}\,(-Ac+Bb)}{-Ac^{3}+Bbc^{2}}+x\right)}{2}+\frac{-3Ab^{2}+x^{4}\left(-15A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**7)*(-A*c + B*b)*log(-b**4*sqrt(-c**3/b**7)*(-A*c + B*b)/(-A*
c**3 + B*b*c**2) + x)/2 + sqrt(-c**3/b**7)*(-A*c + B*b)*log(b**4*sqrt(-c**3
/b**7)*(-A*c + B*b)/(-A*c**3 + B*b*c**2) + x)/2 + (-3*A*b**2 + x**4*(-15*A*
c**2 + 15*B*b*c) + x**2*(5*A*b*c - 5*B*b**2))/(15*b**3*x**5)
```

3.57 $\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)} \, dx$

Optimal. Leaf size=92

$$-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{c^2\log(x)(bB-Ac)}{b^4}+\frac{c(bB-Ac)}{2b^3x^2}-\frac{bB-Ac}{4b^2x^4}-\frac{A}{6bx^6}$$

```
[Out] -1/6*A/b/x^6+1/4*(A*c-B*b)/b^2/x^4+1/2*c*(-A*c+B*b)/b^3/x^2+c^2*(-A*c+B*b)*
ln(x)/b^4-1/2*c^2*(-A*c+B*b)*ln(c*x^2+b)/b^4
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{c^2\log(x)(bB-Ac)}{b^4}+\frac{c(bB-Ac)}{2b^3x^2}-\frac{bB-Ac}{4b^2x^4}-\frac{A}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -A/(6*b*x^6) - (b*B - A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) + (c^2
*(b*B - A*c)*Log[x])/b^4 - (c^2*(b*B - A*c)*Log[b + c*x^2])/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)} \, dx &= \int \frac{A+Bx^2}{x^7\left(b+cx^2\right)} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^4}+\frac{bB-Ac}{b^2x^3}-\frac{c(bB-Ac)}{b^3x^2}+\frac{c^2(bB-Ac)}{b^4x}-\frac{c^3(bB-Ac)}{b^4(b+cx)}\right) dx, x, x^2\right) \\
&= -\frac{A}{6bx^6}-\frac{bB-Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{c^2(bB-Ac)\log(x)}{b^4}-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 96, normalized size = 1.04

$$\frac{\left(Ac^3-bBc^2\right)\log\left(b+cx^2\right)}{2b^4}+\frac{\log(x)\left(bBc^2-Ac^3\right)}{b^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{Ac-bB}{4b^2x^4}-\frac{A}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*A/(b*x^6) + (-(b*B) + A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) +
 ((b*B*c^2 - A*c^3)*Log[x])/b^4 + ((-(b*B*c^2) + A*c^3)*Log[b + c*x^2])/(2*
b^4)
```

fricas [A] time = 0.56, size = 98, normalized size = 1.07

$$-\frac{6\left(Bbc^2-Ac^3\right)x^6\log\left(cx^2+b\right)-12\left(Bbc^2-Ac^3\right)x^6\log(x)-6\left(Bb^2c-Abc^2\right)x^4+2\,Ab^3+3\left(Bb^3-Ab^2c\right)x^2}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*b*c^2 - A*c^3)*x^6*log(c*x^2 + b) - 12*(B*b*c^2 - A*c^3)*x^6*lo
g(x) - 6*(B*b^2*c - A*b*c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 - A*b^2*c)*x^2)/(b^4*
x^6)
```

giac [A] time = 0.16, size = 126, normalized size = 1.37

$$\frac{\left(Bbc^2-Ac^3\right)\log\left(x^2\right)}{2\,b^4}-\frac{\left(Bbc^3-Ac^4\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^4c}-\frac{11\,Bbc^2x^6-11\,Ac^3x^6-6\,Bb^2cx^4+6\,Abc^2x^4+3\,Bb^3x^2-3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b*c^2 - A*c^3)*log(x^2)/b^4 - 1/2*(B*b*c^3 - A*c^4)*log(abs(c*x^2 +
b))/(b^4*c) - 1/12*(11*B*b*c^2*x^6 - 11*A*c^3*x^6 - 6*B*b^2*c*x^4 + 6*A*b*c
^2*x^4 + 3*B*b^3*x^2 - 3*A*b^2*c*x^2 + 2*A*b^3)/(b^4*x^6)
```

maple [A] time = 0.05, size = 107, normalized size = 1.16

$$-\frac{A\,c^3\ln(x)}{b^4}+\frac{A\,c^3\ln\left(c\,x^2+b\right)}{2b^4}+\frac{B\,c^2\ln(x)}{b^3}-\frac{B\,c^2\ln\left(c\,x^2+b\right)}{2b^3}-\frac{A\,c^2}{2b^3x^2}+\frac{Bc}{2b^2x^2}+\frac{Ac}{4b^2x^4}-\frac{B}{4b\,x^4}-\frac{A}{6b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*c^3/b^4*ln(c*x^2+b)*A-1/2*c^2/b^3*ln(c*x^2+b)*B-1/6*A/b/x^6+1/4/b^2/x^4
*A*c-1/4/b/x^4*B-1/2*c^2/b^3/x^2*A+1/2*c/b^2/x^2*B-c^3/b^4*ln(x)*A+c^2/b^3*
ln(x)*B
```

maxima [A] time = 1.28, size = 96, normalized size = 1.04

$$-\frac{\left(Bbc^2-Ac^3\right)\log\left(cx^2+b\right)}{2\,b^4}+\frac{\left(Bbc^2-Ac^3\right)\log\left(x^2\right)}{2\,b^4}+\frac{6\left(Bbc-Ac^2\right)x^4-2\,Ab^2-3\left(Bb^2-Abc\right)x^2}{12\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b*c^2 - A*c^3)*log(c*x^2 + b)/b^4 + 1/2*(B*b*c^2 - A*c^3)*log(x^2)/
b^4 + 1/12*(6*(B*b*c - A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 - A*b*c)*x^2)/(b^3*x
^6)
```

mupad [B] time = 0.15, size = 92, normalized size = 1.00

$$\frac{\ln\left(c\,x^2+b\right)\,\left(A\,c^3-B\,b\,c^2\right)}{2\,b^4}-\frac{\frac{A}{6\,b}-\frac{x^2\,(A\,c-B\,b)}{4\,b^2}+\frac{c\,x^4\,(A\,c-B\,b)}{2\,b^3}}{x^6}-\frac{\ln(x)\,\left(A\,c^3-B\,b\,c^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(b + c*x^2)*(A*c^3 - B*b*c^2))/(2*b^4) - (A/(6*b) - (x^2*(A*c - B*b))/(
4*b^2) + (c*x^4*(A*c - B*b))/(2*b^3))/x^6 - (log(x)*(A*c^3 - B*b*c^2))/b^4
```

sympy [A] time = 1.41, size = 88, normalized size = 0.96

$$\frac{-2Ab^2+x^4\left(-6Ac^2+6Bbc\right)+x^2\left(3Abc-3Bb^2\right)}{12b^3x^6}+\frac{c^2\left(-Ac+Bb\right)\log\left(x\right)}{b^4}-\frac{c^2\left(-Ac+Bb\right)\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2),x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 6*B*b*c) + x**2*(3*A*b*c - 3*B*b**2))/(12*b*
*3*x**6) + c**2*(-A*c + B*b)*log(x)/b**4 - c**2*(-A*c + B*b)*log(b/c + x**2
)/(2*b**4)
```

3.58 $\int \frac{x^{12}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=133

$$\frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}-\frac{b^3x(bB-Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^2x(4bB-3Ac)}{c^5}+\frac{bx^3(3bB-2Ac)}{3c^4}-\frac{x^5(2bB-Ac)}{5c^3}+\frac{Bx^7}{7c^2}$$

```
[Out] -b^2*(-3*A*c+4*B*b)*x/c^5+1/3*b*(-2*A*c+3*B*b)*x^3/c^4-1/5*(-A*c+2*B*b)*x^5
/c^3+1/7*B*x^7/c^2-1/2*b^3*(-A*c+B*b)*x/c^5/(c*x^2+b)+1/2*b^(5/2)*(-7*A*c+9
*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

Rubi [A] time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 1810, 205}

$$-\frac{b^3x(bB-Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^2x(4bB-3Ac)}{c^5}+\frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}-\frac{x^5(2bB-Ac)}{5c^3}+\frac{bx^3(3bB-2Ac)}{3c^4}+\frac{Bx^7}{7c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) - ((2*b*B
- A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) - (b^3*(b*B - A*c)*x)/(2*c^5*(b + c*x
^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^{12}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{x^8\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$
$$=-\frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)}-\frac{\int \frac{-b^3(bB-Ac)+2b^2c(bB-Ac)x^2-2bc^2(bB-Ac)x^4+2c^3(bB-Ac)x^6-2Bc^4x^8}{b+cx^2}dx}{2c^5}$$
$$=-\frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)}-\frac{\int \left(2b^2(4bB-3Ac)-2bc(3bB-2Ac)x^2+2c^2(2bB-Ac)x^4-2Bc^3x^6\right.}{2c^5}$$
$$=-\frac{b^2(4bB-3Ac)x}{c^5}+\frac{b(3bB-2Ac)x^3}{3c^4}-\frac{(2bB-Ac)x^5}{5c^3}+\frac{Bx^7}{7c^2}-\frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)}+\frac{\left(b^3(9b\right.}{}$$
$$=-\frac{b^2(4bB-3Ac)x}{c^5}+\frac{b(3bB-2Ac)x^3}{3c^4}-\frac{(2bB-Ac)x^5}{5c^3}+\frac{Bx^7}{7c^2}-\frac{b^3(bB-Ac)x}{2c^5\left(b+cx^2\right)}+\frac{b^{5/2}(9b}{}$$

Mathematica [A] time = 0.11, size = 134, normalized size = 1.01

$$\frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}-\frac{b^2x(4bB-3Ac)}{c^5}+\frac{x\left(Ab^3c-b^4B\right)}{2c^5\left(b+cx^2\right)}+\frac{bx^3(3bB-2Ac)}{3c^4}+\frac{x^5(Ac-2bB)}{5c^3}+\frac{Bx^7}{7c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) + ((-2*b*B
 + A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) + ((-(b^4*B) + A*b^3*c)*x)/(2*c^5*(b
 + c*x^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11
/2))
```

fricas [A] time = 0.87, size = 350, normalized size = 2.63

$$\left[\frac{60\,Bc^4x^9-12\left(9\,Bbc^3-7\,Ac^4\right)x^7+28\left(9\,Bb^2c^2-7\,Abc^3\right)x^5-140\left(9\,Bb^3c-7\,Ab^2c^2\right)x^3-105\left(9\,Bb^4-7\,Ab^3\right.}{420\left(c^6x^2+bc^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*B*c^4*x^9 - 12*(9*B*b*c^3 - 7*A*c^4)*x^7 + 28*(9*B*b^2*c^2 - 7*A
*b*c^3)*x^5 - 140*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 7*A*b^3*c
+ (9*B*b^3*c - 7*A*b^2*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
 b)/(c*x^2 + b)) - 210*(9*B*b^4 - 7*A*b^3*c)*x)/(c^6*x^2 + b*c^5), 1/210*(3
0*B*c^4*x^9 - 6*(9*B*b*c^3 - 7*A*c^4)*x^7 + 14*(9*B*b^2*c^2 - 7*A*b*c^3)*x^
5 - 70*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 + 105*(9*B*b^4 - 7*A*b^3*c + (9*B*b^3*
c - 7*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 105*(9*B*b^4 - 7*
A*b^3*c)*x)/(c^6*x^2 + b*c^5)]
```

giac [A] time = 0.17, size = 139, normalized size = 1.05

$$\frac{\left(9\,Bb^4-7\,Ab^3c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^5}-\frac{Bb^4x-Ab^3cx}{2\left(cx^2+b\right)c^5}+\frac{15\,Bc^{12}x^7-42\,Bbc^{11}x^5+21\,Ac^{12}x^5+105\,Bb^2c^{10}x^3-70\,Abc}{105\,c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*b^4 - 7*A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) - 1/2*(B*b^
4*x - A*b^3*c*x)/((c*x^2 + b)*c^5) + 1/105*(15*B*c^12*x^7 - 42*B*b*c^11*x^5
 + 21*A*c^12*x^5 + 105*B*b^2*c^10*x^3 - 70*A*b*c^11*x^3 - 420*B*b^3*c^9*x +
 315*A*b^2*c^10*x)/c^14
```

maple [A] time = 0.05, size = 155, normalized size = 1.17

$$\frac{B\,x^7}{7c^2}+\frac{A\,x^5}{5c^2}-\frac{2Bb\,x^5}{5c^3}-\frac{2Ab\,x^3}{3c^3}+\frac{B\,b^2x^3}{c^4}+\frac{A\,b^3x}{2\left(c\,x^2+b\right)c^4}-\frac{7A\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^4}-\frac{B\,b^4x}{2\left(c\,x^2+b\right)c^5}+\frac{9B\,b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/7*B*x^7/c^2+1/5/c^2*A*x^5-2/5/c^3*B*x^5*b-2/3/c^3*A*x^3*b+1/c^4*B*x^3*b^2
+3/c^4*A*b^2*x-4/c^5*B*b^3*x+1/2*b^3/c^4*x/(c*x^2+b)*A-1/2*b^4/c^5*x/(c*x^2
+b)*B-7/2*b^3/c^4/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A+9/2*b^4/c^5/(b*c)
^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 2.94, size = 136, normalized size = 1.02

$$-\frac{\left(Bb^4-Ab^3c\right)x}{2\left(c^6x^2+bc^5\right)}+\frac{\left(9\,Bb^4-7\,Ab^3c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^5}+\frac{15\,Bc^3x^7-21\left(2\,Bbc^2-Ac^3\right)x^5+35\left(3\,Bb^2c-2\,Abc^2\right)x^3-10}{105\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^4 - A*b^3*c)*x/(c^6*x^2 + b*c^5) + 1/2*(9*B*b^4 - 7*A*b^3*c)*arct
an(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/105*(15*B*c^3*x^7 - 21*(2*B*b*c^2 - A
*c^3)*x^5 + 35*(3*B*b^2*c - 2*A*b*c^2)*x^3 - 105*(4*B*b^3 - 3*A*b^2*c)*x)/c
^5
```

mupad [B] time = 0.10, size = 203, normalized size = 1.53

$$x\left(\frac{2\,b\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right)}{c}-\frac{b^2\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c^2}\right)+x^5\left(\frac{A}{5\,c^2}-\frac{2\,B\,b}{5\,c^3}\right)-x^3\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{3\,c}+\frac{B\,b^2}{3\,c^4}\right)+\frac{B\,x^7}{7\,c^2}-\frac{x\left(\frac{B\,b^4}{2}-\frac{A\,b}{2}\right.}{c^6\,x^2+b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x*((2*b*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^2)/c^4))/c - (b^2*(A/c^2 - (2
*B*b)/c^3))/c^2) + x^5*(A/(5*c^2) - (2*B*b)/(5*c^3)) - x^3*((2*b*(A/c^2 - (
2*B*b)/c^3))/(3*c) + (B*b^2)/(3*c^4)) + (B*x^7)/(7*c^2) - (x*((B*b^4)/2 - (
A*b^3*c)/2))/(b*c^5 + c^6*x^2) + (b^(5/2)*atan((b^(5/2)*c^(1/2)*x*(7*A*c -
9*B*b))/(9*B*b^4 - 7*A*b^3*c))*(7*A*c - 9*B*b))/(2*c^(11/2))
```

sympy [A] time = 1.01, size = 238, normalized size = 1.79

$$\frac{Bx^7}{7c^2}+x^5\left(\frac{A}{5c^2}-\frac{2Bb}{5c^3}\right)+x^3\left(-\frac{2Ab}{3c^3}+\frac{Bb^2}{c^4}\right)+x\left(\frac{3Ab^2}{c^4}-\frac{4Bb^3}{c^5}\right)+\frac{x\left(Ab^3c-Bb^4\right)}{2bc^5+2c^6x^2}-\frac{\sqrt{-\frac{b^5}{c^{11}}}\left(-7Ac+9Bb\right)\log\left(-\frac{c^5\sqrt{-}}{-}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**7/(7*c**2) + x**5*(A/(5*c**2) - 2*B*b/(5*c**3)) + x**3*(-2*A*b/(3*c**3
) + B*b**2/c**4) + x*(3*A*b**2/c**4 - 4*B*b**3/c**5) + x*(A*b**3*c - B*b**4
)/(2*b*c**5 + 2*c**6*x**2) - sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)*log(-c**5*s
qrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7*A*b**2*c + 9*B*b**3) + x)/4 + sqrt(-b
**5/c**11)*(-7*A*c + 9*B*b)*log(c**5*sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7
*A*b**2*c + 9*B*b**3) + x)/4
```

3.59 $\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{b^3(bB-Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^2(4bB-3Ac)\log\left(b+cx^2\right)}{2c^5}+\frac{bx^2(3bB-2Ac)}{2c^4}-\frac{x^4(2bB-Ac)}{4c^3}+\frac{Bx^6}{6c^2}$$

```
[Out] 1/2*b*(-2*A*c+3*B*b)*x^2/c^4-1/4*(-A*c+2*B*b)*x^4/c^3+1/6*B*x^6/c^2-1/2*b^3
*(-A*c+B*b)/c^5/(c*x^2+b)-1/2*b^2*(-3*A*c+4*B*b)*ln(c*x^2+b)/c^5
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b^3(bB-Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^2(4bB-3Ac)\log\left(b+cx^2\right)}{2c^5}-\frac{x^4(2bB-Ac)}{4c^3}+\frac{bx^2(3bB-2Ac)}{2c^4}+\frac{Bx^6}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x^2)/(2*c^4) - ((2*b*B - A*c)*x^4)/(4*c^3) + (B*x^6)/(6*
c^2) - (b^3*(b*B - A*c))/(2*c^5*(b + c*x^2)) - (b^2*(4*b*B - 3*A*c)*Log[b +
 c*x^2])/(2*c^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^7\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b(3bB-2Ac)}{c^4} + \frac{(-2bB+Ac)x}{c^3} + \frac{Bx^2}{c^2} + \frac{b^3(bB-Ac)}{c^4(b+cx)^2} - \frac{b^2(4bB-3Ac)}{c^4(b+cx)}\right)\right.$$
$$= \frac{b(3bB-2Ac)x^2}{2c^4} - \frac{(2bB-Ac)x^4}{4c^3} + \frac{Bx^6}{6c^2} - \frac{b^3(bB-Ac)}{2c^5\left(b+cx^2\right)} - \frac{b^2(4bB-3Ac)\log\left(b+cx^2\right)}{2c^5}$$

Mathematica [A] time = 0.08, size = 93, normalized size = 0.89

$$\frac{\frac{6b^3(Ac-bB)}{b+cx^2} + 6b^2(3Ac-4bB)\log\left(b+cx^2\right) + 3c^2x^4(Ac-2bB) + 6bcx^2(3bB-2Ac) + 2Bc^3x^6}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (6*b*c*(3*b*B - 2*A*c)*x^2 + 3*c^2*(-2*b*B + A*c)*x^4 + 2*B*c^3*x^6 + (6*b^
3*(-(b*B) + A*c))/(b + c*x^2) + 6*b^2*(-4*b*B + 3*A*c)*Log[b + c*x^2])/(12*
c^5)
```

fricas [A] time = 0.95, size = 148, normalized size = 1.41

$$\frac{2\,Bc^4x^8 - \left(4\,Bbc^3 - 3\,Ac^4\right)x^6 - 6\,Bb^4 + 6\,Ab^3c + 3\left(4\,Bb^2c^2 - 3\,Abc^3\right)x^4 + 6\left(3\,Bb^3c - 2\,Ab^2c^2\right)x^2 - 6\left(4\,Bb^4 - \right.}{12\left(c^6x^2 + bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^4*x^8 - (4*B*b*c^3 - 3*A*c^4)*x^6 - 6*B*b^4 + 6*A*b^3*c + 3*(4*
B*b^2*c^2 - 3*A*b*c^3)*x^4 + 6*(3*B*b^3*c - 2*A*b^2*c^2)*x^2 - 6*(4*B*b^4 -
 3*A*b^3*c + (4*B*b^3*c - 3*A*b^2*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^2 + b*c^
5)
```

giac [A] time = 0.15, size = 135, normalized size = 1.29

$$-\frac{\left(4\,Bb^3 - 3\,Ab^2c\right)\log\left(\left|cx^2+b\right|\right)}{2\,c^5} + \frac{2\,Bc^4x^6 - 6\,Bbc^3x^4 + 3\,Ac^4x^4 + 18\,Bb^2c^2x^2 - 12\,Abc^3x^2}{12\,c^6} + \frac{4\,Bb^3cx^2 - 3\,Ab^2}{2\,(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*b^3 - 3*A*b^2*c)*log(abs(c*x^2 + b))/c^5 + 1/12*(2*B*c^4*x^6 - 6*
B*b*c^3*x^4 + 3*A*c^4*x^4 + 18*B*b^2*c^2*x^2 - 12*A*b*c^3*x^2)/c^6 + 1/2*(4
*B*b^3*c*x^2 - 3*A*b^2*c^2*x^2 + 3*B*b^4 - 2*A*b^3*c)/((c*x^2 + b)*c^5)
```

maple [A] time = 0.05, size = 122, normalized size = 1.16

$$\frac{B\,x^6}{6c^2} + \frac{A\,x^4}{4c^2} - \frac{Bb\,x^4}{2c^3} - \frac{Ab\,x^2}{c^3} + \frac{3B\,b^2x^2}{2c^4} + \frac{A\,b^3}{2\left(c\,x^2+b\right)c^4} + \frac{3A\,b^2\ln\left(c\,x^2+b\right)}{2c^4} - \frac{B\,b^4}{2\left(c\,x^2+b\right)c^5} - \frac{2B\,b^3\ln\left(c\,x^2+b\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/6*B*x^6/c^2+1/4/c^2*A*x^4-1/2/c^3*B*x^4*b-1/c^3*A*x^2*b+3/2/c^4*B*x^2*b^2
+3/2*b^2/c^4*ln(c*x^2+b)*A-2*b^3/c^5*ln(c*x^2+b)*B+1/2*b^3/c^4/(c*x^2+b)*A-
1/2*b^4/c^5/(c*x^2+b)*B
```

maxima [A] time = 1.38, size = 107, normalized size = 1.02

$$-\frac{Bb^4 - Ab^3c}{2\left(c^6x^2 + bc^5\right)} + \frac{2\,Bc^2x^6 - 3\left(2\,Bbc - Ac^2\right)x^4 + 6\left(3\,Bb^2 - 2\,Abc\right)x^2}{12\,c^4} - \frac{\left(4\,Bb^3 - 3\,Ab^2c\right)\log\left(cx^2 + b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^4 - A*b^3*c)/(c^6*x^2 + b*c^5) + 1/12*(2*B*c^2*x^6 - 3*(2*B*b*c -
 A*c^2)*x^4 + 6*(3*B*b^2 - 2*A*b*c)*x^2)/c^4 - 1/2*(4*B*b^3 - 3*A*b^2*c)*lo
g(c*x^2 + b)/c^5
```

mupad [B] time = 0.10, size = 121, normalized size = 1.15

$$x^4\left(\frac{A}{4\,c^2} - \frac{B\,b}{2\,c^3}\right) - x^2\left(\frac{b\left(\frac{A}{c^2} - \frac{2\,B\,b}{c^3}\right)}{c} + \frac{B\,b^2}{2\,c^4}\right) + \frac{B\,x^6}{6\,c^2} - \frac{\ln\left(c\,x^2 + b\right)\left(4\,B\,b^3 - 3\,A\,b^2\,c\right)}{2\,c^5} - \frac{B\,b^4 - A\,b^3\,c}{2\,c\,\left(c^5\,x^2 + b\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4*(A/(4*c^2) - (B*b)/(2*c^3)) - x^2*((b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^2
)/(2*c^4)) + (B*x^6)/(6*c^2) - (log(b + c*x^2)*(4*B*b^3 - 3*A*b^2*c))/(2*c^
5) - (B*b^4 - A*b^3*c)/(2*c*(b*c^4 + c^5*x^2))
```

sympy [A] time = 0.93, size = 104, normalized size = 0.99

$$\frac{Bx^6}{6c^2} - \frac{b^2\left(-3Ac + 4Bb\right)\log\left(b + cx^2\right)}{2c^5} + x^4\left(\frac{A}{4c^2} - \frac{Bb}{2c^3}\right) + x^2\left(-\frac{Ab}{c^3} + \frac{3Bb^2}{2c^4}\right) + \frac{Ab^3c - Bb^4}{2bc^5 + 2c^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**6/(6*c**2) - b**2*(-3*A*c + 4*B*b)*log(b + c*x**2)/(2*c**5) + x**4*(A/
(4*c**2) - B*b/(2*c**3)) + x**2*(-A*b/c**3 + 3*B*b**2/(2*c**4)) + (A*b**3*c
 - B*b**4)/(2*b*c**5 + 2*c**6*x**2)
```

3.60 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}+\frac{b^2x(bB-Ac)}{2c^4\left(b+cx^2\right)}+\frac{bx(3bB-2Ac)}{c^4}-\frac{x^3(2bB-Ac)}{3c^3}+\frac{Bx^5}{5c^2}$$

```
[Out] b*(-2*A*c+3*B*b)*x/c^4-1/3*(-A*c+2*B*b)*x^3/c^3+1/5*B*x^5/c^2+1/2*b^2*(-A*c
+B*b)*x/c^4/(c*x^2+b)-1/2*b^(3/2)*(-5*A*c+7*B*b)*arctan(x*c^(1/2)/b^(1/2))/
c^(9/2)
```

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 1810, 205}

$$\frac{b^2x(bB-Ac)}{2c^4\left(b+cx^2\right)}-\frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}-\frac{x^3(2bB-Ac)}{3c^3}+\frac{bx(3bB-2Ac)}{c^4}+\frac{Bx^5}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 - ((2*b*B - A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2) +
 (b^2*(b*B - A*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^6\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{\int \frac{b^2(bB-Ac)-2bc(bB-Ac)x^2+2c^2(bB-Ac)x^4-2Bc^3x^6}{b+cx^2}\,dx}{2c^4}$$
$$= \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{\int \left(-2b(3bB-2Ac)+2c(2bB-Ac)x^2-2Bc^2x^4+\frac{7b^3B-5Ab^2c}{b+cx^2}\right)dx}{2c^4}$$
$$= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{\left(b^2(7bB-5Ac)\right)\int \frac{1}{b+cx^2}\,dx}{2c^4}$$
$$= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4\left(b+cx^2\right)} - \frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 1.01

$$-\frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}} - \frac{x\left(Ab^2c-b^3B\right)}{2c^4\left(b+cx^2\right)} + \frac{bx(3bB-2Ac)}{c^4} + \frac{x^3(Ac-2bB)}{3c^3} + \frac{Bx^5}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 + ((-2*b*B + A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2)
- ((-(b^3*B) + A*b^2*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

fricas [A] time = 1.15, size = 298, normalized size = 2.71

$$\left[\frac{12\,Bc^3x^7-4\left(7\,Bbc^2-5\,Ac^3\right)x^5+20\left(7\,Bb^2c-5\,Abc^2\right)x^3-15\left(7\,Bb^3-5\,Ab^2c+\left(7\,Bb^2c-5\,Abc^2\right)x^2\right)\sqrt{-\frac{b}{c}}\log}{60\left(c^5x^2+bc^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*c^3*x^7 - 4*(7*B*b*c^2 - 5*A*c^3)*x^5 + 20*(7*B*b^2*c - 5*A*b*c
^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(7*B*b^3 - 5*A*b^2*c)
*x)/(c^5*x^2 + b*c^4), 1/30*(6*B*c^3*x^7 - 2*(7*B*b*c^2 - 5*A*c^3)*x^5 + 10
*(7*B*b^2*c - 5*A*b*c^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b
*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*(7*B*b^3 - 5*A*b^2*c)*x)/
(c^5*x^2 + b*c^4)]
```

giac [A] time = 0.16, size = 115, normalized size = 1.05

$$-\frac{\left(7\,Bb^3-5\,Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^4}+\frac{Bb^3x-Ab^2cx}{2\left(cx^2+b\right)c^4}+\frac{3\,Bc^8x^5-10\,Bbc^7x^3+5\,Ac^8x^3+45\,Bb^2c^6x-30\,Abc^7x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*(B*b
^3*x - A*b^2*c*x)/((c*x^2 + b)*c^4) + 1/15*(3*B*c^8*x^5 - 10*B*b*c^7*x^3 +
5*A*c^8*x^3 + 45*B*b^2*c^6*x - 30*A*b*c^7*x)/c^10
```

maple [A] time = 0.05, size = 132, normalized size = 1.20

$$\frac{B x^5}{5c^2}+\frac{A x^3}{3c^2}-\frac{2Bb x^3}{3c^3}-\frac{A b^2 x}{2\left(c x^2+b\right)c^3}+\frac{5A b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\, c^3}+\frac{B b^3 x}{2\left(c x^2+b\right)c^4}-\frac{7B b^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\, c^4}-\frac{2Abx}{c^3}+\frac{3B b^2 x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*B*x^5/c^2+1/3/c^2*A*x^3-2/3/c^3*B*x^3*b-2/c^3*A*b*x+3/c^4*B*b^2*x-1/2*b
^2/c^3*x/(c*x^2+b)*A+1/2*b^3/c^4*x/(c*x^2+b)*B+5/2*b^2/c^3/(b*c)^(1/2)*arct
an(1/(b*c)^(1/2)*c*x)*A-7/2*b^3/c^4/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 3.02, size = 112, normalized size = 1.02

$$\frac{\left(Bb^3-Ab^2c\right)x}{2\left(c^5x^2+bc^4\right)}-\frac{\left(7\,Bb^3-5\,Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^4}+\frac{3\,Bc^2x^5-5\left(2\,Bbc-Ac^2\right)x^3+15\left(3\,Bb^2-2\,Abc\right)x}{15\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)*x/(c^5*x^2 + b*c^4) - 1/2*(7*B*b^3 - 5*A*b^2*c)*arcta
n(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/15*(3*B*c^2*x^5 - 5*(2*B*b*c - A*c^2)*
x^3 + 15*(3*B*b^2 - 2*A*b*c)*x)/c^4
```

mupad [B] time = 0.09, size = 141, normalized size = 1.28

$$x^3\left(\frac{A}{3\,c^2}-\frac{2\,B\,b}{3\,c^3}\right)-x\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right)+\frac{B\,x^5}{5\,c^2}+\frac{x\left(\frac{B\,b^3}{2}-\frac{A\,b^2\,c}{2}\right)}{c^5\,x^2+b\,c^4}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(5\,A\,c-7\,B\,b)}{7\,B\,b^3-5\,A\,b^2\,c}\right)(5\,A\,c-7\,B\,b)}{2\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3*(A/(3*c^2) - (2*B*b)/(3*c^3)) - x*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*b
^2)/c^4) + (B*x^5)/(5*c^2) + (x*((B*b^3)/2 - (A*b^2*c)/2))/(b*c^4 + c^5*x^2
) - (b^(3/2)*atan((b^(3/2)*c^(1/2)*x*(5*A*c - 7*B*b))/(7*B*b^3 - 5*A*b^2*c)
)*(5*A*c - 7*B*b))/(2*c^(9/2))
```

sympy [B] time = 0.76, size = 211, normalized size = 1.92

$$\frac{Bx^5}{5c^2}+x^3\left(\frac{A}{3c^2}-\frac{2Bb}{3c^3}\right)+x\left(-\frac{2Ab}{c^3}+\frac{3Bb^2}{c^4}\right)+\frac{x\left(-Ab^2c+Bb^3\right)}{2bc^4+2c^5x^2}+\frac{\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**5/(5*c**2) + x**3*(A/(3*c**2) - 2*B*b/(3*c**3)) + x*(-2*A*b/c**3 + 3*B
*b**2/c**4) + x*(-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2) + sqrt(-b**3/
c**9)*(-5*A*c + 7*B*b)*log(-c**4*sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*
c + 7*B*b**2) + x)/4 - sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)*log(c**4*sqrt(-b**
3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*c + 7*B*b**2) + x)/4
```

3.61 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=83

$$\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}-\frac{x^2(2bB-Ac)}{2c^3}+\frac{Bx^4}{4c^2}$$

```
[Out] -1/2*(-A*c+2*B*b)*x^2/c^3+1/4*B*x^4/c^2+1/2*b^2*(-A*c+B*b)/c^4/(c*x^2+b)+1/
2*b*(-2*A*c+3*B*b)*ln(c*x^2+b)/c^4
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}-\frac{x^2(2bB-Ac)}{2c^3}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}+\frac{Bx^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((2*b*B - A*c)*x^2)/(2*c^3) + (B*x^4)/(4*c^2) + (b^2*(b*B - A*c))/(2*c^4*(
b + c*x^2)) + (b*(3*b*B - 2*A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^9 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^5 \left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-2bB+Ac}{c^3} + \frac{Bx}{c^2} - \frac{b^2(bB-Ac)}{c^3(b+cx)^2} + \frac{b(3bB-2Ac)}{c^3(b+cx)}\right) dx, x, x^2\right)$$
$$= -\frac{(2bB-Ac)x^2}{2c^3} + \frac{Bx^4}{4c^2} + \frac{b^2(bB-Ac)}{2c^4\left(b+cx^2\right)} + \frac{b(3bB-2Ac)\log\left(b+cx^2\right)}{2c^4}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.87

$$\frac{\frac{2b^2(bB-Ac)}{b+cx^2} + 2cx^2(Ac-2bB) + 2b(3bB-2Ac)\log\left(b+cx^2\right) + Bc^2x^4}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(-2*b*B + A*c)*x^2 + B*c^2*x^4 + (2*b^2*(b*B - A*c))/(b + c*x^2) + 2*b
*(3*b*B - 2*A*c)*Log[b + c*x^2])/(4*c^4)
```

fricas [A] time = 0.79, size = 121, normalized size = 1.46

$$\frac{Bc^3x^6 - \left(3\,Bbc^2 - 2\,Ac^3\right)x^4 + 2\,Bb^3 - 2\,Ab^2c - 2\left(2\,Bb^2c - Abc^2\right)x^2 + 2\left(3\,Bb^3 - 2\,Ab^2c + \left(3\,Bb^2c - 2\,Abc^2\right)x^2 \right.}{4\left(c^5x^2 + bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^6 - (3*B*b*c^2 - 2*A*c^3)*x^4 + 2*B*b^3 - 2*A*b^2*c - 2*(2*B*b
^2*c - A*b*c^2)*x^2 + 2*(3*B*b^3 - 2*A*b^2*c + (3*B*b^2*c - 2*A*b*c^2)*x^2)
*log(c*x^2 + b))/(c^5*x^2 + b*c^4)
```

giac [A] time = 0.17, size = 106, normalized size = 1.28

$$\frac{\left(3\,Bb^2 - 2\,Abc\right)\log\left(\left|cx^2+b\right|\right)}{2\,c^4} + \frac{Bc^2x^4 - 4\,Bbcx^2 + 2\,Ac^2x^2}{4\,c^4} - \frac{3\,Bb^2cx^2 - 2\,Abc^2x^2 + 2\,Bb^3 - Ab^2c}{2\left(cx^2+b\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^2 - 2*A*b*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(B*c^2*x^4 - 4*B*b*c*
x^2 + 2*A*c^2*x^2)/c^4 - 1/2*(3*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 2*B*b^3 - A*b
^2*c)/((c*x^2 + b)*c^4)
```

maple [A] time = 0.05, size = 98, normalized size = 1.18

$$\frac{B\,x^4}{4c^2} + \frac{A\,x^2}{2c^2} - \frac{Bb\,x^2}{c^3} - \frac{A\,b^2}{2\left(c\,x^2+b\right)c^3} - \frac{Ab\ln\left(c\,x^2+b\right)}{c^3} + \frac{B\,b^3}{2\left(c\,x^2+b\right)c^4} + \frac{3B\,b^2\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/4*B*x^4/c^2+1/2/c^2*A*x^2-1/c^3*B*x^2*b-b/c^3*ln(c*x^2+b)*A+3/2*b^2/c^4*l
n(c*x^2+b)*B-1/2*b^2/c^3/(c*x^2+b)*A+1/2*b^3/c^4/(c*x^2+b)*B
```

maxima [A] time = 1.32, size = 82, normalized size = 0.99

$$\frac{Bb^3 - Ab^2c}{2\left(c^5x^2 + bc^4\right)} + \frac{Bcx^4 - 2\,(2\,Bb - Ac)x^2}{4\,c^3} + \frac{\left(3\,Bb^2 - 2\,Abc\right)\log\left(cx^2 + b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)/(c^5*x^2 + b*c^4) + 1/4*(B*c*x^4 - 2*(2*B*b - A*c)*x^
2)/c^3 + 1/2*(3*B*b^2 - 2*A*b*c)*log(c*x^2 + b)/c^4
```

mupad [B] time = 0.07, size = 86, normalized size = 1.04

$$x^2\left(\frac{A}{2\,c^2} - \frac{B\,b}{c^3}\right) + \frac{\ln\left(c\,x^2 + b\right)\,\left(3\,B\,b^2 - 2\,A\,b\,c\right)}{2\,c^4} + \frac{B\,x^4}{4\,c^2} + \frac{B\,b^3 - A\,b^2\,c}{2\,c\,\left(c^4\,x^2 + b\,c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2*(A/(2*c^2) - (B*b)/c^3) + (log(b + c*x^2)*(3*B*b^2 - 2*A*b*c))/(2*c^4)
+ (B*x^4)/(4*c^2) + (B*b^3 - A*b^2*c)/(2*c*(b*c^3 + c^4*x^2))
```

sympy [A] time = 0.70, size = 78, normalized size = 0.94

$$\frac{Bx^4}{4c^2} + \frac{b\,(-2Ac + 3Bb)\log\left(b + cx^2\right)}{2c^4} + x^2\left(\frac{A}{2c^2} - \frac{Bb}{c^3}\right) + \frac{-Ab^2c + Bb^3}{2bc^4 + 2c^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**4/(4*c**2) + b*(-2*A*c + 3*B*b)*log(b + c*x**2)/(2*c**4) + x**2*(A/(2*
c**2) - B*b/c**3) + (-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2)
```

3.62 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{b}\,(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{bx(bB-Ac)}{2c^3\,(b+cx^2)}-\frac{x(2bB-Ac)}{c^3}+\frac{Bx^3}{3c^2}$$

```
[Out] -(-A*c+2*B*b)*x/c^3+1/3*B*x^3/c^2-1/2*b*(-A*c+B*b)*x/c^3/(c*x^2+b)+1/2*(-3*
A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(7/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 1153, 205}

$$-\frac{bx(bB-Ac)}{2c^3\,(b+cx^2)}-\frac{x(2bB-Ac)}{c^3}+\frac{\sqrt{b}\,(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}+\frac{Bx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(((2*b*B - A*c)*x)/c^3) + (B*x^3)/(3*c^2) - (b*(b*B - A*c)*x)/(2*c^3*(b +
c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^4\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)} - \frac{\int \frac{-b(bB-Ac)+2c(bB-Ac)x^2-2Bc^2x^4}{b+cx^2}\,dx}{2c^3}$$
$$= -\frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)} - \frac{\int \left(2(2bB-Ac)-2Bcx^2+\frac{-5b^2B+3Abc}{b+cx^2}\right)dx}{2c^3}$$
$$= -\frac{(2bB-Ac)x}{c^3} + \frac{Bx^3}{3c^2} - \frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)} + \frac{(b(5bB-3Ac))\int \frac{1}{b+cx^2}\,dx}{2c^3}$$
$$= -\frac{(2bB-Ac)x}{c^3} + \frac{Bx^3}{3c^2} - \frac{b(bB-Ac)x}{2c^3\left(b+cx^2\right)} + \frac{\sqrt{b}\,(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 1.00

$$\frac{x\left(Abc-b^2B\right)}{2c^3\left(b+cx^2\right)} + \frac{\sqrt{b}\,(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}} + \frac{x(Ac-2bB)}{c^3} + \frac{Bx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*b*B + A*c)*x)/c^3 + (B*x^3)/(3*c^2) + ((-(b^2*B) + A*b*c)*x)/(2*c^3*(b
 + c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/
2))
```

fricas [A] time = 0.94, size = 240, normalized size = 2.70

$$\left[\frac{4\,Bc^2x^5 - 4\left(5\,Bbc-3\,Ac^2\right)x^3 - 3\left(5\,Bb^2-3\,Abc+\left(5\,Bbc-3\,Ac^2\right)x^2\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right) - 6\left(5\,Bb^2-3\,A\right.}{12\left(c^4x^2+bc^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*c^2*x^5 - 4*(5*B*b*c - 3*A*c^2)*x^3 - 3*(5*B*b^2 - 3*A*b*c + (5*
B*b*c - 3*A*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2
+ b)) - 6*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3), 1/6*(2*B*c^2*x^5 - 2*(5
*B*b*c - 3*A*c^2)*x^3 + 3*(5*B*b^2 - 3*A*b*c + (5*B*b*c - 3*A*c^2)*x^2)*sqr
t(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3)
]
```

giac [A] time = 0.16, size = 88, normalized size = 0.99

$$\frac{\left(5\,Bb^2-3\,Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3} - \frac{Bb^2x-Abcx}{2\left(cx^2+b\right)c^3} + \frac{Bc^4x^3-6\,Bbc^3x+3\,Ac^4x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*(B*b^2*
x - A*b*c*x)/((c*x^2 + b)*c^3) + 1/3*(B*c^4*x^3 - 6*B*b*c^3*x + 3*A*c^4*x)/
c^6
```

maple [A] time = 0.06, size = 105, normalized size = 1.18

$$\frac{B\,x^3}{3c^2}+\frac{Abx}{2\left(c\,x^2+b\right)c^2}-\frac{3Ab\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^2}-\frac{B\,b^2x}{2\left(c\,x^2+b\right)c^3}+\frac{5B\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^3}+\frac{Ax}{c^2}-\frac{2Bbx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/3*B*x^3/c^2+1/c^2*A*x-2/c^3*b*B*x+1/2*b/c^2*x/(c*x^2+b)*A-1/2*b^2/c^3*x/(
c*x^2+b)*B-3/2*b/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A+5/2*b^2/c^3/(b
*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 3.03, size = 85, normalized size = 0.96

$$-\frac{\left(Bb^2-Abc\right)x}{2\left(c^4x^2+bc^3\right)}+\frac{\left(5\,Bb^2-3\,Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3}+\frac{Bcx^3-3\,(2\,Bb-Ac)x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*x/(c^4*x^2 + b*c^3) + 1/2*(5*B*b^2 - 3*A*b*c)*arctan(c
*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/3*(B*c*x^3 - 3*(2*B*b - A*c)*x)/c^3
```

mupad [B] time = 0.11, size = 104, normalized size = 1.17

$$x\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)-\frac{x\,\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3}+\frac{B\,x^3}{3\,c^2}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-5\,B\,b)}{5\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-5\,B\,b)}{2\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x*(A/c^2 - (2*B*b)/c^3) - (x*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + (
B*x^3)/(3*c^2) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 5*B*b))/(5*B*b^2
 - 3*A*b*c))*(3*A*c - 5*B*b))/(2*c^(7/2))
```

sympy [A] time = 0.69, size = 129, normalized size = 1.45

$$\frac{Bx^3}{3c^2}+x\left(\frac{A}{c^2}-\frac{2Bb}{c^3}\right)+\frac{x\left(Abc-Bb^2\right)}{2bc^3+2c^4x^2}-\frac{\sqrt{-\frac{b}{c^7}}\,(-3Ac+5Bb)\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{4}+\frac{\sqrt{-\frac{b}{c^7}}\,(-3Ac+5Bb)\log\left(c^3\sqrt{\right.}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**3/(3*c**2) + x*(A/c**2 - 2*B*b/c**3) + x*(A*b*c - B*b**2)/(2*b*c**3 +
2*c**4*x**2) - sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(-c**3*sqrt(-b/c**7) + x)/
4 + sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(c**3*sqrt(-b/c**7) + x)/4
```

3.63 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{b(bB-Ac)}{2c^3(b+cx^2)}-\frac{(2bB-Ac)\log(b+cx^2)}{2c^3}+\frac{Bx^2}{2c^2}$$

```
[Out] 1/2*B*x^2/c^2-1/2*b*(-A*c+B*b)/c^3/(c*x^2+b)-1/2*(-A*c+2*B*b)*ln(c*x^2+b)/c
^3
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{b(bB-Ac)}{2c^3(b+cx^2)}-\frac{(2bB-Ac)\log(b+cx^2)}{2c^3}+\frac{Bx^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x^2)/(2*c^2) - (b*(b*B - A*c))/(2*c^3*(b + c*x^2)) - ((2*b*B - A*c)*Log[
b + c*x^2])/(2*c^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{x^3\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c^2}+\frac{b(bB-Ac)}{c^2(b+cx)^2}+\frac{-2bB+Ac}{c^2(b+cx)}\right)dx,x,x^2\right)$$
$$=\frac{Bx^2}{2c^2}-\frac{b(bB-Ac)}{2c^3\left(b+cx^2\right)}-\frac{(2bB-Ac)\log\left(b+cx^2\right)}{2c^3}$$

Mathematica [**A**] time = 0.04, size = 50, normalized size = 0.82

$$\frac{\frac{b(Ac-bB)}{b+cx^2}+(Ac-2bB)\log\left(b+cx^2\right)+Bcx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*c*x^2 + (b*(-(b*B) + A*c))/(b + c*x^2) + (-2*b*B + A*c)*Log[b + c*x^2])/
(2*c^3)
```

fricas [**A**] time = 0.88, size = 81, normalized size = 1.33

$$\frac{Bc^2x^4+Bbcx^2-Bb^2+Abc-\left(2\,Bb^2-Abc+\left(2\,Bbc-Ac^2\right)x^2\right)\log\left(cx^2+b\right)}{2\left(c^4x^2+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*c^2*x^4 + B*b*c*x^2 - B*b^2 + A*b*c - (2*B*b^2 - A*b*c + (2*B*b*c -
A*c^2)*x^2)*log(c*x^2 + b))/(c^4*x^2 + b*c^3)
```

giac [**A**] time = 0.16, size = 70, normalized size = 1.15

$$\frac{Bx^2}{2\,c^2}-\frac{(2\,Bb-Ac)\log\left(\left|cx^2+b\right|\right)}{2\,c^3}+\frac{2\,Bbcx^2-Ac^2x^2+Bb^2}{2\left(cx^2+b\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(2*B*b - A*c)*log(abs(c*x^2 + b))/c^3 + 1/2*(2*B*b*c*x^
2 - A*c^2*x^2 + B*b^2)/((c*x^2 + b)*c^3)
```

maple [**A**] time = 0.06, size = 74, normalized size = 1.21

$$\frac{B\,x^2}{2c^2}+\frac{Ab}{2\left(c\,x^2+b\right)c^2}+\frac{A\ln\left(c\,x^2+b\right)}{2c^2}-\frac{B\,b^2}{2\left(c\,x^2+b\right)c^3}-\frac{Bb\ln\left(c\,x^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*B*x^2/c^2+1/2/c^2*ln(c*x^2+b)*A-1/c^3*ln(c*x^2+b)*b*B+1/2/c^2*b/(c*x^2+
b)*A-1/2/c^3*b^2/(c*x^2+b)*B
```

maxima [A] time = 1.31, size = 60, normalized size = 0.98

$$\frac{Bx^2}{2\,c^2} - \frac{Bb^2 - Abc}{2\left(c^4x^2 + bc^3\right)} - \frac{(2\,Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(B*b^2 - A*b*c)/(c^4*x^2 + b*c^3) - 1/2*(2*B*b - A*c)*l
og(c*x^2 + b)/c^3
```

mupad [B] time = 0.12, size = 62, normalized size = 1.02

$$\frac{B\,x^2}{2\,c^2} + \frac{\ln\left(c\,x^2 + b\right)\,(A\,c - 2\,B\,b)}{2\,c^3} - \frac{B\,b^2 - A\,b\,c}{2\,c\,\left(c^3\,x^2 + b\,c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x^2)/(2*c^2) + (log(b + c*x^2)*(A*c - 2*B*b))/(2*c^3) - (B*b^2 - A*b*c)/
(2*c*(b*c^2 + c^3*x^2))
```

sympy [A] time = 0.60, size = 56, normalized size = 0.92

$$\frac{Bx^2}{2c^2} + \frac{Abc - Bb^2}{2bc^3 + 2c^4x^2} - \frac{(-Ac + 2Bb)\log\left(b + cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**2/(2*c**2) + (A*b*c - B*b**2)/(2*b*c**3 + 2*c**4*x**2) - (-A*c + 2*B*b
)*log(b + c*x**2)/(2*c**3)
```

3.64 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}}+\frac{x(bB-Ac)}{2c^2\left(b+cx^2\right)}+\frac{Bx}{c^2}$$

```
[Out] B*x/c^2+1/2*(-A*c+B*b)*x/c^2/(c*x^2+b)-1/2*(-A*c+3*B*b)*arctan(x*c^(1/2)/b^
(1/2))/c^(5/2)/b^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 388, 205}

$$\frac{x(bB-Ac)}{2c^2\left(b+cx^2\right)}-\frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}}+\frac{Bx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x)/c^2 + ((b*B - A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6 \left(A + Bx^2\right)}{\left(bx^2 + cx^4\right)^2} \, dx = \int \frac{x^2 \left(A + Bx^2\right)}{\left(b + cx^2\right)^2} \, dx$$

$$= \frac{(bB - Ac)x}{2c^2 \left(b + cx^2\right)} - \frac{\int \frac{bB - Ac - 2Bcx^2}{b + cx^2} \, dx}{2c^2}$$

$$= \frac{Bx}{c^2} + \frac{(bB - Ac)x}{2c^2 \left(b + cx^2\right)} - \frac{(3bB - Ac) \int \frac{1}{b + cx^2} \, dx}{2c^2}$$

$$= \frac{Bx}{c^2} + \frac{(bB - Ac)x}{2c^2 \left(b + cx^2\right)} - \frac{(3bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.00

$$-\frac{(3bB - Ac) \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}} - \frac{x(Ac - bB)}{2c^2 \left(b + cx^2\right)} + \frac{Bx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x)/c^2 - ((-(b*B) + A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

fricas [A] time = 0.72, size = 208, normalized size = 3.06

$$\left[\frac{4\,Bbc^2x^3 + \left(3\,Bb^2 - Abc + \left(3\,Bbc - Ac^2\right)x^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2 - 2\,\sqrt{-bc}\,x - b}{cx^2 + b}\right) + 2\left(3\,Bb^2c - Abc^2\right)x}{4\left(bc^4x^2 + b^2c^3\right)}, \frac{2\,Bbc^2x^3 - \left(3\,Bb^2 - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*b*c^2*x^3 + (3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^2*c - A*b*c^2)*x)/
(b*c^4*x^2 + b^2*c^3), 1/2*(2*B*b*c^2*x^3 - (3*B*b^2 - A*b*c + (3*B*b*c - A
*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + (3*B*b^2*c - A*b*c^2)*x)/(b*c^
4*x^2 + b^2*c^3)]
```

giac [A] time = 0.19, size = 59, normalized size = 0.87

$$\frac{Bx}{c^2} - \frac{(3\,Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2} + \frac{Bbx - Acx}{2\left(cx^2 + b\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*(B*
b*x - A*c*x)/((c*x^2 + b)*c^2)
```

maple [A] time = 0.05, size = 82, normalized size = 1.21

$$-\frac{Ax}{2\left(c\,x^2 + b\right)c} + \frac{A\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c} + \frac{Bbx}{2\left(c\,x^2 + b\right)c^2} - \frac{3Bb\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^2} + \frac{Bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] B*x/c^2-1/2/c*x/(c*x^2+b)*A+1/2/c^2*x/(c*x^2+b)*b*B+1/2/c/(b*c)^(1/2)*arcta
n(1/(b*c)^(1/2)*c*x)*A-3/2/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*b*B
```

maxima [A] time = 3.00, size = 61, normalized size = 0.90

$$\frac{(Bb-Ac)x}{2\left(c^3x^2+bc^2\right)}+\frac{Bx}{c^2}-\frac{(3\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)*x/(c^3*x^2 + b*c^2) + B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*
x/sqrt(b*c))/(sqrt(b*c)*c^2)
```

mupad [B] time = 0.13, size = 59, normalized size = 0.87

$$\frac{B\,x}{c^2}-\frac{x\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-3\,B\,b\right)}{2\,\sqrt{b}\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x)/c^2 - (x*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (atan((c^(1/2)*x)/b
^(1/2))*(A*c - 3*B*b))/(2*b^(1/2)*c^(5/2))
```

sympy [A] time = 0.55, size = 114, normalized size = 1.68

$$\frac{Bx}{c^2}+\frac{x\,(-Ac+Bb)}{2bc^2+2c^3x^2}+\frac{\sqrt{-\frac{1}{bc^5}}\,(-Ac+3Bb)\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{bc^5}}\,(-Ac+3Bb)\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x/c**2 + x*(-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2) + sqrt(-1/(b*c**5))*(-A*
c + 3*B*b)*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/4 - sqrt(-1/(b*c**5))*(-A*c +
 3*B*b)*log(b*c**2*sqrt(-1/(b*c**5)) + x)/4
```

3.65 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=41

$$\frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

```
[Out] 1/2*(-A*c+B*b)/c^2/(c*x^2+b)+1/2*B*ln(c*x^2+b)/c^2
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 43}

$$\frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{x(A+Bx^2)}{(b+cx^2)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(b+cx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-bB+Ac}{c(b+cx)^2}+\frac{B}{c(b+cx)}\right)dx,x,x^2\right)\\
&= \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{bB-Ac}{2c^2\left(b+cx^2\right)}+\frac{B\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

fricas [A] time = 0.87, size = 44, normalized size = 1.07

$$\frac{Bb-Ac+\left(Bcx^2+Bb\right)\log\left(cx^2+b\right)}{2\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 1/2*(B*b - A*c + (B*c*x^2 + B*b)*log(c*x^2 + b))/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.19, size = 37, normalized size = 0.90

$$\frac{B\log\left(\left|cx^2+b\right|\right)}{2\,c^2}-\frac{Bx^2+A}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 1/2*B*log(abs(c*x^2 + b))/c^2 - 1/2*(B*x^2 + A)/((c*x^2 + b)*c)
```

maple [A] time = 0.05, size = 47, normalized size = 1.15

$$-\frac{A}{2\left(c\,x^2+b\right)c}+\frac{Bb}{2\left(c\,x^2+b\right)c^2}+\frac{B\ln\left(c\,x^2+b\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
[Out] 1/2*B*ln(c*x^2+b)/c^2-1/2/c/(c*x^2+b)*A+1/2/c^2/(c*x^2+b)*b*B
```

maxima [A] time = 1.31, size = 40, normalized size = 0.98

$$\frac{Bb-Ac}{2\left(c^3x^2+bc^2\right)}+\frac{B\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 1/2*(B*b - A*c)/(c^3*x^2 + b*c^2) + 1/2*B*log(c*x^2 + b)/c^2
```

mupad [B] time = 0.09, size = 37, normalized size = 0.90

$$\frac{B\,\ln\left(c\,x^2+b\right)}{2\,c^2}-\frac{A\,c-B\,b}{2\,c^2\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
[Out] (B*log(b + c*x^2))/(2*c^2) - (A*c - B*b)/(2*c^2*(b + c*x^2))
```

sympy [A] time = 0.37, size = 36, normalized size = 0.88

$$\frac{B\log\left(b+cx^2\right)}{2c^2}+\frac{-Ac+Bb}{2bc^2+2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] B*log(b + c*x**2)/(2*c**2) + (-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2)
```

3.66 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\left(b+cx^2\right)}$$

```
[Out] -1/2*(-A*c+B*b)*x/b/c/(c*x^2+b)+1/2*(A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(
3/2)/c^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 385, 205}

$$\frac{(Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*x)/(2*b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/(2*b^(3/2)*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{\left(b+cx^2\right)^2}\,dx\\
&= -\frac{(bB-Ac)x}{2bc\left(b+cx^2\right)}+\frac{(bB+Ac)\int \frac{1}{b+cx^2}\,dx}{2bc}\\
&= -\frac{(bB-Ac)x}{2bc\left(b+cx^2\right)}+\frac{(bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 1.00

$$\frac{(Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*x)/(b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(3/2)*c^(3/2))
```

fricas [A] time = 0.65, size = 182, normalized size = 2.89

$$\left[-\frac{\left(Bb^2+Abc+\left(Bbc+Ac^2\right)x^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)+2\left(Bb^2c-Abc^2\right)x}{4\left(b^2c^3x^2+b^3c^2\right)}, \frac{\left(Bb^2+Abc+\left(Bbc+Ac^2\right)x^2\right)\sqrt{bc}}{2\left(b^2c^3x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt
(-b*c)*x - b)/(c*x^2 + b)) + 2*(B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^
2), 1/2*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x
/b) - (B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^2)]
```

giac [A] time = 0.18, size = 57, normalized size = 0.90

$$\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,bc}-\frac{Bbx-Acx}{2\left(cx^2+b\right)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) - 1/2*(B*b*x - A*c*x)
/((c*x^2 + b)*b*c)
```

maple [A] time = 0.06, size = 68, normalized size = 1.08

$$\frac{A\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b}+\frac{B\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,c}+\frac{(Ac-bB)\,x}{2\left(c\,x^2+b\right)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A
+1/2/c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 2.96, size = 57, normalized size = 0.90

$$-\frac{(Bb-Ac)x}{2\left(bc^2x^2+b^2c\right)}+\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*x/(b*c^2*x^2 + b^2*c) + 1/2*(B*b + A*c)*arctan(c*x/sqrt(b*
c))/(sqrt(b*c)*b*c)
```

mupad [B] time = 0.12, size = 51, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)(A\,c+B\,b)}{2\,b^{3/2}\,c^{3/2}}+\frac{x\,(A\,c-B\,b)}{2\,b\,c\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atan((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(2*b^(3/2)*c^(3/2)) + (x*(A*c - B*b
))/(2*b*c*(b + c*x^2))
```

sympy [B] time = 0.42, size = 112, normalized size = 1.78

$$\frac{x\,(Ac-Bb)}{2b^2c+2bc^2x^2}-\frac{\sqrt{-\frac{1}{b^3c^3}}\,(Ac+Bb)\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{b^3c^3}}\,(Ac+Bb)\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] x*(A*c - B*b)/(2*b**2*c + 2*b*c**2*x**2) - sqrt(-1/(b**3*c**3))*(A*c + B*b)
*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/4 + sqrt(-1/(b**3*c**3))*(A*c + B*b)
*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/4
```

3.67 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{A\log(b+cx^2)}{2b^2}+\frac{A\log(x)}{b^2}-\frac{bB-Ac}{2bc(b+cx^2)}$$

```
[Out] 1/2*(A*c-B*b)/b/c/(c*x^2+b)+A*ln(x)/b^2-1/2*A*ln(c*x^2+b)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{A\log(b+cx^2)}{2b^2}+\frac{A\log(x)}{b^2}-\frac{bB-Ac}{2bc(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*B - A*c)/(2*b*c*(b + c*x^2)) + (A*Log[x])/b^2 - (A*Log[b + c*x^2])/(2*b
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{b^2x}+\frac{bB-Ac}{b(b+cx)^2}-\frac{Ac}{b^2(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{bB-Ac}{2bc\left(b+cx^2\right)}+\frac{A\log(x)}{b^2}-\frac{A\log\left(b+cx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.90

$$\frac{\frac{b(Ac-bB)}{c\left(b+cx^2\right)}-A\log\left(b+cx^2\right)+2A\log(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((b*(-(b*B) + A*c))/(c*(b + c*x^2)) + 2*A*Log[x] - A*Log[b + c*x^2])/(2*b^2
)
```

fricas [A] time = 0.88, size = 70, normalized size = 1.37

$$-\frac{Bb^2-Abc+\left(Ac^2x^2+Abc\right)\log\left(cx^2+b\right)-2\left(Ac^2x^2+Abc\right)\log(x)}{2\left(b^2c^2x^2+b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(B*b^2 - A*b*c + (A*c^2*x^2 + A*b*c)*log(c*x^2 + b) - 2*(A*c^2*x^2 + A
*b*c)*log(x))/(b^2*c^2*x^2 + b^3*c)
```

giac [A] time = 0.16, size = 52, normalized size = 1.02

$$-\frac{A\log\left(\left|cx^2+b\right|\right)}{2\,b^2}+\frac{A\log\left(|x|\right)}{b^2}-\frac{Bb^2-Abc}{2\left(cx^2+b\right)b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(abs(c*x^2 + b))/b^2 + A*log(abs(x))/b^2 - 1/2*(B*b^2 - A*b*c)/((
c*x^2 + b)*b^2*c)
```

maple [A] time = 0.06, size = 53, normalized size = 1.04

$$\frac{A}{2\left(c\,x^2+b\right)b}+\frac{A\ln(x)}{b^2}-\frac{A\ln\left(c\,x^2+b\right)}{2b^2}-\frac{B}{2\left(c\,x^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*A*ln(c*x^2+b)/b^2+1/2/b/(c*x^2+b)*A-1/2/c/(c*x^2+b)*B+A*ln(x)/b^2
```

maxima [A] time = 1.31, size = 51, normalized size = 1.00

$$-\frac{Bb - Ac}{2\left(bc^2x^2 + b^2c\right)} - \frac{A\log\left(cx^2 + b\right)}{2\,b^2} + \frac{A\log\left(x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)/(b*c^2*x^2 + b^2*c) - 1/2*A*log(c*x^2 + b)/b^2 + 1/2*A*log
(x^2)/b^2
```

mupad [B] time = 0.16, size = 47, normalized size = 0.92

$$\frac{A\,\ln(x)}{b^2} - \frac{A\,\ln\left(c\,x^2 + b\right)}{2\,b^2} + \frac{A\,c - B\,b}{2\,b\,c\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (A*log(x))/b^2 - (A*log(b + c*x^2))/(2*b^2) + (A*c - B*b)/(2*b*c*(b + c*x^2
))
```

sympy [A] time = 0.44, size = 46, normalized size = 0.90

$$\frac{A\log\left(x\right)}{b^2} - \frac{A\log\left(\frac{b}{c} + x^2\right)}{2b^2} + \frac{Ac - Bb}{2b^2c + 2bc^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] A*log(x)/b**2 - A*log(b/c + x**2)/(2*b**2) + (A*c - B*b)/(2*b**2*c + 2*b*c*
*2*x**2)
```

3.68 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=70

$$\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}+\frac{x(bB-Ac)}{2b^2\left(b+cx^2\right)}-\frac{A}{b^2x}$$

```
[Out] -A/b^2/x+1/2*(-A*c+B*b)*x/b^2/(c*x^2+b)+1/2*(-3*A*c+B*b)*arctan(x*c^(1/2)/b
^(1/2))/b^(5/2)/c^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 456, 453, 205}

$$\frac{x(bB-Ac)}{2b^2\left(b+cx^2\right)}+\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}-\frac{A}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^2 \left(b+cx^2\right)^2}\,dx$$
$$= \frac{(bB-Ac)x}{2b^2 \left(b+cx^2\right)} - \frac{1}{2}\int \frac{-\frac{2A}{b} - \frac{(bB-Ac)x^2}{b^2}}{x^2 \left(b+cx^2\right)}\,dx$$
$$= -\frac{A}{b^2 x} + \frac{(bB-Ac)x}{2b^2 \left(b+cx^2\right)} + \frac{(bB-3Ac)\int \frac{1}{b+cx^2}\,dx}{2b^2}$$
$$= -\frac{A}{b^2 x} + \frac{(bB-Ac)x}{2b^2 \left(b+cx^2\right)} + \frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.00

$$\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}} + \frac{x(bB-Ac)}{2b^2 \left(b+cx^2\right)} - \frac{A}{b^2 x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

fricas [A] time = 1.03, size = 210, normalized size = 3.00

$$\left[-\frac{4\,Ab^2c - 2\left(Bb^2c - 3\,Abc^2\right)x^2 - \left(\left(Bbc - 3\,Ac^2\right)x^3 + \left(Bb^2 - 3\,Abc\right)x\right)\sqrt{-bc}\,\log\left(\frac{cx^2 + 2\,\sqrt{-bc}\,x - b}{cx^2 + b}\right)}{4\left(b^3c^2x^3 + b^4cx\right)}, -\frac{2\,Ab^2c - \left(Bb^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*b^2*c - 2*(B*b^2*c - 3*A*b*c^2)*x^2 - ((B*b*c - 3*A*c^2)*x^3 + (
B*b^2 - 3*A*b*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
))/(b^3*c^2*x^3 + b^4*c*x), -1/2*(2*A*b^2*c - (B*b^2*c - 3*A*b*c^2)*x^2 - (
(B*b*c - 3*A*c^2)*x^3 + (B*b^2 - 3*A*b*c)*x)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b^3*c^2*x^3 + b^4*c*x)]
```

giac [A] time = 0.21, size = 62, normalized size = 0.89

$$\frac{(Bb - 3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2} + \frac{Bbx^2 - 3\,Acx^2 - 2\,Ab}{2\left(cx^3 + bx\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b - 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/2*(B*b*x^2 - 3*
A*c*x^2 - 2*A*b)/((c*x^3 + b*x)*b^2)
```

maple [A] time = 0.06, size = 85, normalized size = 1.21

$$-\frac{Acx}{2\left(c\,x^2+b\right)b^2} - \frac{3Ac\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^2} + \frac{Bx}{2\left(c\,x^2+b\right)b} + \frac{B\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b} - \frac{A}{b^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*x/(c*x^2+b)*A*c+1/2/b*x/(c*x^2+b)*B-3/2/b^2/(b*c)^(1/2)*arctan(1/(
b*c)^(1/2)*c*x)*A*c+1/2/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B-A/b^2/x
```

maxima [A] time = 2.98, size = 63, normalized size = 0.90

$$\frac{(Bb - 3\,Ac)x^2 - 2\,Ab}{2\left(b^2cx^3 + b^3x\right)} + \frac{(Bb - 3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 3*A*c)*x^2 - 2*A*b)/(b^2*c*x^3 + b^3*x) + 1/2*(B*b - 3*A*c)*arc
tan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2)
```

mupad [B] time = 0.13, size = 63, normalized size = 0.90

$$-\frac{\frac{A}{b} + \frac{x^2\,(3\,A\,c - B\,b)}{2\,b^2}}{c\,x^3 + b\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c - B\,b)}{2\,b^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - (A/b + (x^2*(3*A*c - B*b))/(2*b^2))/(b*x + c*x^3) - (atan((c^(1/2)*x)/b^(
1/2))*(3*A*c - B*b))/(2*b^(5/2)*c^(1/2))
```

sympy [A] time = 0.51, size = 114, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^5c}}\,(-3Ac + Bb)\log\left(-b^3\sqrt{-\frac{1}{b^5c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{b^5c}}\,(-3Ac + Bb)\log\left(b^3\sqrt{-\frac{1}{b^5c}} + x\right)}{4} + \frac{-2Ab + x^2\,(-3Ac + Bb)}{2b^3x + 2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-1/(b**5*c))*(-3*A*c + B*b)*log(-b**3*sqrt(-1/(b**5*c)) + x)/4 + sqrt
(-1/(b**5*c))*(-3*A*c + B*b)*log(b**3*sqrt(-1/(b**5*c)) + x)/4 + (-2*A*b +
x**2*(-3*A*c + B*b))/(2*b**3*x + 2*b**2*c*x**3)
```

3.69 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)(bB-2Ac)}{b^3}+\frac{bB-Ac}{2b^2\left(b+cx^2\right)}-\frac{A}{2b^2x^2}$$

```
[Out] -1/2*A/b^2/x^2+1/2*(-A*c+B*b)/b^2/(c*x^2+b)+(-2*A*c+B*b)*ln(x)/b^3-1/2*(-2*
A*c+B*b)*ln(c*x^2+b)/b^3
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 77}

$$\frac{bB-Ac}{2b^2\left(b+cx^2\right)}-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)(bB-2Ac)}{b^3}-\frac{A}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -A/(2*b^2*x^2) + (b*B - A*c)/(2*b^2*(b + c*x^2)) + ((b*B - 2*A*c)*Log[x])/b
^3 - ((b*B - 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)^2}\,dx\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{A}{b^2x^2}+\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)}{b^2(b+cx)^2}-\frac{c(bB-2Ac)}{b^3(b+cx)}\right)dx,x,x^2\right)\\
&= -\frac{A}{2b^2x^2}+\frac{bB-Ac}{2b^2\left(b+cx^2\right)}+\frac{(bB-2Ac)\log(x)}{b^3}-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.88

$$
\frac{\frac{b(bB-Ac)}{b+cx^2}+(2Ac-bB)\log\left(b+cx^2\right)+2\log(x)(bB-2Ac)-\frac{Ab}{x^2}}{2b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-((A*b)/x^2) + (b*(b*B - A*c))/(b + c*x^2) + 2*(b*B - 2*A*c)*Log[x] + (-(b
*B) + 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

fricas [A] time = 0.80, size = 117, normalized size = 1.60

$$
-\frac{Ab^2-\left(Bb^2-2\,Abc\right)x^2+\left(\left(Bbc-2\,Ac^2\right)x^4+\left(Bb^2-2\,Abc\right)x^2\right)\log\left(cx^2+b\right)-2\left(\left(Bbc-2\,Ac^2\right)x^4+\left(Bb^2-\right.\right.}{2\left(b^3cx^4+b^4x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2 - (B*b^2 - 2*A*b*c)*x^2 + ((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A
*b*c)*x^2)*log(c*x^2 + b) - 2*((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A*b*c)*x^
2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

giac [A] time = 0.16, size = 80, normalized size = 1.10

$$
\frac{(Bb-2\,Ac)\log\left(|x|\right)}{b^3}+\frac{Bbx^2-2\,Acx^2-Ab}{2\left(cx^4+bx^2\right)b^2}-\frac{\left(Bbc-2\,Ac^2\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^3c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*log(abs(x))/b^3 + 1/2*(B*b*x^2 - 2*A*c*x^2 - A*b)/((c*x^4 + b
*x^2)*b^2) - 1/2*(B*b*c - 2*A*c^2)*log(abs(c*x^2 + b))/(b^3*c)
```

maple [A] time = 0.07, size = 86, normalized size = 1.18

$$
-\frac{Ac}{2\left(c\,x^2+b\right)b^2}-\frac{2Ac\ln(x)}{b^3}+\frac{Ac\ln\left(c\,x^2+b\right)}{b^3}+\frac{B}{2\left(c\,x^2+b\right)b}+\frac{B\ln(x)}{b^2}-\frac{B\ln\left(c\,x^2+b\right)}{2b^2}-\frac{A}{2b^2x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/b^3*c*ln(c*x^2+b)*A-1/2/b^2*ln(c*x^2+b)*B-1/2/b^2*c/(c*x^2+b)*A+1/2/b/(c*
x^2+b)*B-1/2*A/b^2/x^2-2/b^3*ln(x)*A*c+1/b^2*ln(x)*B
```

maxima [A] time = 1.33, size = 76, normalized size = 1.04

$$\frac{(Bb-2\,Ac)x^2-Ab}{2\left(b^2cx^4+b^3x^2\right)}-\frac{(Bb-2\,Ac)\log\left(cx^2+b\right)}{2\,b^3}+\frac{(Bb-2\,Ac)\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2) - 1/2*(B*b - 2*A*c)*log
(c*x^2 + b)/b^3 + 1/2*(B*b - 2*A*c)*log(x^2)/b^3
```

mupad [B] time = 0.15, size = 78, normalized size = 1.07

$$\frac{\ln\left(c\,x^2+b\right)\,\left(2\,A\,c-B\,b\right)}{2\,b^3}-\frac{\frac{A}{2\,b}+\frac{x^2\,\left(2\,A\,c-B\,b\right)}{2\,b^2}}{c\,x^4+b\,x^2}-\frac{\ln(x)\,\left(2\,A\,c-B\,b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (log(b + c*x^2)*(2*A*c - B*b))/(2*b^3) - (A/(2*b) + (x^2*(2*A*c - B*b))/(2*
b^2))/(b*x^2 + c*x^4) - (log(x)*(2*A*c - B*b))/b^3
```

sympy [A] time = 0.88, size = 70, normalized size = 0.96

$$\frac{-Ab+x^2\left(-2Ac+Bb\right)}{2b^3x^2+2b^2cx^4}+\frac{\left(-2Ac+Bb\right)\log\left(x\right)}{b^3}-\frac{\left(-2Ac+Bb\right)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-A*b + x**2*(-2*A*c + B*b))/(2*b**3*x**2 + 2*b**2*c*x**4) + (-2*A*c + B*b)
*log(x)/b**3 - (-2*A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

3.70 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=90

$$-\frac{\sqrt{c}\,(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{b^3x}-\frac{A}{3b^2x^3}$$

```
[Out] -1/3*A/b^2/x^3+(2*A*c-B*b)/b^3/x-1/2*c*(-A*c+B*b)*x/b^3/(c*x^2+b)-1/2*(-5*A
*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(7/2)
```

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1593, 456, 1261, 205}

$$-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{b^3x}-\frac{\sqrt{c}\,(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{A}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -A/(3*b^2*x^3) - (b*B - 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b + c*x^
2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)} - \frac{1}{2}c\int \frac{-\frac{2A}{bc}-\frac{2(bB-Ac)x^2}{b^2c}+\frac{(bB-Ac)x^4}{b^3}}{x^4\left(b+cx^2\right)}\,dx$$
$$= -\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)} - \frac{1}{2}c\int \left(-\frac{2A}{b^2cx^4}-\frac{2(bB-2Ac)}{b^3cx^2}+\frac{3bB-5Ac}{b^3\left(b+cx^2\right)}\right)dx$$
$$= -\frac{A}{3b^2x^3}-\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{(c(3bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{2b^3}$$
$$= -\frac{A}{3b^2x^3}-\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{\sqrt{c}\,(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 1.00

$$-\frac{\sqrt{c}\,(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}+\frac{2Ac-bB}{b^3x}-\frac{A}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/3*A/(b^2*x^3) + (-(b*B) + 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b +
 c*x^2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2)
)
```

fricas [A] time = 0.89, size = 250, normalized size = 2.78

$$\left[-\frac{6\left(3\,Bbc-5\,Ac^2\right)x^4+4\,Ab^2+4\left(3\,Bb^2-5\,Abc\right)x^2+3\left(\left(3\,Bbc-5\,Ac^2\right)x^5+\left(3\,Bb^2-5\,Abc\right)x^3\right)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+\cdots}{\cdots}\right.}{12\left(b^3cx^5+b^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*b*c - 5*A*c^2)*x^4 + 4*A*b^2 + 4*(3*B*b^2 - 5*A*b*c)*x^2 + 3
*((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(-c/b)*log((c*x^2
+ 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^3*c*x^5 + b^4*x^3), -1/6*(3*(3*B*b
*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2 + 3*((3*B*b*c - 5*A
*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^3*c*
x^5 + b^4*x^3)]
```

giac [A] time = 0.16, size = 85, normalized size = 0.94

$$-\frac{\left(3\,Bbc-5\,Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^3}-\frac{Bbcx-Ac^2x}{2\left(cx^2+b\right)b^3}-\frac{3\,Bbx^2-6\,Acx^2+Ab}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/2*(B*b*c
*x - A*c^2*x)/((c*x^2 + b)*b^3) - 1/3*(3*B*b*x^2 - 6*A*c*x^2 + A*b)/(b^3*x^
3)
```

maple [A] time = 0.07, size = 110, normalized size = 1.22

$$\frac{A\,c^2 x}{2\left(c\,x^2+b\right)b^3}+\frac{5A\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^3}-\frac{Bcx}{2\left(c\,x^2+b\right)b^2}-\frac{3Bc\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^2}+\frac{2Ac}{b^3x}-\frac{B}{b^2x}-\frac{A}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x/(c*x^2+b)*A-1/2/b^2*c*x/(c*x^2+b)*B+5/2/b^3*c^2/(b*c)^(1/2)*a
rctan(1/(b*c)^(1/2)*c*x)*A-3/2/b^2*c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*
B-1/3*A/b^2/x^3+2/b^3/x*A*c-1/b^2/x*B
```

maxima [A] time = 3.00, size = 93, normalized size = 1.03

$$-\frac{3\left(3\,Bbc-5\,Ac^2\right)x^4+2\,Ab^2+2\left(3\,Bb^2-5\,Abc\right)x^2}{6\left(b^3cx^5+b^4x^3\right)}-\frac{\left(3\,Bbc-5\,Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*B*b*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2)/(b^3
*c*x^5 + b^4*x^3) - 1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c
)*b^3)
```

mupad [B] time = 0.14, size = 83, normalized size = 0.92

$$\frac{\frac{x^2\,(5\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{c\,x^4\,(5\,A\,c-3\,B\,b)}{2\,b^3}}{c\,x^5+b\,x^3}+\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(5\,A\,c-3\,B\,b)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^2*(5*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (c*x^4*(5*A*c - 3*B*b))/(2*b^3))
/(b*x^3 + c*x^5) + (c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(5*A*c - 3*B*b))/(2*b
^(7/2))
```

sympy [B] time = 0.60, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4}-\frac{\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4}+\frac{-2Ab^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(-b**4*sqrt(-c/b**7)*(-5*A*c + 3*B*b)/(-5
*A*c**2 + 3*B*b*c) + x)/4 - sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(b**4*sqrt(-c
/b**7)*(-5*A*c + 3*B*b)/(-5*A*c**2 + 3*B*b*c) + x)/4 + (-2*A*b**2 + x**4*(1
5*A*c**2 - 9*B*b*c) + x**2*(10*A*b*c - 6*B*b**2))/(6*b**4*x**3 + 6*b**3*c*x
**5)
```

3.71 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=97

$$\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}-\frac{c\log(x)(2bB-3Ac)}{b^4}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{2b^3x^2}-\frac{A}{4b^2x^4}$$

```
[Out] -1/4*A/b^2/x^4+1/2*(2*A*c-B*b)/b^3/x^2-1/2*c*(-A*c+B*b)/b^3/(c*x^2+b)-c*(-3
*A*c+2*B*b)*ln(x)/b^4+1/2*c*(-3*A*c+2*B*b)*ln(c*x^2+b)/b^4
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{2b^3x^2}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}-\frac{c\log(x)(2bB-3Ac)}{b^4}-\frac{A}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -A/(4*b^2*x^4) - (b*B - 2*A*c)/(2*b^3*x^2) - (c*(b*B - A*c))/(2*b^3*(b + c*
x^2)) - (c*(2*b*B - 3*A*c)*Log[x])/b^4 + (c*(2*b*B - 3*A*c)*Log[b + c*x^2])
/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^2x^3}+\frac{bB-2Ac}{b^3x^2}-\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^2}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)}\right)dx,\right.$$
$$= -\frac{A}{4b^2x^4}-\frac{bB-2Ac}{2b^3x^2}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{c(2bB-3Ac)\log(x)}{b^4}+\frac{c(2bB-3Ac)\log\left(b+c\right.}{2b^4}$$

Mathematica [A] time = 0.11, size = 85, normalized size = 0.88

$$-\frac{\frac{Ab^2}{x^4}+\frac{2bc(bB-Ac)}{b+cx^2}+\frac{2b(bB-2Ac)}{x^2}+2c(3Ac-2bB)\log\left(b+cx^2\right)-4c\log(x)(3Ac-2bB)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((A*b^2)/x^4 + (2*b*(b*B - 2*A*c))/x^2 + (2*b*c*(b*B - A*c))/(b + c*x^
2) - 4*c*(-2*b*B + 3*A*c)*Log[x] + 2*c*(-2*b*B + 3*A*c)*Log[b + c*x^2])/b^4
```

fricas [A] time = 0.93, size = 154, normalized size = 1.59

$$-\frac{2\left(2\,Bb^2c-3\,Abc^2\right)x^4+Ab^3+\left(2\,Bb^3-3\,Ab^2c\right)x^2-2\left(\left(2\,Bbc^2-3\,Ac^3\right)x^6+\left(2\,Bb^2c-3\,Abc^2\right)x^4\right)\log\left(cx^2+\right.}{4\left(b^4cx^6+b^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*b^2*c - 3*A*b*c^2)*x^4 + A*b^3 + (2*B*b^3 - 3*A*b^2*c)*x^2 - 2
*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(c*x^2 + b) +
 4*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(x))/(b^4*c
*x^6 + b^5*x^4)
```

giac [A] time = 0.32, size = 150, normalized size = 1.55

$$-\frac{\left(2\,Bbc-3\,Ac^2\right)\log\left(x^2\right)}{2\,b^4}+\frac{\left(2\,Bbc^2-3\,Ac^3\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^4c}-\frac{2\,Bbc^2x^2-3\,Ac^3x^2+3\,Bb^2c-4\,Abc^2}{2\left(cx^2+b\right)b^4}+\frac{6\,Bbcx^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*b*c - 3*A*c^2)*log(x^2)/b^4 + 1/2*(2*B*b*c^2 - 3*A*c^3)*log(abs(c
*x^2 + b))/(b^4*c) - 1/2*(2*B*b*c^2*x^2 - 3*A*c^3*x^2 + 3*B*b^2*c - 4*A*b*c
^2)/((c*x^2 + b)*b^4) + 1/4*(6*B*b*c*x^4 - 9*A*c^2*x^4 - 2*B*b^2*x^2 + 4*A*
b*c*x^2 - A*b^2)/(b^4*x^4)
```

maple [A] time = 0.06, size = 114, normalized size = 1.18

$$\frac{A\,c^2}{2\left(c\,x^2+b\right)b^3}+\frac{3A\,c^2\ln(x)}{b^4}-\frac{3A\,c^2\ln\left(c\,x^2+b\right)}{2b^4}-\frac{Bc}{2\left(c\,x^2+b\right)b^2}-\frac{2Bc\ln(x)}{b^3}+\frac{Bc\ln\left(c\,x^2+b\right)}{b^3}+\frac{Ac}{b^3x^2}-\frac{B}{2b^2x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -3/2/b^4*c^2*ln(c*x^2+b)*A+1/b^3*c*ln(c*x^2+b)*B+1/2/b^3*c^2/(c*x^2+b)*A-1/
2/b^2*c/(c*x^2+b)*B-1/4*A/b^2/x^4+1/b^3/x^2*A*c-1/2/b^2/x^2*B+3*c^2/b^4*ln(
x)*A-2*c/b^3*ln(x)*B
```

maxima [A] time = 1.33, size = 106, normalized size = 1.09

$$-\frac{2\left(2\,Bbc-3\,Ac^{2}\right)x^{4}+Ab^{2}+\left(2\,Bb^{2}-3\,Abc\right)x^{2}}{4\left(b^{3}cx^{6}+b^{4}x^{4}\right)}+\frac{\left(2\,Bbc-3\,Ac^{2}\right)\log\left(cx^{2}+b\right)}{2\,b^{4}}-\frac{\left(2\,Bbc-3\,Ac^{2}\right)\log\left(x^{2}\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*b*c - 3*A*c^2)*x^4 + A*b^2 + (2*B*b^2 - 3*A*b*c)*x^2)/(b^3*c*x
^6 + b^4*x^4) + 1/2*(2*B*b*c - 3*A*c^2)*log(c*x^2 + b)/b^4 - 1/2*(2*B*b*c -
 3*A*c^2)*log(x^2)/b^4
```

mupad [B] time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^{2}\,(3\,A\,c-2\,B\,b)}{4\,b^{2}}-\frac{A}{4\,b}+\frac{c\,x^{4}\,(3\,A\,c-2\,B\,b)}{2\,b^{3}}}{c\,x^{6}+b\,x^{4}}-\frac{\ln\left(c\,x^{2}+b\right)\,\left(3\,A\,c^{2}-2\,B\,b\,c\right)}{2\,b^{4}}+\frac{\ln(x)\,\left(3\,A\,c^{2}-2\,B\,b\,c\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(3*A*c - 2*B*b))/(4*b^2) - A/(4*b) + (c*x^4*(3*A*c - 2*B*b))/(2*b^3))
/(b*x^4 + c*x^6) - (log(b + c*x^2)*(3*A*c^2 - 2*B*b*c))/(2*b^4) + (log(x)*(
3*A*c^2 - 2*B*b*c))/b^4
```

sympy [A] time = 1.04, size = 100, normalized size = 1.03

$$\frac{-Ab^{2}+x^{4}\left(6Ac^{2}-4Bbc\right)+x^{2}\left(3Abc-2Bb^{2}\right)}{4b^{4}x^{4}+4b^{3}cx^{6}}-\frac{c\left(-3Ac+2Bb\right)\log\left(x\right)}{b^{4}}+\frac{c\left(-3Ac+2Bb\right)\log\left(\frac{b}{c}+x^{2}\right)}{2b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-A*b**2 + x**4*(6*A*c**2 - 4*B*b*c) + x**2*(3*A*b*c - 2*B*b**2))/(4*b**4*x
**4 + 4*b**3*c*x**6) - c*(-3*A*c + 2*B*b)*log(x)/b**4 + c*(-3*A*c + 2*B*b)*
log(b/c + x**2)/(2*b**4)
```

3.72 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=111

$$\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}+\frac{c(2bB-3Ac)}{b^4x}-\frac{bB-2Ac}{3b^3x^3}-\frac{A}{5b^2x^5}$$

```
[Out] -1/5*A/b^2/x^5+1/3*(2*A*c-B*b)/b^3/x^3+c*(-3*A*c+2*B*b)/b^4/x+1/2*c^2*(-A*c
+B*b)*x/b^4/(c*x^2+b)+1/2*c^(3/2)*(-7*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/
b^(9/2)
```

Rubi [A] time = 0.20, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 456, 1802, 205}

$$\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}-\frac{A}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -A/(5*b^2*x^5) - (b*B - 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*x) +
(c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*ArcTan[(
Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx=\int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^2}\,dx$$
$$=\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}-\frac{1}{2}c^2\int \frac{-\frac{2A}{bc^2}-\frac{2(bB-Ac)x^2}{b^2c^2}+\frac{2(bB-Ac)x^4}{b^3c}-\frac{(bB-Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)}\,dx$$
$$=\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}-\frac{1}{2}c^2\int \left(-\frac{2A}{b^2c^2x^6}-\frac{2(bB-2Ac)}{b^3c^2x^4}+\frac{2(2bB-3Ac)}{b^4cx^2}+\frac{-5bB+7Ac}{b^4\left(b+cx^2\right)}\right)dx$$
$$=-\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{\left(c^2(5bB-7Ac)\right)\int \frac{1}{b+cx^2}\,dx}{2b^4}$$
$$=-\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}$$

Mathematica [A] time = 0.10, size = 112, normalized size = 1.01

$$\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}+\frac{c(2bB-3Ac)}{b^4x}+\frac{2Ac-bB}{3b^3x^3}-\frac{A}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/5*A/(b^2*x^5) + (-(b*B) + 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*
x) + (c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

fricas [A] time = 0.81, size = 308, normalized size = 2.77

$$\left[\frac{30\left(5Bbc^2-7Ac^3\right)x^6+20\left(5Bb^2c-7Abc^2\right)x^4-12Ab^3-4\left(5Bb^3-7Ab^2c\right)x^2-15\left(\left(5Bbc^2-7Ac^3\right)x^7+\left(5B\right.\right.}{60\left(b^4cx^7+b^5x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*b*c^2 - 7*A*c^3)*x^6 + 20*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 12*A
*b^3 - 4*(5*B*b^3 - 7*A*b^2*c)*x^2 - 15*((5*B*b*c^2 - 7*A*c^3)*x^7 + (5*B*b
^2*c - 7*A*b*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2
 + b)))/(b^4*c*x^7 + b^5*x^5), 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B
*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2 + 15*((5*B*
b*c^2 - 7*A*c^3)*x^7 + (5*B*b^2*c - 7*A*b*c^2)*x^5)*sqrt(c/b)*arctan(x*sqrt
(c/b)))/(b^4*c*x^7 + b^5*x^5)]
```

giac [A] time = 0.17, size = 112, normalized size = 1.01

$$\frac{\left(5Bbc^2-7Ac^3\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b^4}+\frac{Bbc^2x-Ac^3x}{2\left(cx^2+b\right)b^4}+\frac{30Bbcx^4-45Ac^2x^4-5Bb^2x^2+10Abcx^2-3Ab^2}{15b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b*c^2 - 7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/2*(B*b*
c^2*x - A*c^3*x)/((c*x^2 + b)*b^4) + 1/15*(30*B*b*c*x^4 - 45*A*c^2*x^4 - 5*
B*b^2*x^2 + 10*A*b*c*x^2 - 3*A*b^2)/(b^4*x^5)
```

maple [A] time = 0.06, size = 136, normalized size = 1.23

$$-\frac{A\,c^3 x}{2\left(c\,x^2+b\right)b^4}-\frac{7A\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^4}+\frac{B\,c^2 x}{2\left(c\,x^2+b\right)b^3}+\frac{5B\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^3}-\frac{3A\,c^2}{b^4 x}+\frac{2Bc}{b^3 x}+\frac{2Ac}{3b^3x^3}-\frac{B}{3b^2x^3}-\frac{A}{5b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x/(c*x^2+b)*A+1/2/b^3*c^2*x/(c*x^2+b)*B-7/2/b^4*c^3/(b*c)^(1/2
)*arctan(1/(b*c)^(1/2)*c*x)*A+5/2/b^3*c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*
c*x)*B-1/5*A/b^2/x^5+2/3/b^3/x^3*A*c-1/3/b^2/x^3*B-3*c^2/b^4/x*A+2*c/b^3/x*
B
```

maxima [A] time = 2.94, size = 119, normalized size = 1.07

$$\frac{15\left(5\,Bbc^2-7\,Ac^3\right)x^6+10\left(5\,Bb^2c-7\,Abc^2\right)x^4-6\,Ab^3-2\left(5\,Bb^3-7\,Ab^2c\right)x^2}{30\left(b^4cx^7+b^5x^5\right)}+\frac{\left(5\,Bbc^2-7\,Ac^3\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b
^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c*x^7 + b^5*x^5) + 1/2*(5*B*b*c^2 -
7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

mupad [B] time = 0.16, size = 104, normalized size = 0.94

$$-\frac{\frac{A}{5\,b}-\frac{x^2\,\left(7\,A\,c-5\,B\,b\right)}{15\,b^2}+\frac{c^2\,x^6\,\left(7\,A\,c-5\,B\,b\right)}{2\,b^4}+\frac{c\,x^4\,\left(7\,A\,c-5\,B\,b\right)}{3\,b^3}}{c\,x^7+b\,x^5}-\frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(7\,A\,c-5\,B\,b\right)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x)
```

```
[Out] - (A/(5*b) - (x^2*(7*A*c - 5*B*b))/(15*b^2) + (c^2*x^6*(7*A*c - 5*B*b))/(2*
b^4) + (c*x^4*(7*A*c - 5*B*b))/(3*b^3))/(b*x^5 + c*x^7) - (c^(3/2)*atan((c^
(1/2)*x)/b^(1/2))*(7*A*c - 5*B*b))/(2*b^(9/2))
```

sympy [B] time = 0.70, size = 218, normalized size = 1.96

$$-\frac{\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}+\frac{\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}+\frac{-6Ab^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log(-b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B
*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log
(b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + (
-6*A*b**3 + x**6*(-105*A*c**3 + 75*B*b*c**2) + x**4*(-70*A*b*c**2 + 50*B*b*
*2*c) + x**2*(14*A*b**2*c - 10*B*b**3))/(30*b**5*x**5 + 30*b**4*c*x**7)
```

3.73 $\int \frac{x^{14}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{11/2}}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}+\frac{b^2x(17bB-13Ac)}{8c^5\left(b+cx^2\right)}+\frac{3bx(2bB-Ac)}{c^5}-\frac{x^3(3bB-Ac)}{3c^4}+\frac{Bx^5}{5c^3}$$

```
[Out] 3*b*(-A*c+2*B*b)*x/c^5-1/3*(-A*c+3*B*b)*x^3/c^4+1/5*B*x^5/c^3-1/4*b^3*(-A*c
+B*b)*x/c^5/(c*x^2+b)^2+1/8*b^2*(-13*A*c+17*B*b)*x/c^5/(c*x^2+b)-7/8*b^(3/2
)*(-5*A*c+9*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

Rubi [A] time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 455, 1814, 1810, 205}

$$\frac{b^2x(17bB-13Ac)}{8c^5\left(b+cx^2\right)}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}-\frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{11/2}}-\frac{x^3(3bB-Ac)}{3c^4}+\frac{3bx(2bB-Ac)}{c^5}+\frac{Bx^5}{5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*b*(2*b*B - A*c)*x)/c^5 - ((3*b*B - A*c)*x^3)/(3*c^4) + (B*x^5)/(5*c^3) -
 (b^3*(b*B - A*c)*x)/(4*c^5*(b + c*x^2)^2) + (b^2*(17*b*B - 13*A*c)*x)/(8*c
^5*(b + c*x^2)) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*c^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{14}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^8\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} - \frac{\int \frac{-b^3(bB-Ac)+4b^2c(bB-Ac)x^2-4bc^2(bB-Ac)x^4+4c^3(bB-Ac)x^6-4Bc^4x^8}{\left(b+cx^2\right)^2}\,dx}{4c^5}$$

$$= -\frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} + \frac{\int \frac{-b^3(15bB-11Ac)+8b^2c(3bB-2Ac)x^2-8bc^2(2bB-Ac)x^4+8b}{b+cx^2}}{8bc^5}$$

$$= -\frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} + \frac{\int \left(24b^2(2bB-Ac) - 8bc(3bB-Ac)x^2 + 8bBc\right.}{8bc^5}$$

$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} - \frac{\left(7b^2(9\right.}{}$$

$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2} + \frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)} - \frac{7b^{3/2}(9}{}$$

Mathematica [A] time = 0.14, size = 133, normalized size = 0.95

$$\frac{x\left(-525b^3c\left(A-3Bx^2\right)+7b^2c^2x^2\left(72Bx^2-125A\right)-8bc^3x^4\left(35A+9Bx^2\right)+8c^4x^6\left(5A+3Bx^2\right)+945b^4B\right)}{120c^5\left(b+cx^2\right)^2} - $$

Antiderivative was successfully verified.

```
[In] Integrate[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(945*b^4*B - 525*b^3*c*(A - 3*B*x^2) + 8*c^4*x^6*(5*A + 3*B*x^2) - 8*b*c
^3*x^4*(35*A + 9*B*x^2) + 7*b^2*c^2*x^2*(-125*A + 72*B*x^2)))/(120*c^5*(b +
 c*x^2)^2) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(
11/2))
```

fricas [A] time = 1.05, size = 416, normalized size = 2.97

$$\left[\frac{48\,Bc^4x^9 - 16\left(9\,Bbc^3 - 5\,Ac^4\right)x^7 + 112\left(9\,Bb^2c^2 - 5\,Abc^3\right)x^5 + 350\left(9\,Bb^3c - 5\,Ab^2c^2\right)x^3 - 105\left(9\,Bb^4 - 5\,Ab\right.}{240\left(c^7x^4 + 2\,bc^6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*B*c^4*x^9 - 16*(9*B*b*c^3 - 5*A*c^4)*x^7 + 112*(9*B*b^2*c^2 - 5*
A*b*c^3)*x^5 + 350*(9*B*b^3*c - 5*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c
```

```
 + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 + 2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(-b
/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 210*(9*B*b^4 - 5*A*b^
3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5), 1/120*(24*B*c^4*x^9 - 8*(9*B*b*c
^3 - 5*A*c^4)*x^7 + 56*(9*B*b^2*c^2 - 5*A*b*c^3)*x^5 + 175*(9*B*b^3*c - 5*A
*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 +
2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 105*(9
*B*b^4 - 5*A*b^3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)]
```

giac [A] time = 0.18, size = 138, normalized size = 0.99

$$-\frac{7\left(9\,Bb^3-5\,Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^5}+\frac{17\,Bb^3cx^3-13\,Ab^2c^2x^3+15\,Bb^4x-11\,Ab^3cx}{8\left(cx^2+b\right)^2c^5}+\frac{3\,Bc^{12}x^5-15\,Bbc^{11}x^3+5\,Ac}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/8*(17*
B*b^3*c*x^3 - 13*A*b^2*c^2*x^3 + 15*B*b^4*x - 11*A*b^3*c*x)/((c*x^2 + b)^2*
c^5) + 1/15*(3*B*c^12*x^5 - 15*B*b*c^11*x^3 + 5*A*c^12*x^3 + 90*B*b^2*c^10*
x - 45*A*b*c^11*x)/c^15
```

maple [A] time = 0.06, size = 174, normalized size = 1.24

$$-\frac{13A\,b^2x^3}{8\left(c\,x^2+b\right)^2c^3}+\frac{17B\,b^3x^3}{8\left(c\,x^2+b\right)^2c^4}+\frac{B\,x^5}{5c^3}-\frac{11A\,b^3x}{8\left(c\,x^2+b\right)^2c^4}+\frac{A\,x^3}{3c^3}+\frac{15B\,b^4x}{8\left(c\,x^2+b\right)^2c^5}-\frac{Bb\,x^3}{c^4}+\frac{35A\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/5*B*x^5/c^3+1/3/c^3*A*x^3-1/c^4*B*x^3*b-3/c^4*A*b*x+6/c^5*B*b^2*x-13/8*b^
2/c^3/(c*x^2+b)^2*A*x^3+17/8*b^3/c^4/(c*x^2+b)^2*B*x^3-11/8*b^3/c^4/(c*x^2+
b)^2*A*x+15/8*b^4/c^5/(c*x^2+b)^2*B*x+35/8*b^2/c^4/(b*c)^(1/2)*arctan(1/(b*
c)^(1/2)*c*x)*A-63/8*b^3/c^5/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 2.90, size = 147, normalized size = 1.05

$$\frac{\left(17\,Bb^3c-13\,Ab^2c^2\right)x^3+\left(15\,Bb^4-11\,Ab^3c\right)x}{8\left(c^7x^4+2\,bc^6x^2+b^2c^5\right)}-\frac{7\left(9\,Bb^3-5\,Ab^2c\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^5}+\frac{3\,Bc^2x^5-5\left(3\,Bbc-Ac^2\right)x^3+}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((17*B*b^3*c - 13*A*b^2*c^2)*x^3 + (15*B*b^4 - 11*A*b^3*c)*x)/(c^7*x^4
+ 2*b*c^6*x^2 + b^2*c^5) - 7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/
(sqrt(b*c)*c^5) + 1/15*(3*B*c^2*x^5 - 5*(3*B*b*c - A*c^2)*x^3 + 45*(2*B*b^2
 - A*b*c)*x)/c^5
```

mupad [B] time = 0.12, size = 177, normalized size = 1.26

$$\frac{x\left(\frac{15\,B\,b^4}{8}-\frac{11\,A\,b^3\,c}{8}\right)-x^3\left(\frac{13\,A\,b^2\,c^2}{8}-\frac{17\,B\,b^3\,c}{8}\right)}{b^2\,c^5+2\,b\,c^6\,x^2+c^7\,x^4}-x\left(\frac{3\,b\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right)}{c}+\frac{3\,B\,b^2}{c^5}\right)+x^3\left(\frac{A}{3\,c^3}-\frac{B\,b}{c^4}\right)+\frac{B\,x^5}{5\,c^3}-\frac{7\,b^{3/2}\,\mathrm{atan}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x*((15*B*b^4)/8 - (11*A*b^3*c)/8) - x^3*((13*A*b^2*c^2)/8 - (17*B*b^3*c)/8
))/(b^2*c^5 + c^7*x^4 + 2*b*c^6*x^2) - x*((3*b*(A/c^3 - (3*B*b)/c^4))/c + (
3*B*b^2)/c^5) + x^3*(A/(3*c^3) - (B*b)/c^4) + (B*x^5)/(5*c^3) - (7*b^(3/2)*
atan((b^(3/2)*c^(1/2)*x*(5*A*c - 9*B*b))/(9*B*b^3 - 5*A*b^2*c))*(5*A*c - 9*
B*b))/(8*c^(11/2))
```

sympy [A] time = 1.40, size = 252, normalized size = 1.80

$$\frac{Bx^5}{5c^3}+x^3\left(\frac{A}{3c^3}-\frac{Bb}{c^4}\right)+x\left(-\frac{3Ab}{c^4}+\frac{6Bb^2}{c^5}\right)+\frac{7\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)\log\left(-\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16}-\frac{7\sqrt{-\frac{b^3}{c^{11}}}\,(-5\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**5/(5*c**3) + x**3*(A/(3*c**3) - B*b/c**4) + x*(-3*A*b/c**4 + 6*B*b**2/
c**5) + 7*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)*log(-7*c**5*sqrt(-b**3/c**11)*
(-5*A*c + 9*B*b)/(-35*A*b*c + 63*B*b**2) + x)/16 - 7*sqrt(-b**3/c**11)*(-5*
A*c + 9*B*b)*log(7*c**5*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)/(-35*A*b*c + 63*
B*b**2) + x)/16 + (x**3*(-13*A*b**2*c**2 + 17*B*b**3*c) + x*(-11*A*b**3*c +
 15*B*b**4))/(8*b**2*c**5 + 16*b*c**6*x**2 + 8*c**7*x**4)
```

3.74 $\int \frac{x^{13}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=111

$$-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}+\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right)}{2c^5}-\frac{x^2(3bB-Ac)}{2c^4}+\frac{Bx^4}{4c^3}$$

```
[Out] -1/2*(-A*c+3*B*b)*x^2/c^4+1/4*B*x^4/c^3-1/4*b^3*(-A*c+B*b)/c^5/(c*x^2+b)^2+
1/2*b^2*(-3*A*c+4*B*b)/c^5/(c*x^2+b)+3/2*b*(-A*c+2*B*b)*ln(c*x^2+b)/c^5
```

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}-\frac{x^2(3bB-Ac)}{2c^4}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right)}{2c^5}+\frac{Bx^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((3*b*B - A*c)*x^2)/(2*c^4) + (B*x^4)/(4*c^3) - (b^3*(b*B - A*c))/(4*c^5*(
b + c*x^2)^2) + (b^2*(4*b*B - 3*A*c))/(2*c^5*(b + c*x^2)) + (3*b*(2*b*B - A
*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^7\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^3}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-3bB+Ac}{c^4}+\frac{Bx}{c^3}+\frac{b^3(bB-Ac)}{c^4(b+cx)^3}-\frac{b^2(4bB-3Ac)}{c^4(b+cx)^2}+\frac{3b(2bB-Ac)}{c^4(b+cx)}\right)dx,\right.$$
$$=-\frac{(3bB-Ac)x^2}{2c^4}+\frac{Bx^4}{4c^3}-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}+\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right.}{2c^5}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.85

$$\frac{\frac{b^3(Ac-bB)}{\left(b+cx^2\right)^2}+\frac{2b^2(4bB-3Ac)}{b+cx^2}+2cx^2(Ac-3bB)+6b(2bB-Ac)\log\left(b+cx^2\right)+Bc^2x^4}{4c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*c*(-3*b*B + A*c)*x^2 + B*c^2*x^4 + (b^3*(-(b*B) + A*c))/(b + c*x^2)^2 +
(2*b^2*(4*b*B - 3*A*c))/(b + c*x^2) + 6*b*(2*b*B - A*c)*Log[b + c*x^2])/(4*
c^5)
```

fricas [A] time = 0.77, size = 179, normalized size = 1.61

$$\frac{Bc^4x^8-2\left(2\,Bbc^3-Ac^4\right)x^6+7\,Bb^4-5\,Ab^3c-\left(11\,Bb^2c^2-4\,Abc^3\right)x^4+2\left(Bb^3c-2\,Ab^2c^2\right)x^2+6\left(2\,Bb^4-Ab\right.}{4\left(c^7x^4+2\,bc^6x^2+b^2c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^4*x^8 - 2*(2*B*b*c^3 - A*c^4)*x^6 + 7*B*b^4 - 5*A*b^3*c - (11*B*b^
2*c^2 - 4*A*b*c^3)*x^4 + 2*(B*b^3*c - 2*A*b^2*c^2)*x^2 + 6*(2*B*b^4 - A*b^3
*c + (2*B*b^2*c^2 - A*b*c^3)*x^4 + 2*(2*B*b^3*c - A*b^2*c^2)*x^2)*log(c*x^2
 + b))/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)
```

giac [A] time = 0.18, size = 132, normalized size = 1.19

$$\frac{3\left(2\,Bb^2-Abc\right)\log\left(\left|cx^2+b\right|\right)}{2\,c^5}+\frac{Bc^3x^4-6\,Bbc^2x^2+2\,Ac^3x^2}{4\,c^6}-\frac{18\,Bb^2c^2x^4-9\,Abc^3x^4+28\,Bb^3cx^2-12\,Ab^2c^2x}{4\left(cx^2+b\right)^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*b^2 - A*b*c)*log(abs(c*x^2 + b))/c^5 + 1/4*(B*c^3*x^4 - 6*B*b*c^2*
x^2 + 2*A*c^3*x^2)/c^6 - 1/4*(18*B*b^2*c^2*x^4 - 9*A*b*c^3*x^4 + 28*B*b^3*c
*x^2 - 12*A*b^2*c^2*x^2 + 11*B*b^4 - 4*A*b^3*c)/((c*x^2 + b)^2*c^5)
```

maple [A] time = 0.06, size = 134, normalized size = 1.21

$$\frac{B\,x^4}{4c^3}+\frac{A\,b^3}{4\left(c\,x^2+b\right)^2c^4}+\frac{A\,x^2}{2c^3}-\frac{B\,b^4}{4\left(c\,x^2+b\right)^2c^5}-\frac{3Bb\,x^2}{2c^4}-\frac{3A\,b^2}{2\left(c\,x^2+b\right)c^4}-\frac{3Ab\ln\left(c\,x^2+b\right)}{2c^4}+\frac{2B\,b^3}{\left(c\,x^2+b\right)c^5}+\frac{3B\,b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*B*x^4/c^3-3/2/c^4*B*x^2*b+1/2/c^3*A*x^2-3/2*b/c^4*ln(c*x^2+b)*A+3*b^2/c
^5*ln(c*x^2+b)*B-3/2*b^2/c^4/(c*x^2+b)*A+2*b^3/c^5/(c*x^2+b)*B+1/4*b^3/c^4/
(c*x^2+b)^2*A-1/4*b^4/c^5/(c*x^2+b)^2*B
```

maxima [A] time = 1.41, size = 116, normalized size = 1.05

$$\frac{7\,Bb^4 - 5\,Ab^3c + 2\left(4\,Bb^3c - 3\,Ab^2c^2\right)x^2}{4\left(c^7x^4 + 2\,bc^6x^2 + b^2c^5\right)} + \frac{Bcx^4 - 2\,(3\,Bb - Ac)x^2}{4\,c^4} + \frac{3\left(2\,Bb^2 - Abc\right)\log\left(cx^2 + b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*b^4 - 5*A*b^3*c + 2*(4*B*b^3*c - 3*A*b^2*c^2)*x^2)/(c^7*x^4 + 2*b*
c^6*x^2 + b^2*c^5) + 1/4*(B*c*x^4 - 2*(3*B*b - A*c)*x^2)/c^4 + 3/2*(2*B*b^2
 - A*b*c)*log(c*x^2 + b)/c^5
```

mupad [B] time = 0.12, size = 118, normalized size = 1.06

$$\frac{\frac{7\,B\,b^4-5\,A\,b^3\,c}{4\,c}+x^2\,\left(2\,B\,b^3-\frac{3\,A\,b^2\,c}{2}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+x^2\,\left(\frac{A}{2\,c^3}-\frac{3\,B\,b}{2\,c^4}\right)+\frac{\ln\left(c\,x^2+b\right)\,\left(6\,B\,b^2-3\,A\,b\,c\right)}{2\,c^5}+\frac{B\,x^4}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*B*b^4 - 5*A*b^3*c)/(4*c) + x^2*(2*B*b^3 - (3*A*b^2*c)/2))/(b^2*c^4 + c^
6*x^4 + 2*b*c^5*x^2) + x^2*(A/(2*c^3) - (3*B*b)/(2*c^4)) + (log(b + c*x^2)*
(6*B*b^2 - 3*A*b*c))/(2*c^5) + (B*x^4)/(4*c^3)
```

sympy [A] time = 1.52, size = 119, normalized size = 1.07

$$\frac{Bx^4}{4c^3}+\frac{3b\,(-Ac+2Bb)\log\left(b+cx^2\right)}{2c^5}+x^2\left(\frac{A}{2c^3}-\frac{3Bb}{2c^4}\right)+\frac{-5Ab^3c+7Bb^4+x^2\left(-6Ab^2c^2+8Bb^3c\right)}{4b^2c^5+8bc^6x^2+4c^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**4/(4*c**3) + 3*b*(-A*c + 2*B*b)*log(b + c*x**2)/(2*c**5) + x**2*(A/(2*
c**3) - 3*B*b/(2*c**4)) + (-5*A*b**3*c + 7*B*b**4 + x**2*(-6*A*b**2*c**2 +
8*B*b**3*c))/(4*b**2*c**5 + 8*b*c**6*x**2 + 4*c**7*x**4)
```

3.75 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=118

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}+\frac{5\sqrt{b}(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{bx(13bB-9Ac)}{8c^4(b+cx^2)}-\frac{x(3bB-Ac)}{c^4}+\frac{Bx^3}{3c^3}$$

```
[Out] -(-A*c+3*B*b)*x/c^4+1/3*B*x^3/c^3+1/4*b^2*(-A*c+B*b)*x/c^4/(c*x^2+b)^2-1/8*
b*(-9*A*c+13*B*b)*x/c^4/(c*x^2+b)+5/8*(-3*A*c+7*B*b)*arctan(x*c^(1/2)/b^(1/
2))*b^(1/2)/c^(9/2)
```

Rubi [A] time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 455, 1814, 1153, 205}

$$\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{bx(13bB-9Ac)}{8c^4(b+cx^2)}-\frac{x(3bB-Ac)}{c^4}+\frac{5\sqrt{b}(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}+\frac{Bx^3}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(((3*b*B - A*c)*x)/c^4) + (B*x^3)/(3*c^3) + (b^2*(b*B - A*c)*x)/(4*c^4*(b
+ c*x^2)^2) - (b*(13*b*B - 9*A*c)*x)/(8*c^4*(b + c*x^2)) + (5*Sqrt[b]*(7*b*
B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^6\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx\\
&= \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2}-\frac{\int \frac{b^2(bB-Ac)-4bc(bB-Ac)x^2+4c^2(bB-Ac)x^4-4Bc^3x^6}{\left(b+cx^2\right)^2}\,dx}{4c^4}\\
&= \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2}-\frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)}+\frac{\int \frac{b^2(11bB-7Ac)-8bc(2bB-Ac)x^2+8bBc^2x^4}{b+cx^2}\,dx}{8bc^4}\\
&= \frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2}-\frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)}+\frac{\int \left(-8b(3bB-Ac)+8bBcx^2+\frac{5\left(7b^3B-3Ab^2c\right)}{b+cx^2}\right)dx}{8bc^4}\\
&= -\frac{(3bB-Ac)x}{c^4}+\frac{Bx^3}{3c^3}+\frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2}-\frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)}+\frac{(5b(7bB-3Ac))\int \frac{1}{b+cx^2}\,dx}{8c^4}\\
&= -\frac{(3bB-Ac)x}{c^4}+\frac{Bx^3}{3c^3}+\frac{b^2(bB-Ac)x}{4c^4\left(b+cx^2\right)^2}-\frac{b(13bB-9Ac)x}{8c^4\left(b+cx^2\right)}+\frac{5\sqrt{b}\,(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 113, normalized size = 0.96

$$\frac{5b^2cx\left(9A-35Bx^2\right)+bc^2x^3\left(75A-56Bx^2\right)+8c^3x^5\left(3A+Bx^2\right)-105b^3Bx}{24c^4\left(b+cx^2\right)^2}+\frac{5\sqrt{b}\,(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-105*b^3*B*x + b*c^2*x^3*(75*A - 56*B*x^2) + 5*b^2*c*x*(9*A - 35*B*x^2) +
8*c^3*x^5*(3*A + B*x^2))/(24*c^4*(b + c*x^2)^2) + (5*Sqrt[b]*(7*b*B - 3*A*c
)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

fricas [A] time = 0.88, size = 358, normalized size = 3.03

$$\left[\frac{16\,Bc^3x^7-16\left(7\,Bbc^2-3\,Ac^3\right)x^5-50\left(7\,Bb^2c-3\,Abc^2\right)x^3-15\left(\left(7\,Bbc^2-3\,Ac^3\right)x^4+7\,Bb^3-3\,Ab^2c+2\left(7\,Bb^2\right.\right.}{48\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*c^3*x^7 - 16*(7*B*b*c^2 - 3*A*c^3)*x^5 - 50*(7*B*b^2*c - 3*A*b*
c^2)*x^3 - 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2
*c - 3*A*b*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
 b)) - 30*(7*B*b^3 - 3*A*b^2*c)*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*
(8*B*c^3*x^7 - 8*(7*B*b*c^2 - 3*A*c^3)*x^5 - 25*(7*B*b^2*c - 3*A*b*c^2)*x^3
 + 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2*c - 3*A
*b*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 15*(7*B*b^3 - 3*A*b^2*c)*x
)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

giac [A] time = 0.16, size = 111, normalized size = 0.94

$$\frac{5\left(7\,Bb^2-3\,Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4}-\frac{13\,Bb^2cx^3-9\,Abc^2x^3+11\,Bb^3x-7\,Ab^2cx}{8\left(cx^2+b\right)^2c^4}+\frac{Bc^6x^3-9\,Bbc^5x+3\,Ac^6x}{3\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*B*b
^2*c*x^3 - 9*A*b*c^2*x^3 + 11*B*b^3*x - 7*A*b^2*c*x)/((c*x^2 + b)^2*c^4) +
1/3*(B*c^6*x^3 - 9*B*b*c^5*x + 3*A*c^6*x)/c^9
```

maple [A] time = 0.06, size = 147, normalized size = 1.25

$$\frac{9Ab\,x^3}{8\left(c\,x^2+b\right)^2c^2}-\frac{13B\,b^2x^3}{8\left(c\,x^2+b\right)^2c^3}+\frac{7A\,b^2x}{8\left(c\,x^2+b\right)^2c^3}-\frac{11B\,b^3x}{8\left(c\,x^2+b\right)^2c^4}+\frac{B\,x^3}{3c^3}-\frac{15Ab\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^3}+\frac{35B\,b^2\arctan\left(\right.}{8\sqrt{bc}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/3*B*x^3/c^3+1/c^3*A*x-3/c^4*b*B*x+9/8*b/c^2/(c*x^2+b)^2*A*x^3-13/8*b^2/c^
3/(c*x^2+b)^2*B*x^3+7/8*b^2/c^3/(c*x^2+b)^2*A*x-11/8*b^3/c^4/(c*x^2+b)^2*B*
x-15/8*b/c^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*A+35/8*b^2/c^4/(b*c)^(1/
2)*arctan(1/(b*c)^(1/2)*c*x)*B
```

maxima [A] time = 3.02, size = 120, normalized size = 1.02

$$-\frac{\left(13\,Bb^2c-9\,Abc^2\right)x^3+\left(11\,Bb^3-7\,Ab^2c\right)x}{8\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}+\frac{5\left(7\,Bb^2-3\,Abc\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4}+\frac{Bcx^3-3\,(3\,Bb-Ac)x}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*B*b^2*c - 9*A*b*c^2)*x^3 + (11*B*b^3 - 7*A*b^2*c)*x)/(c^6*x^4 + 2
*b*c^5*x^2 + b^2*c^4) + 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt
(b*c)*c^4) + 1/3*(B*c*x^3 - 3*(3*B*b - A*c)*x)/c^4
```

mupad [B] time = 0.08, size = 138, normalized size = 1.17

$$\frac{x^3\left(\frac{9\,A\,b\,c^2}{8}-\frac{13\,B\,b^2\,c}{8}\right)-x\left(\frac{11\,B\,b^3}{8}-\frac{7\,A\,b^2\,c}{8}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+x\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right)+\frac{B\,x^3}{3\,c^3}+\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-7\,B\,b)}{7\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-7\,B}{8\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^3*((9*A*b*c^2)/8 - (13*B*b^2*c)/8) - x*((11*B*b^3)/8 - (7*A*b^2*c)/8))/(
b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x*(A/c^3 - (3*B*b)/c^4) + (B*x^3)/(3*c^3
) + (5*b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 7*B*b))/(7*B*b^2 - 3*A*b*c)
)*(3*A*c - 7*B*b))/(8*c^(9/2))
```

sympy [A] time = 1.30, size = 214, normalized size = 1.81

$$\frac{Bx^3}{3c^3}+x\left(\frac{A}{c^3}-\frac{3Bb}{c^4}\right)-\frac{5\sqrt{-\frac{b}{c^9}}\,(-3Ac+7Bb)\log\left(-\frac{5c^4\sqrt{-\frac{b}{c^9}}\,(-3Ac+7Bb)}{-15Ac+35Bb}+x\right)}{16}+\frac{5\sqrt{-\frac{b}{c^9}}\,(-3Ac+7Bb)\log\left(\frac{5c^4\sqrt{-\frac{b}{c^9}}\,(-3A}{-15Ac+3}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**3/(3*c**3) + x*(A/c**3 - 3*B*b/c**4) - 5*sqrt(-b/c**9)*(-3*A*c + 7*B*b
)*log(-5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(-15*A*c + 35*B*b) + x)/16 + 5
*sqrt(-b/c**9)*(-3*A*c + 7*B*b)*log(5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(
-15*A*c + 35*B*b) + x)/16 + (x**3*(9*A*b*c**2 - 13*B*b**2*c) + x*(7*A*b**2*
c - 11*B*b**3))/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4)
```

3.76 $\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}+\frac{Bx^2}{2c^3}$$

```
[Out] 1/2*B*x^2/c^3+1/4*b^2*(-A*c+B*b)/c^4/(c*x^2+b)^2-1/2*b*(-2*A*c+3*B*b)/c^4/(
c*x^2+b)-1/2*(-A*c+3*B*b)*ln(c*x^2+b)/c^4
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}+\frac{Bx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^2*(b*B - A*c))/(4*c^4*(b + c*x^2)^2) - (b*(3*b*B - 2*A
*c))/(2*c^4*(b + c*x^2)) - ((3*b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^5\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{B}{c^3} - \frac{b^2(bB-Ac)}{c^3(b+cx)^3} + \frac{b(3bB-2Ac)}{c^3(b+cx)^2} + \frac{-3bB+Ac}{c^3(b+cx)}\right) dx, x, x^2\right)$$
$$= \frac{Bx^2}{2c^3} + \frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2} - \frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)} - \frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 1.03

$$\frac{2Abc-3b^2B}{2c^4\left(b+cx^2\right)} + \frac{b^3B-Ab^2c}{4c^4\left(b+cx^2\right)^2} + \frac{(Ac-3bB)\log\left(b+cx^2\right)}{2c^4} + \frac{Bx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^3*B - A*b^2*c)/(4*c^4*(b + c*x^2)^2) + (-3*b^2*B + 2*A
*b*c)/(2*c^4*(b + c*x^2)) + ((-3*b*B + A*c)*Log[b + c*x^2])/(2*c^4)
```

fricas [A] time = 0.99, size = 142, normalized size = 1.60

$$\frac{2\,Bc^3x^6 + 4\,Bbc^2x^4 - 5\,Bb^3 + 3\,Ab^2c - 4\left(Bb^2c - Abc^2\right)x^2 - 2\left(\left(3\,Bbc^2 - Ac^3\right)x^4 + 3\,Bb^3 - Ab^2c + 2\left(3\,Bb^2c - Ab\right.\right.}{4\left(c^6x^4 + 2\,bc^5x^2 + b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*c^3*x^6 + 4*B*b*c^2*x^4 - 5*B*b^3 + 3*A*b^2*c - 4*(B*b^2*c - A*b*c
^2)*x^2 - 2*((3*B*b*c^2 - A*c^3)*x^4 + 3*B*b^3 - A*b^2*c + 2*(3*B*b^2*c - A
*b*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

giac [A] time = 0.16, size = 93, normalized size = 1.04

$$\frac{Bx^2}{2\,c^3} - \frac{(3\,Bb - Ac)\log\left(\left|cx^2+b\right|\right)}{2\,c^4} + \frac{9\,Bbc^2x^4 - 3\,Ac^3x^4 + 12\,Bb^2cx^2 - 2\,Abc^2x^2 + 4\,Bb^3}{4\left(cx^2+b\right)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(9*B*b*c^2*
x^4 - 3*A*c^3*x^4 + 12*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 4*B*b^3)/((c*x^2 + b)^
2*c^4)
```

maple [A] time = 0.06, size = 109, normalized size = 1.22

$$-\frac{A\,b^2}{4\left(c\,x^2+b\right)^2c^3} + \frac{B\,b^3}{4\left(c\,x^2+b\right)^2c^4} + \frac{B\,x^2}{2c^3} + \frac{Ab}{\left(c\,x^2+b\right)c^3} + \frac{A\ln\left(c\,x^2+b\right)}{2c^3} - \frac{3B\,b^2}{2\left(c\,x^2+b\right)c^4} - \frac{3Bb\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*B*x^2/c^3+1/2/c^3*ln(c*x^2+b)*A-3/2/c^4*ln(c*x^2+b)*b*B+1/c^3*b/(c*x^2+
b)*A-3/2/c^4*b^2/(c*x^2+b)*B-1/4/c^3*b^2/(c*x^2+b)^2*A+1/4/c^4*b^3/(c*x^2+b
)^2*B
```

maxima [A] time = 1.36, size = 94, normalized size = 1.06

$$-\frac{5\,Bb^3 - 3\,Ab^2c + 2\left(3\,Bb^2c - 2\,Abc^2\right)x^2}{4\left(c^6x^4 + 2\,bc^5x^2 + b^2c^4\right)} + \frac{Bx^2}{2\,c^3} - \frac{(3\,Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*b^3 - 3*A*b^2*c + 2*(3*B*b^2*c - 2*A*b*c^2)*x^2)/(c^6*x^4 + 2*b*c
^5*x^2 + b^2*c^4) + 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(c*x^2 + b)/c^4
```

mupad [B] time = 0.09, size = 95, normalized size = 1.07

$$\frac{B\,x^2}{2\,c^3} - \frac{x^2\,\left(\frac{3\,B\,b^2}{2} - A\,b\,c\right) + \frac{5\,B\,b^3 - 3\,A\,b^2\,c}{4\,c}}{b^2\,c^3 + 2\,b\,c^4\,x^2 + c^5\,x^4} + \frac{\ln\left(c\,x^2 + b\right)\,\left(A\,c - 3\,B\,b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (B*x^2)/(2*c^3) - (x^2*((3*B*b^2)/2 - A*b*c) + (5*B*b^3 - 3*A*b^2*c)/(4*c))
/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (log(b + c*x^2)*(A*c - 3*B*b))/(2*c^4)
```

sympy [A] time = 1.29, size = 94, normalized size = 1.06

$$\frac{Bx^2}{2c^3} + \frac{3Ab^2c - 5Bb^3 + x^2\left(4Abc^2 - 6Bb^2c\right)}{4b^2c^4 + 8bc^5x^2 + 4c^6x^4} - \frac{(-Ac + 3Bb)\log\left(b + cx^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**2/(2*c**3) + (3*A*b**2*c - 5*B*b**3 + x**2*(4*A*b*c**2 - 6*B*b**2*c))/
(4*b**2*c**4 + 8*b*c**5*x**2 + 4*c**6*x**4) - (-A*c + 3*B*b)*log(b + c*x**2
)/(2*c**4)
```

3.77 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=95

$$-\frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}+\frac{x(9bB-5Ac)}{8c^3(b+cx^2)}-\frac{bx(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{Bx}{c^3}$$

```
[Out] B*x/c^3-1/4*b*(-A*c+B*b)*x/c^3/(c*x^2+b)^2+1/8*(-5*A*c+9*B*b)*x/c^3/(c*x^2+
b)-3/8*(-A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(7/2)/b^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 455, 1157, 388, 205}

$$\frac{x(9bB-5Ac)}{8c^3(b+cx^2)}-\frac{bx(bB-Ac)}{4c^3(b+cx^2)^2}-\frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}+\frac{Bx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x)/c^3 - (b*(b*B - A*c)*x)/(4*c^3*(b + c*x^2)^2) + ((9*b*B - 5*A*c)*x)/(
8*c^3*(b + c*x^2)) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[
b]*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^4\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx\\ &= -\frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2}-\frac{\int \frac{-b(bB-Ac)+4c(bB-Ac)x^2-4Bc^2x^4}{\left(b+cx^2\right)^2}\,dx}{4c^3}\\ &= -\frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2}+\frac{(9bB-5Ac)x}{8c^3\left(b+cx^2\right)}+\frac{\int \frac{-b(7bB-3Ac)+8bBcx^2}{b+cx^2}\,dx}{8bc^3}\\ &= \frac{Bx}{c^3}-\frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2}+\frac{(9bB-5Ac)x}{8c^3\left(b+cx^2\right)}-\frac{(3(5bB-Ac))\int \frac{1}{b+cx^2}\,dx}{8c^3}\\ &= \frac{Bx}{c^3}-\frac{b(bB-Ac)x}{4c^3\left(b+cx^2\right)^2}+\frac{(9bB-5Ac)x}{8c^3\left(b+cx^2\right)}-\frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 92, normalized size = 0.97

$$\frac{x\left(b\left(25Bcx^2-3Ac\right)+c^2x^2\left(8Bx^2-5A\right)+15b^2B\right)}{8c^3\left(b+cx^2\right)^2}-\frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(15*b^2*B + c^2*x^2*(-5*A + 8*B*x^2) + b*(-3*A*c + 25*B*c*x^2)))/(8*c^3*
(b + c*x^2)^2) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]*c
^(7/2))
```

fricas [A] time = 0.82, size = 328, normalized size = 3.45

$$\left[\frac{16\,Bbc^3x^5+10\left(5\,Bb^2c^2-Abc^3\right)x^3+3\left(\left(5\,Bbc^2-Ac^3\right)x^4+5\,Bb^3-Ab^2c+2\left(5\,Bb^2c-Abc^2\right)x^2\right)\sqrt{-bc}\,\log\left(\right.}{16\left(bc^6x^4+2\,b^2c^5x^2+b^3c^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*B*b*c^3*x^5 + 10*(5*B*b^2*c^2 - A*b*c^3)*x^3 + 3*((5*B*b*c^2 - A*
c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*c - A*b*c^2)*x^2)*sqrt(-b*c)*log(
(c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 6*(5*B*b^3*c - A*b^2*c^2)*x)/(b
*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4), 1/8*(8*B*b*c^3*x^5 + 5*(5*B*b^2*c^2 -
A*b*c^3)*x^3 - 3*((5*B*b*c^2 - A*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*
c - A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + 3*(5*B*b^3*c - A*b^2*c^
2)*x)/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4)]
```

giac [A] time = 0.18, size = 80, normalized size = 0.84

$$\frac{Bx}{c^3} - \frac{3\,(5\,Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^3} + \frac{9\,Bbcx^3 - 5\,Ac^2x^3 + 7\,Bb^2x - 3\,Abcx}{8\,\left(cx^2 + b\right)^2 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/8*(9*
B*b*c*x^3 - 5*A*c^2*x^3 + 7*B*b^2*x - 3*A*b*c*x)/((c*x^2 + b)^2*c^3)
```

maple [A] time = 0.05, size = 122, normalized size = 1.28

$$-\frac{5A\,x^3}{8\left(c\,x^2+b\right)^2 c}+\frac{9Bb\,x^3}{8\left(c\,x^2+b\right)^2 c^2}-\frac{3Abx}{8\left(c\,x^2+b\right)^2 c^2}+\frac{7B\,b^2x}{8\left(c\,x^2+b\right)^2 c^3}+\frac{3A\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^2}-\frac{15Bb\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^3}+\frac{Bx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] B*x/c^3-5/8/c/(c*x^2+b)^2*A*x^3+9/8/c^2/(c*x^2+b)^2*B*x^3*b-3/8/c^2/(c*x^2+
b)^2*A*b*x+7/8/c^3/(c*x^2+b)^2*B*b^2*x+3/8/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(
1/2)*c*x)*A-15/8/c^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*b*B
```

maxima [A] time = 2.94, size = 94, normalized size = 0.99

$$\frac{\left(9\,Bbc - 5\,Ac^2\right)x^3 + \left(7\,Bb^2 - 3\,Abc\right)x}{8\,\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)} + \frac{Bx}{c^3} - \frac{3\,(5\,Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*B*b*c - 5*A*c^2)*x^3 + (7*B*b^2 - 3*A*b*c)*x)/(c^5*x^4 + 2*b*c^4*x^
2 + b^2*c^3) + B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)
*c^3)
```

mupad [B] time = 0.15, size = 92, normalized size = 0.97

$$\frac{B\,x}{c^3} - \frac{x^3\,\left(\frac{5\,A\,c^2}{8} - \frac{9\,B\,b\,c}{8}\right) - x\,\left(\frac{7\,B\,b^2}{8} - \frac{3\,A\,b\,c}{8}\right)}{b^2\,c^3 + 2\,b\,c^4\,x^2 + c^5\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c - 5\,B\,b)}{8\,\sqrt{b}\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (B*x)/c^3 - (x^3*((5*A*c^2)/8 - (9*B*b*c)/8) - x*((7*B*b^2)/8 - (3*A*b*c)/8
))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (3*atan((c^(1/2)*x)/b^(1/2))*(A*c -
5*B*b))/(8*b^(1/2)*c^(7/2))
```

sympy [B] time = 1.11, size = 194, normalized size = 2.04

$$\frac{Bx}{c^3}+\frac{3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)\log\left(-\frac{3bc^3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16}-\frac{3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)\log\left(\frac{3bc^3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16}+\frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x/c**3 + 3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)*log(-3*b*c**3*sqrt(-1/(b*c**7
))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/16 - 3*sqrt(-1/(b*c**7))*(-A*c + 5
*B*b)*log(3*b*c**3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/
16 + (x**3*(-5*A*c**2 + 9*B*b*c) + x*(-3*A*b*c + 7*B*b**2))/(8*b**2*c**3 +
16*b*c**4*x**2 + 8*c**5*x**4)
```

3.78 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}$$

```
[Out] -1/4*b*(-A*c+B*b)/c^3/(c*x^2+b)^2+1/2*(-A*c+2*B*b)/c^3/(c*x^2+b)+1/2*B*ln(c
*x^2+b)/c^3
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{1584, 446, 77\}$

$$-\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*(b*B - A*c))/(4*c^3*(b + c*x^2)^2) + (2*b*B - A*c)/(2*c^3*(b + c*x^2))
+ (B*Log[b + c*x^2])/(2*c^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^9 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3} \, dx = \int \frac{x^3 \left(A+Bx^2\right)}{\left(b+cx^2\right)^3} \, dx$$
$$= \frac{1}{2} \text{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^3} \, dx, x, x^2\right)$$
$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^2(b+cx)^3} + \frac{-2bB+Ac}{c^2(b+cx)^2} + \frac{B}{c^2(b+cx)}\right) dx, x, x^2\right)$$
$$= -\frac{b(bB-Ac)}{4c^3\left(b+cx^2\right)^2} + \frac{2bB-Ac}{2c^3\left(b+cx^2\right)} + \frac{B\log\left(b+cx^2\right)}{2c^3}$$

Mathematica [**A**] time = 0.03, size = 64, normalized size = 0.96

$$\frac{-bc\left(A-4Bx^2\right)-2Ac^2x^2+3b^2B+2B\left(b+cx^2\right)^2\log\left(b+cx^2\right)}{4c^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*b^2*B - 2*A*c^2*x^2 - b*c*(A - 4*B*x^2) + 2*B*(b + c*x^2)^2*Log[b + c*x^
2])/(4*c^3*(b + c*x^2)^2)
```

fricas [**A**] time = 0.78, size = 89, normalized size = 1.33

$$\frac{3\,Bb^2-Abc+2\left(2\,Bbc-Ac^2\right)x^2+2\left(Bc^2x^4+2\,Bbcx^2+Bb^2\right)\log\left(cx^2+b\right)}{4\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2 + 2*(B*c^2*x^4 + 2*B*b*c*x^2
 + B*b^2)*log(c*x^2 + b))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

giac [**A**] time = 0.19, size = 55, normalized size = 0.82

$$\frac{B\log\left(\left|cx^2+b\right|\right)}{2\,c^3}-\frac{3\,Bcx^4+2\,Bbx^2+2\,Acx^2+Ab}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(c*x^2 + b))/c^3 - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*
b)/((c*x^2 + b)^2*c^2)
```

maple [**A**] time = 0.05, size = 80, normalized size = 1.19

$$\frac{Ab}{4\left(c\,x^2+b\right)^2c^2}-\frac{B\,b^2}{4\left(c\,x^2+b\right)^2c^3}-\frac{A}{2\left(c\,x^2+b\right)c^2}+\frac{Bb}{\left(c\,x^2+b\right)c^3}+\frac{B\ln\left(c\,x^2+b\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*B*ln(c*x^2+b)/c^3-1/2/c^2/(c*x^2+b)*A+1/c^3/(c*x^2+b)*b*B+1/4*b/c^2/(c*
x^2+b)^2*A-1/4*b^2/c^3/(c*x^2+b)^2*B
```

maxima [A] time = 1.33, size = 72, normalized size = 1.07

$$\frac{3\,Bb^2 - Abc + 2\left(2\,Bbc - Ac^2\right)x^2}{4\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)} + \frac{B\log\left(cx^2 + b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^
2*c^3) + 1/2*B*log(c*x^2 + b)/c^3
```

mupad [B] time = 0.11, size = 70, normalized size = 1.04

$$\frac{\frac{3\,B\,b^2-A\,b\,c}{4\,c^3}-\frac{x^2\,\left(A\,c-2\,B\,b\right)}{2\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{B\,\ln\left(c\,x^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*B*b^2 - A*b*c)/(4*c^3) - (x^2*(A*c - 2*B*b))/(2*c^2))/(b^2 + c^2*x^4 +
2*b*c*x^2) + (B*log(b + c*x^2))/(2*c^3)
```

sympy [A] time = 0.93, size = 70, normalized size = 1.04

$$\frac{B\log\left(b + cx^2\right)}{2c^3} + \frac{-Abc + 3Bb^2 + x^2\left(-2Ac^2 + 4Bbc\right)}{4b^2c^3 + 8bc^4x^2 + 4c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*log(b + c*x**2)/(2*c**3) + (-A*b*c + 3*B*b**2 + x**2*(-2*A*c**2 + 4*B*b*c
))/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4)
```

3.79 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=90

$$\frac{(Ac+3bB)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}-\frac{x(5bB-Ac)}{8bc^2\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4c^2\left(b+cx^2\right)^2}$$

```
[Out] 1/4*(-A*c+B*b)*x/c^2/(c*x^2+b)^2-1/8*(-A*c+5*B*b)*x/b/c^2/(c*x^2+b)+1/8*(A*
c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(5/2)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 455, 385, 205}

$$\frac{(Ac+3bB)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}-\frac{x(5bB-Ac)}{8bc^2\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*B - A*c)*x)/(4*c^2*(b + c*x^2)^2) - ((5*b*B - A*c)*x)/(8*b*c^2*(b + c*x
^2)) + ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(3/2)*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^8 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^2 \left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= \frac{(bB-Ac)x}{4c^2\left(b+cx^2\right)^2} - \frac{\int \frac{bB-Ac-4Bcx^2}{\left(b+cx^2\right)^2}\,dx}{4c^2}$$

$$= \frac{(bB-Ac)x}{4c^2\left(b+cx^2\right)^2} - \frac{(5bB-Ac)x}{8bc^2\left(b+cx^2\right)} + \frac{(3bB+Ac)\int \frac{1}{b+cx^2}\,dx}{8bc^2}$$

$$= \frac{(bB-Ac)x}{4c^2\left(b+cx^2\right)^2} - \frac{(5bB-Ac)x}{8bc^2\left(b+cx^2\right)} + \frac{(3bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}$$

Mathematica [A] time = 0.12, size = 83, normalized size = 0.92

$$\frac{\frac{(Ac+3bB)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}} + \frac{\sqrt{c}\,x\left(-bc\left(A+5Bx^2\right)+Ac^2x^2-3b^2B\right)}{b\left(b+cx^2\right)^2}}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[c]*x*(-3*b^2*B + A*c^2*x^2 - b*c*(A + 5*B*x^2)))/(b*(b + c*x^2)^2) +
 ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2))/(8*c^(5/2))
```

fricas [A] time = 0.99, size = 301, normalized size = 3.34

$$\left[-\frac{2\left(5\,Bb^2c^2-Abc^3\right)x^3+\left(\left(3\,Bbc^2+Ac^3\right)x^4+3\,Bb^3+Ab^2c+2\left(3\,Bb^2c+Abc^2\right)x^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)+2}{16\left(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*B*b^2*c^2 - A*b*c^3)*x^3 + ((3*B*b*c^2 + A*c^3)*x^4 + 3*B*b^3
+ A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*
c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*
c^4*x^2 + b^4*c^3), -1/8*((5*B*b^2*c^2 - A*b*c^3)*x^3 - ((3*B*b*c^2 + A*c^3
)*x^4 + 3*B*b^3 + A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(b*c)*arctan(s
qrt(b*c)*x/b) + (3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*c^4*x^2 + b
^4*c^3)]
```

giac [A] time = 0.16, size = 78, normalized size = 0.87

$$\frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,bc^2} - \frac{5\,Bbcx^3-Ac^2x^3+3\,Bb^2x+Abcx}{8\left(cx^2+b\right)^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2) - 1/8*(5*B*b*c*x^
3 - A*c^2*x^3 + 3*B*b^2*x + A*b*c*x)/((c*x^2 + b)^2*b*c^2)
```

maple [A] time = 0.06, size = 89, normalized size = 0.99

$$\frac{A\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;bc}+\frac{3B\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^2}+\frac{\frac{(Ac-5bB)x^3}{8bc}-\frac{(Ac+3bB)x}{8c^2}}{\left(c\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8*(A*c-5*B*b)/b/c*x^3-1/8*(A*c+3*B*b)/c^2*x)/(c*x^2+b)^2+1/8/c/b/(b*c)^(
1/2)*arctan(1/(b*c)^(1/2)*c*x)*A+3/8/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c
*x)*B
```

maxima [A] time = 2.98, size = 92, normalized size = 1.02

$$-\frac{\left(5\,Bbc-Ac^2\right)x^3+\left(3\,Bb^2+Abc\right)x}{8\left(bc^4x^4+2\,b^2c^3x^2+b^3c^2\right)}+\frac{\left(3\,Bb+Ac\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b*c - A*c^2)*x^3 + (3*B*b^2 + A*b*c)*x)/(b*c^4*x^4 + 2*b^2*c^3*x
^2 + b^3*c^2) + 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2)
```

mupad [B] time = 0.15, size = 82, normalized size = 0.91

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\left(A\,c+3\,B\,b\right)}{8\,b^{3/2}\,c^{5/2}}-\frac{\frac{x\,\left(A\,c+3\,B\,b\right)}{8\,c^2}-\frac{x^3\,\left(A\,c-5\,B\,b\right)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/b^(1/2))*(A*c + 3*B*b))/(8*b^(3/2)*c^(5/2)) - ((x*(A*c +
3*B*b))/(8*c^2) - (x^3*(A*c - 5*B*b))/(8*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

sympy [A] time = 0.76, size = 155, normalized size = 1.72

$$-\frac{\sqrt{-\frac{1}{b^3c^5}}\,(Ac+3Bb)\log\left(-b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^3c^5}}\,(Ac+3Bb)\log\left(b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16}+\frac{x^3\left(Ac^2-5Bbc\right)+}{8b^3c^2+16b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(-b**2*c**2*sqrt(-1/(b**3*c**5)) + x
)/16 + sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(b**2*c**2*sqrt(-1/(b**3*c**5)
) + x)/16 + (x**3*(A*c**2 - 5*B*b*c) + x*(-A*b*c - 3*B*b**2))/(8*b**3*c**2
+ 16*b**2*c**3*x**2 + 8*b*c**4*x**4)
```

3.80 $\int \frac{x^7\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=32

$$\frac{\left(A+Bx^2\right)^2}{4\left(b+cx^2\right)^2(bB-Ac)}$$

[Out] 1/4*(B*x^2+A)^2/(-A*c+B*b)/(c*x^2+b)^2

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 444, 37}

$$\frac{\left(A+Bx^2\right)^2}{4\left(b+cx^2\right)^2(bB-Ac)}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]

[Out] (A + B*x^2)^2/(4*(b*B - A*c)*(b + c*x^2)^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3} dx &= \int \frac{x\left(A+Bx^2\right)}{\left(b+cx^2\right)^3} dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(b+cx)^3} dx, x, x^2\right) \\
&= \frac{\left(A+Bx^2\right)^2}{4(bB-Ac)\left(b+cx^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.94

$$-\frac{c\left(A+2Bx^2\right)+bB}{4c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*B + c*(A + 2*B*x^2))/(c^2*(b + c*x^2)^2)
```

fricas [A] time = 0.74, size = 42, normalized size = 1.31

$$-\frac{2\,Bcx^2+Bb+Ac}{4\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

giac [A] time = 0.16, size = 28, normalized size = 0.88

$$-\frac{2\,Bcx^2+Bb+Ac}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/((c*x^2 + b)^2*c^2)
```

maple [A] time = 0.05, size = 39, normalized size = 1.22

$$-\frac{B}{2\left(c\,x^2+b\right)c^2}-\frac{Ac-bB}{4\left(c\,x^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2*B/c^2/(c*x^2+b)-1/4*(A*c-B*b)/c^2/(c*x^2+b)^2
```

maxima [A] time = 1.35, size = 42, normalized size = 1.31

$$-\frac{2\,Bcx^2+Bb+Ac}{4\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

mupad [B] time = 0.08, size = 44, normalized size = 1.38

$$-\frac{\frac{A\,c+B\,b}{4\,c^2}+\frac{B\,x^2}{2\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -((A*c + B*b)/(4*c^2) + (B*x^2)/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

sympy [**A**] time = 0.54, size = 42, normalized size = 1.31

$$\frac{-Ac - Bb - 2Bcx^2}{4b^2c^2 + 8bc^3x^2 + 4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-A*c - B*b - 2*B*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

3.81 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=92

$$\frac{(3Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}+\frac{x(3Ac+bB)}{8b^2c(b+cx^2)}-\frac{x(bB-Ac)}{4bc(b+cx^2)^2}$$

```
[Out] -1/4*(-A*c+B*b)*x/b/c/(c*x^2+b)^2+1/8*(3*A*c+B*b)*x/b^2/c/(c*x^2+b)+1/8*(3*
A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(5/2)/c^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 385, 199, 205}

$$\frac{(3Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}+\frac{x(3Ac+bB)}{8b^2c(b+cx^2)}-\frac{x(bB-Ac)}{4bc(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*x)/(4*b*c*(b + c*x^2)^2) + ((b*B + 3*A*c)*x)/(8*b^2*c*(b + c*
x^2)) + ((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{\left(b+cx^2\right)^3}\,dx$$
$$= -\frac{(bB-Ac)x}{4bc\left(b+cx^2\right)^2} + \frac{(bB+3Ac)\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4bc}$$
$$= -\frac{(bB-Ac)x}{4bc\left(b+cx^2\right)^2} + \frac{(bB+3Ac)x}{8b^2c\left(b+cx^2\right)} + \frac{(bB+3Ac)\int \frac{1}{b+cx^2}\,dx}{8b^2c}$$
$$= -\frac{(bB-Ac)x}{4bc\left(b+cx^2\right)^2} + \frac{(bB+3Ac)x}{8b^2c\left(b+cx^2\right)} + \frac{(bB+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 0.91

$$\frac{(3Ac+bB)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}} + \frac{x\left(bc\left(5A+Bx^2\right)+3Ac^2x^2+b^2(-B)\right)}{8b^2c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(-(b^2*B) + 3*A*c^2*x^2 + b*c*(5*A + B*x^2)))/(8*b^2*c*(b + c*x^2)^2) +
((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

fricas [A] time = 1.06, size = 300, normalized size = 3.26

$$\left[\frac{2\left(Bb^2c^2+3\,Abc^3\right)x^3-\left(\left(Bbc^2+3\,Ac^3\right)x^4+Bb^3+3\,Ab^2c+2\left(Bb^2c+3\,Abc^2\right)x^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)-2\,(}{16\left(b^3c^4x^4+2\,b^4c^3x^2+b^5c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(B*b^2*c^2 + 3*A*b*c^3)*x^3 - ((B*b*c^2 + 3*A*c^3)*x^4 + B*b^3 + 3
*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c
)*x - b)/(c*x^2 + b)) - 2*(B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c
^3*x^2 + b^5*c^2), 1/8*((B*b^2*c^2 + 3*A*b*c^3)*x^3 + ((B*b*c^2 + 3*A*c^3)*
x^4 + B*b^3 + 3*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqr
t(b*c)*x/b) - (B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c^3*x^2 + b^5
*c^2)]
```

giac [A] time = 0.16, size = 78, normalized size = 0.85

$$\frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^2c} + \frac{Bbcx^3+3\,Ac^2x^3-Bb^2x+5\,Abcx}{8\left(cx^2+b\right)^2b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c) + 1/8*(B*b*c*x^3
+ 3*A*c^2*x^3 - B*b^2*x + 5*A*b*c*x)/((c*x^2 + b)^2*b^2*c)
```

maple [A] time = 0.06, size = 90, normalized size = 0.98

$$\frac{3A\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,b^2}+\frac{B\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,bc}+\frac{\frac{(3Ac+bB)x^3}{8b^2}+\frac{(5Ac-bB)x}{8bc}}{\left(c\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8*(3*A*c+B*b)/b^2*x^3+1/8*(5*A*c-B*b)/b/c*x)/(c*x^2+b)^2+3/8/b^2/(b*c)^(
1/2)*arctan(1/(b*c)^(1/2)*c*x)*A+1/8/b/c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c
*x)*B
```

maxima [A] time = 2.95, size = 92, normalized size = 1.00

$$\frac{\left(Bbc+3\,Ac^2\right)x^3-\left(Bb^2-5\,Abc\right)x}{8\left(b^2c^3x^4+2\,b^3c^2x^2+b^4c\right)}+\frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b*c + 3*A*c^2)*x^3 - (B*b^2 - 5*A*b*c)*x)/(b^2*c^3*x^4 + 2*b^3*c^2*
x^2 + b^4*c) + 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c)
```

mupad [B] time = 0.14, size = 82, normalized size = 0.89

$$\frac{\frac{x^3\,(3\,A\,c+B\,b)}{8\,b^2}+\frac{x\,(5\,A\,c-B\,b)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c+B\,b)}{8\,b^{5/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(3*A*c + B*b))/(8*b^2) + (x*(5*A*c - B*b))/(8*b*c))/(b^2 + c^2*x^4 +
2*b*c*x^2) + (atan((c^(1/2)*x)/b^(1/2))*(3*A*c + B*b))/(8*b^(5/2)*c^(3/2))
```

sympy [A] time = 0.60, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^5c^3}}\,(3Ac+Bb)\log\left(-b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^5c^3}}\,(3Ac+Bb)\log\left(b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16}+\frac{x^3\left(3Ac^2+Bbc\right)+x}{8b^4c+16b^3c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(-b**3*c*sqrt(-1/(b**5*c**3)) + x)/1
6 + sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(b**3*c*sqrt(-1/(b**5*c**3)) + x)
/16 + (x**3*(3*A*c**2 + B*b*c) + x*(5*A*b*c - B*b**2))/(8*b**4*c + 16*b**3*
c**2*x**2 + 8*b**2*c**3*x**4)
```

3.82 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{A\log(b+cx^2)}{2b^3}+\frac{A\log(x)}{b^3}+\frac{A}{2b^2(b+cx^2)}-\frac{bB-Ac}{4bc(b+cx^2)^2}$$

```
[Out] 1/4*(A*c-B*b)/b/c/(c*x^2+b)^2+1/2*A/b^2/(c*x^2+b)+A*ln(x)/b^3-1/2*A*ln(c*x^
2+b)/b^3
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{A}{2b^2(b+cx^2)}-\frac{A\log(b+cx^2)}{2b^3}+\frac{A\log(x)}{b^3}-\frac{bB-Ac}{4bc(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*B - A*c)/(4*b*c*(b + c*x^2)^2) + A/(2*b^2*(b + c*x^2)) + (A*Log[x])/b^3
 - (A*Log[b + c*x^2])/(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^5 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3} \, dx = \int \frac{A+Bx^2}{x \left(b+cx^2\right)^3} \, dx$$
$$= \frac{1}{2} \text{Subst}\left(\int \frac{A+Bx}{x(b+cx)^3} \, dx, x, x^2\right)$$
$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{A}{b^3x} + \frac{bB-Ac}{b(b+cx)^3} - \frac{Ac}{b^2(b+cx)^2} - \frac{Ac}{b^3(b+cx)}\right) dx, x, x^2\right)$$
$$= -\frac{bB-Ac}{4bc\left(b+cx^2\right)^2} + \frac{A}{2b^2\left(b+cx^2\right)} + \frac{A\log(x)}{b^3} - \frac{A\log\left(b+cx^2\right)}{2b^3}$$

Mathematica [**A**] time = 0.07, size = 59, normalized size = 0.87

$$\frac{\frac{b\left(3Abc+2Ac^2x^2+b^2(-B)\right)}{c\left(b+cx^2\right)^2} - 2A\log\left(b+cx^2\right) + 4A\log(x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(-(b^2*B) + 3*A*b*c + 2*A*c^2*x^2))/(c*(b + c*x^2)^2) + 4*A*Log[x] - 2*
A*Log[b + c*x^2])/(4*b^3)
```

fricas [**A**] time = 0.69, size = 119, normalized size = 1.75

$$\frac{2\,Abc^2x^2 - Bb^3 + 3\,Ab^2c - 2\left(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c\right)\log\left(cx^2+b\right) + 4\left(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c\right)\log(x)}{4\left(b^3c^3x^4 + 2\,b^4c^2x^2 + b^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*b*c^2*x^2 - B*b^3 + 3*A*b^2*c - 2*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b
^2*c)*log(c*x^2 + b) + 4*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b^2*c)*log(x))/(b^3
*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)
```

giac [**A**] time = 0.16, size = 76, normalized size = 1.12

$$\frac{A\log\left(x^2\right)}{2\,b^3} - \frac{A\log\left(\left|cx^2+b\right|\right)}{2\,b^3} + \frac{3\,Ac^3x^4 + 8\,Abc^2x^2 - Bb^3 + 6\,Ab^2c}{4\left(cx^2+b\right)^2 b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/b^3 - 1/2*A*log(abs(c*x^2 + b))/b^3 + 1/4*(3*A*c^3*x^4 + 8*A
*b*c^2*x^2 - B*b^3 + 6*A*b^2*c)/((c*x^2 + b)^2*b^3*c)
```

maple [**A**] time = 0.06, size = 68, normalized size = 1.00

$$\frac{A}{4\left(c\,x^2+b\right)^2 b} - \frac{B}{4\left(c\,x^2+b\right)^2 c} + \frac{A}{2\left(c\,x^2+b\right)b^2} + \frac{A\ln(x)}{b^3} - \frac{A\ln\left(c\,x^2+b\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2*A*ln(c*x^2+b)/b^3+1/2*A/b^2/(c*x^2+b)+1/4/b/(c*x^2+b)^2*A-1/4/c/(c*x^2
+b)^2*B+A*ln(x)/b^3
```

maxima [A] time = 1.42, size = 77, normalized size = 1.13

$$\frac{2\,Ac^2x^2 - Bb^2 + 3\,Abc}{4\left(b^2c^3x^4 + 2\,b^3c^2x^2 + b^4c\right)} - \frac{A\log\left(cx^2 + b\right)}{2\,b^3} + \frac{A\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*c^2*x^2 - B*b^2 + 3*A*b*c)/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c) -
 1/2*A*log(c*x^2 + b)/b^3 + 1/2*A*log(x^2)/b^3
```

mupad [B] time = 0.18, size = 71, normalized size = 1.04

$$\frac{\frac{3\,A\,c-B\,b}{4\,b\,c}+\frac{A\,c\,x^2}{2\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} - \frac{A\,\ln\left(c\,x^2+b\right)}{2\,b^3} + \frac{A\,\ln(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*A*c - B*b)/(4*b*c) + (A*c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (
A*log(b + c*x^2))/(2*b^3) + (A*log(x))/b^3
```

sympy [A] time = 0.59, size = 75, normalized size = 1.10

$$\frac{A\log\left(x\right)}{b^3} - \frac{A\log\left(\frac{b}{c} + x^2\right)}{2b^3} + \frac{3Abc + 2Ac^2x^2 - Bb^2}{4b^4c + 8b^3c^2x^2 + 4b^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] A*log(x)/b**3 - A*log(b/c + x**2)/(2*b**3) + (3*A*b*c + 2*A*c**2*x**2 - B*b
**2)/(4*b**4*c + 8*b**3*c**2*x**2 + 4*b**2*c**3*x**4)
```

3.83 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=96

$$\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}+\frac{x(3bB-7Ac)}{8b^3\left(b+cx^2\right)}-\frac{A}{b^3x}+\frac{x(bB-Ac)}{4b^2\left(b+cx^2\right)^2}$$

```
[Out] -A/b^3/x+1/4*(-A*c+B*b)*x/b^2/(c*x^2+b)^2+1/8*(-7*A*c+3*B*b)*x/b^3/(c*x^2+b
)+3/8*(-5*A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(7/2)/c^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 456, 453, 205}

$$\frac{x(3bB-7Ac)}{8b^3\left(b+cx^2\right)}+\frac{x(bB-Ac)}{4b^2\left(b+cx^2\right)^2}+\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}-\frac{A}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4 \left(A + Bx^2\right)}{\left(bx^2 + cx^4\right)^3} dx = \int \frac{A + Bx^2}{x^2 \left(b + cx^2\right)^3} dx$$
$$= \frac{(bB - Ac)x}{4b^2 \left(b + cx^2\right)^2} - \frac{1}{4} \int \frac{-\frac{4A}{b} - \frac{3(bB-Ac)x^2}{b^2}}{x^2 \left(b + cx^2\right)^2} dx$$
$$= \frac{(bB - Ac)x}{4b^2 \left(b + cx^2\right)^2} + \frac{(3bB - 7Ac)x}{8b^3 \left(b + cx^2\right)} + \frac{1}{8} \int \frac{\frac{8A}{b^2} + \frac{(3bB-7Ac)x^2}{b^3}}{x^2 \left(b + cx^2\right)} dx$$
$$= -\frac{A}{b^3 x} + \frac{(bB - Ac)x}{4b^2 \left(b + cx^2\right)^2} + \frac{(3bB - 7Ac)x}{8b^3 \left(b + cx^2\right)} + \frac{(3(bB - 5Ac)) \int \frac{1}{b+cx^2} dx}{8b^3}$$
$$= -\frac{A}{b^3 x} + \frac{(bB - Ac)x}{4b^2 \left(b + cx^2\right)^2} + \frac{(3bB - 7Ac)x}{8b^3 \left(b + cx^2\right)} + \frac{3(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}$$

Mathematica [A] time = 0.10, size = 96, normalized size = 1.00

$$\frac{3(bB - 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}} + \frac{x(3bB - 7Ac)}{8b^3 \left(b + cx^2\right)} - \frac{A}{b^3 x} + \frac{x(bB - Ac)}{4b^2 \left(b + cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

fricas [A] time = 1.08, size = 324, normalized size = 3.38

$$\left[-\frac{16\,Ab^3c - 6\left(Bb^2c^2 - 5\,Abc^3\right)x^4 - 10\left(Bb^3c - 5\,Ab^2c^2\right)x^2 - 3\left(\left(Bbc^2 - 5\,Ac^3\right)x^5 + 2\left(Bb^2c - 5\,Abc^2\right)x^3 + \left(Bb^3 -\right.\right.}{16\left(b^4c^3x^5 + 2\,b^5c^2x^3 + b^6cx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*A*b^3*c - 6*(B*b^2*c^2 - 5*A*b*c^3)*x^4 - 10*(B*b^3*c - 5*A*b^2*
c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^
3 - 5*A*b^2*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))
/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x), -1/8*(8*A*b^3*c - 3*(B*b^2*c^2 -
5*A*b*c^3)*x^4 - 5*(B*b^3*c - 5*A*b^2*c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5
 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^3 - 5*A*b^2*c)*x)*sqrt(b*c)*arctan(sq
rt(b*c)*x/b))/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x)]
```

giac [A] time = 0.17, size = 82, normalized size = 0.85

$$\frac{3\,(Bb - 5\,Ac) \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3} - \frac{A}{b^3 x} + \frac{3\,Bbcx^3 - 7\,Ac^2x^3 + 5\,Bb^2x - 9\,Abcx}{8\left(cx^2 + b\right)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - A/(b^3*x) + 1/8*(
3*B*b*c*x^3 - 7*A*c^2*x^3 + 5*B*b^2*x - 9*A*b*c*x)/((c*x^2 + b)^2*b^3)
```

maple [A] time = 0.06, size = 125, normalized size = 1.30

$$-\frac{7Ac^2x^3}{8\left(cx^2+b\right)^2b^3}+\frac{3Bcx^3}{8\left(cx^2+b\right)^2b^2}-\frac{9Acx}{8\left(cx^2+b\right)^2b^2}+\frac{5Bx}{8\left(cx^2+b\right)^2b}-\frac{15Ac\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,b^3}+\frac{3B\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,b^2}-\frac{A}{b^3 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -7/8/b^3/(c*x^2+b)^2*A*x^3*c^2+3/8/b^2/(c*x^2+b)^2*B*x^3*c-9/8/b^2/(c*x^2+b
)^2*A*c*x+5/8/b/(c*x^2+b)^2*B*x-15/8/b^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c
*x)*A*c+3/8/b^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B-A/b^3/x
```

maxima [A] time = 2.97, size = 96, normalized size = 1.00

$$\frac{3\left(Bbc-5\,Ac^2\right)x^4-8\,Ab^2+5\left(Bb^2-5\,Abc\right)x^2}{8\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)}+\frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b*c - 5*A*c^2)*x^4 - 8*A*b^2 + 5*(B*b^2 - 5*A*b*c)*x^2)/(b^3*c^2*
x^5 + 2*b^4*c*x^3 + b^5*x) + 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(
b*c)*b^3)
```

mupad [B] time = 0.18, size = 113, normalized size = 1.18

$$-\frac{\frac{A}{b}+\frac{5\,x^2\,(5\,A\,c-B\,b)}{8\,b^2}+\frac{3\,c\,x^4\,(5\,A\,c-B\,b)}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5}-\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{c}\,x\,(5\,A\,c-B\,b)}{\sqrt{b}\,(15\,A\,c-3\,B\,b)}\right)(5\,A\,c-B\,b)}{8\,b^{7/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (A/b + (5*x^2*(5*A*c - B*b))/(8*b^2) + (3*c*x^4*(5*A*c - B*b))/(8*b^3))/(
b^2*x + c^2*x^5 + 2*b*c*x^3) - (3*atan((3*c^(1/2)*x*(5*A*c - B*b))/(b^(1/2)
*(15*A*c - 3*B*b)))*(5*A*c - B*b))/(8*b^(7/2)*c^(1/2))
```

sympy [B] time = 0.73, size = 194, normalized size = 2.02

$$-\frac{3\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)\log\left(-\frac{3b^4\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)\log\left(\frac{3b^4\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16}+\frac{-8A \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(-3*b**4*sqrt(-1/(b**7*c))*(-5*A*c +
 B*b)/(-15*A*c + 3*B*b) + x)/16 + 3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(3*
b**4*sqrt(-1/(b**7*c))*(-5*A*c + B*b)/(-15*A*c + 3*B*b) + x)/16 + (-8*A*b**
2 + x**4*(-15*A*c**2 + 3*B*b*c) + x**2*(-25*A*b*c + 5*B*b**2))/(8*b**5*x +
16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

3.84 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{\log(x)(bB-3Ac)}{b^4}+\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}-\frac{A}{2b^3x^2}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}$$

```
[Out] -1/2*A/b^3/x^2+1/4*(-A*c+B*b)/b^2/(c*x^2+b)^2+1/2*(-2*A*c+B*b)/b^3/(c*x^2+b
)+(-3*A*c+B*b)*ln(x)/b^4-1/2*(-3*A*c+B*b)*ln(c*x^2+b)/b^4
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{\log(x)(bB-3Ac)}{b^4}-\frac{A}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(2*b^3*x^2) + (b*B - A*c)/(4*b^2*(b + c*x^2)^2) + (b*B - 2*A*c)/(2*b^3*(
b + c*x^2)) + ((b*B - 3*A*c)*Log[x])/b^4 - ((b*B - 3*A*c)*Log[b + c*x^2])/(
2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{A+Bx^2}{x^3\left(b+cx^2\right)^3}\,dx$$
$$=\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^3}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{A}{b^3x^2}+\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)}{b^2(b+cx)^3}-\frac{c(bB-2Ac)}{b^3(b+cx)^2}-\frac{c(bB-3Ac)}{b^4(b+cx)}\right)dx,x,x^2\right)$$
$$=-\frac{A}{2b^3x^2}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}+\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}+\frac{(bB-3Ac)\log(x)}{b^4}-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}$$

Mathematica [A] time = 0.10, size = 86, normalized size = 0.89

$$\frac{\frac{b^2(bB-Ac)}{\left(b+cx^2\right)^2}+\frac{2b(bB-2Ac)}{b+cx^2}-2(bB-3Ac)\log\left(b+cx^2\right)+4\log(x)(bB-3Ac)-\frac{2Ab}{x^2}}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-2*A*b)/x^2 + (b^2*(b*B - A*c))/(b + c*x^2)^2 + (2*b*(b*B - 2*A*c))/(b + c*x^2) + 4*(b*B - 3*A*c)*Log[x] - 2*(b*B - 3*A*c)*Log[b + c*x^2])/(4*b^4)
```

fricas [B] time = 0.88, size = 197, normalized size = 2.03

$$\frac{2\left(Bb^2c-3\,Abc^2\right)x^4-2\,Ab^3+3\left(Bb^3-3\,Ab^2c\right)x^2-2\left(\left(Bbc^2-3\,Ac^3\right)x^6+2\left(Bb^2c-3\,Abc^2\right)x^4+\left(Bb^3-3\,Ab^2c\right)x^2\right)\log\left(cx^2+b\right)+4\left(\left(Bbc^2-3\,Ac^3\right)x^6+2\left(Bb^2c-3\,Abc^2\right)x^4+\left(Bb^3-3\,Ab^2c\right)x^2\right)\log(x)}{4\left(b^4c^2x^6+2\,b^5cx^4+b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)*x^2 - 2*((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2)*x^4 + (B*b^3 - 3*A*b^2*c)*x^2)*log(c*x^2 + b) + 4*((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2)*x^4 + (B*b^3 - 3*A*b^2*c)*x^2)*log(x))/(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)
```

giac [A] time = 0.18, size = 105, normalized size = 1.08

$$\frac{(Bb-3\,Ac)\log\left(|x|\right)}{b^4}-\frac{\left(Bbc-3\,Ac^2\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^4c}+\frac{2\left(Bb^2c-3\,Abc^2\right)x^4-2\,Ab^3+3\left(Bb^3-3\,Ab^2c\right)x^2}{4\left(cx^2+b\right)^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (B*b - 3*A*c)*log(abs(x))/b^4 - 1/2*(B*b*c - 3*A*c^2)*log(abs(c*x^2 + b))/(b^4*c) + 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)*x^2)/((c*x^2 + b)^2*b^4*x^2)
```

maple [A] time = 0.06, size = 118, normalized size = 1.22

$$-\frac{Ac}{4\left(c\,x^2+b\right)^2b^2}+\frac{B}{4\left(c\,x^2+b\right)^2b}-\frac{Ac}{\left(c\,x^2+b\right)b^3}-\frac{3Ac\ln(x)}{b^4}+\frac{3Ac\ln\left(c\,x^2+b\right)}{2b^4}+\frac{B}{2\left(c\,x^2+b\right)b^2}+\frac{B\ln(x)}{b^3}-\frac{B\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 3/2/b^4*c*ln(c*x^2+b)*A-1/2/b^3*ln(c*x^2+b)*B-1/b^3*c/(c*x^2+b)*A+1/2/b^2/(
c*x^2+b)*B-1/4/b^2*c/(c*x^2+b)^2*A+1/4/b/(c*x^2+b)^2*B-1/2*A/b^3/x^2-3/b^4*
ln(x)*A*c+1/b^3*ln(x)*B
```

maxima [A] time = 1.37, size = 109, normalized size = 1.12

$$\frac{2\left(Bbc-3\,Ac^2\right)x^4-2\,Ab^2+3\left(Bb^2-3\,Abc\right)x^2}{4\left(b^3c^2x^6+2\,b^4cx^4+b^5x^2\right)}-\frac{(Bb-3\,Ac)\log\left(cx^2+b\right)}{2\,b^4}+\frac{(Bb-3\,Ac)\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*b*c - 3*A*c^2)*x^4 - 2*A*b^2 + 3*(B*b^2 - 3*A*b*c)*x^2)/(b^3*c^2*
x^6 + 2*b^4*c*x^4 + b^5*x^2) - 1/2*(B*b - 3*A*c)*log(c*x^2 + b)/b^4 + 1/2*(
B*b - 3*A*c)*log(x^2)/b^4
```

mupad [B] time = 0.15, size = 107, normalized size = 1.10

$$\frac{\ln\left(c\,x^2+b\right)\,\left(3\,A\,c-B\,b\right)}{2\,b^4}-\frac{\frac{A}{2\,b}+\frac{3\,x^2\,\left(3\,A\,c-B\,b\right)}{4\,b^2}+\frac{c\,x^4\,\left(3\,A\,c-B\,b\right)}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6}-\frac{\ln(x)\,\left(3\,A\,c-B\,b\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (log(b + c*x^2)*(3*A*c - B*b))/(2*b^4) - (A/(2*b) + (3*x^2*(3*A*c - B*b))/(
4*b^2) + (c*x^4*(3*A*c - B*b))/(2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (
log(x)*(3*A*c - B*b))/b^4
```

sympy [A] time = 1.11, size = 107, normalized size = 1.10

$$\frac{-2Ab^2+x^4\left(-6Ac^2+2Bbc\right)+x^2\left(-9Abc+3Bb^2\right)}{4b^5x^2+8b^4cx^4+4b^3c^2x^6}+\frac{(-3Ac+Bb)\log(x)}{b^4}-\frac{(-3Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 2*B*b*c) + x**2*(-9*A*b*c + 3*B*b**2))/(4*b*
*5*x**2 + 8*b**4*c*x**4 + 4*b**3*c**2*x**6) + (-3*A*c + B*b)*log(x)/b**4 -
(-3*A*c + B*b)*log(b/c + x**2)/(2*b**4)
```

3.85 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=117

$$-\frac{5\sqrt{c}\,(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}-\frac{cx(7bB-11Ac)}{8b^4\left(b+cx^2\right)}-\frac{bB-3Ac}{b^4x}-\frac{cx(bB-Ac)}{4b^3\left(b+cx^2\right)^2}-\frac{A}{3b^3x^3}$$

```
[Out] -1/3*A/b^3/x^3+(3*A*c-B*b)/b^4/x-1/4*c*(-A*c+B*b)*x/b^3/(c*x^2+b)^2-1/8*c*(
-11*A*c+7*B*b)*x/b^4/(c*x^2+b)-5/8*(-7*A*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))
*c^(1/2)/b^(9/2)
```

Rubi [A] time = 0.18, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1584, 456, 1259, 1261, 205}

$$-\frac{cx(7bB-11Ac)}{8b^4\left(b+cx^2\right)}-\frac{cx(bB-Ac)}{4b^3\left(b+cx^2\right)^2}-\frac{bB-3Ac}{b^4x}-\frac{5\sqrt{c}\,(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}-\frac{A}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(3*b^3*x^3) - (b*B - 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b + c*x^
2)^2) - (c*(7*b*B - 11*A*c)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b*B - 7*
A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)^3}\,dx\\
&= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{1}{4}c\int \frac{-\frac{4A}{bc}-\frac{4(bB-Ac)x^2}{b^2c}+\frac{3(bB-Ac)x^4}{b^3}}{x^4\left(b+cx^2\right)^2}\,dx\\
&= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{\int \frac{-8Abc-8c(bB-2Ac)x^2+\frac{c^2(7bB-11Ac)x^4}{b}}{x^4\left(b+cx^2\right)}\,dx}{8b^3c}\\
&= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{\int \left(-\frac{8Ac}{x^4}-\frac{8c(bB-3Ac)}{bx^2}+\frac{5c^2(3bB-7Ac)}{b\left(b+cx^2\right)}\right)dx}{8b^3c}\\
&= -\frac{A}{3b^3x^3}-\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{(5c(3bB-7Ac))\int \frac{1}{b+cx^2}\,dx}{8b^4}\\
&= -\frac{A}{3b^3x^3}-\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 119, normalized size = 1.02

$$-\frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}-\frac{x\left(7bBc-11Ac^2\right)}{8b^4\left(b+cx^2\right)}+\frac{3Ac-bB}{b^4x}-\frac{cx(bB-Ac)}{4b^3\left(b+cx^2\right)^2}-\frac{A}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/3*A/(b^3*x^3) + (-(b*B) + 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b +
 c*x^2)^2) - ((7*b*B*c - 11*A*c^2)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b
*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

fricas [A] time = 0.76, size = 368, normalized size = 3.15

$$\left[-\frac{30\left(3\,Bbc^2-7\,Ac^3\right)x^6+50\left(3\,Bb^2c-7\,Abc^2\right)x^4+16\,Ab^3+16\left(3\,Bb^3-7\,Ab^2c\right)x^2+15\left(\left(3\,Bbc^2-7\,Ac^3\right)x^7+2\right.}{48\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*b*c^2 - 7*A*c^3)*x^6 + 50*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 16*
A*b^3 + 16*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(3
*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(-c/b)*log((c*x^
2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^
3), -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 +
8*A*b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(
3*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(c/b)*arctan(x*
sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

giac [A] time = 0.16, size = 108, normalized size = 0.92

$$-\frac{5\left(3\,Bbc-7\,Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4}-\frac{7\,Bbc^2x^3-11\,Ac^3x^3+9\,Bb^2cx-13\,Abc^2x}{8\left(cx^2+b\right)^2b^4}-\frac{3\,Bbx^2-9\,Acx^2+Ab}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/8*(7*B*b
*c^2*x^3 - 11*A*c^3*x^3 + 9*B*b^2*c*x - 13*A*b*c^2*x)/((c*x^2 + b)^2*b^4) -
 1/3*(3*B*b*x^2 - 9*A*c*x^2 + A*b)/(b^4*x^3)
```

maple [A] time = 0.06, size = 152, normalized size = 1.30

$$\frac{11A\,c^3x^3}{8\left(c\,x^2+b\right)^2b^4}-\frac{7B\,c^2x^3}{8\left(c\,x^2+b\right)^2b^3}+\frac{13A\,c^2x}{8\left(c\,x^2+b\right)^2b^3}-\frac{9Bcx}{8\left(c\,x^2+b\right)^2b^2}+\frac{35A\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^4}-\frac{15Bc\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 11/8/b^4*c^3/(c*x^2+b)^2*A*x^3-7/8/b^3*c^2/(c*x^2+b)^2*B*x^3+13/8/b^3*c^2/(
c*x^2+b)^2*A*x-9/8/b^2*c/(c*x^2+b)^2*B*x+35/8/b^4*c^2/(b*c)^(1/2)*arctan(1/
(b*c)^(1/2)*c*x)*A-15/8/b^3*c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*B-1/3*A
/b^3/x^3+3/b^4/x*A*c-1/b^3/x*B
```

maxima [A] time = 3.11, size = 128, normalized size = 1.09

$$-\frac{15\left(3\,Bbc^2-7\,Ac^3\right)x^6+25\left(3\,Bb^2c-7\,Abc^2\right)x^4+8\,Ab^3+8\left(3\,Bb^3-7\,Ab^2c\right)x^2}{24\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)}-\frac{5\left(3\,Bbc-7\,Ac^2\right)\arctan\left(\right.}{8\,\sqrt{bc}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 8*A*
b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3) -
5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

mupad [B] time = 0.18, size = 114, normalized size = 0.97

$$\frac{\frac{x^2\,(7\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{5\,c^2\,x^6\,(7\,A\,c-3\,B\,b)}{8\,b^4}+\frac{25\,c\,x^4\,(7\,A\,c-3\,B\,b)}{24\,b^3}}{b^2\,x^3+2\,b\,c\,x^5+c^2\,x^7}+\frac{5\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(7\,A\,c-3\,B\,b)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^2*(7*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (5*c^2*x^6*(7*A*c - 3*B*b))/(8*b
^4) + (25*c*x^4*(7*A*c - 3*B*b))/(24*b^3))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5)
+ (5*c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(7*A*c - 3*B*b))/(8*b^(9/2))
```

sympy [B] time = 0.83, size = 226, normalized size = 1.93

$$\frac{5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)\log\left(-\frac{5b^5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16}-\frac{5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)\log\left(\frac{5b^5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16}+\frac{-8Ab^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(-5*b**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)
/(-35*A*c**2 + 15*B*b*c) + x)/16 - 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(5*b
**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)/(-35*A*c**2 + 15*B*b*c) + x)/16 + (-8*A*
b**3 + x**6*(105*A*c**3 - 45*B*b*c**2) + x**4*(175*A*b*c**2 - 75*B*b**2*c)
+ x**2*(56*A*b**2*c - 24*B*b**3))/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**4*
c**2*x**7)
```

3.86 $\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=121

$$\frac{3c(bB-2Ac)\log\left(b+cx^2\right)}{2b^5}-\frac{3c\log(x)(bB-2Ac)}{b^5}-\frac{c(2bB-3Ac)}{2b^4\left(b+cx^2\right)}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\left(b+cx^2\right)^2}-\frac{A}{4b^3x^4}$$

```
[Out] -1/4*A/b^3/x^4+1/2*(3*A*c-B*b)/b^4/x^2-1/4*c*(-A*c+B*b)/b^3/(c*x^2+b)^2-1/2
*c*(-3*A*c+2*B*b)/b^4/(c*x^2+b)-3*c*(-2*A*c+B*b)*ln(x)/b^5+3/2*c*(-2*A*c+B*
b)*ln(c*x^2+b)/b^5
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1584, 446, 77}

$$-\frac{c(2bB-3Ac)}{2b^4\left(b+cx^2\right)}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\left(b+cx^2\right)^2}+\frac{3c(bB-2Ac)\log\left(b+cx^2\right)}{2b^5}-\frac{3c\log(x)(bB-2Ac)}{b^5}-\frac{A}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(4*b^3*x^4) - (b*B - 3*A*c)/(2*b^4*x^2) - (c*(b*B - A*c))/(4*b^3*(b + c*
x^2)^2) - (c*(2*b*B - 3*A*c))/(2*b^4*(b + c*x^2)) - (3*c*(b*B - 2*A*c)*Log[
x])/b^5 + (3*c*(b*B - 2*A*c)*Log[b + c*x^2])/(2*b^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^3}\,dx$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^3}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{b^3x^3}+\frac{bB-3Ac}{b^4x^2}-\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^3}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)^2}+\frac{3c^2(bB}{b^5(b}\right.\right.$$
$$=-\frac{A}{4b^3x^4}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\left(b+cx^2\right)^2}-\frac{c(2bB-3Ac)}{2b^4\left(b+cx^2\right)}-\frac{3c(bB-2Ac)\log(x)}{b^5}+\frac{3c(bB-2A}{}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.89

$$\frac{\frac{b^2c(Ac-bB)}{\left(b+cx^2\right)^2}-\frac{Ab^2}{x^4}+\frac{2bc(3Ac-2bB)}{b+cx^2}-\frac{2b(bB-3Ac)}{x^2}+6c(bB-2Ac)\log\left(b+cx^2\right)+12c\log(x)(2Ac-bB)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-((A*b^2)/x^4) - (2*b*(b*B - 3*A*c))/x^2 + (b^2*c*(-(b*B) + A*c))/(b + c*x
^2)^2 + (2*b*c*(-2*b*B + 3*A*c))/(b + c*x^2) + 12*c*(-(b*B) + 2*A*c)*Log[x]
 + 6*c*(b*B - 2*A*c)*Log[b + c*x^2])/(4*b^5)
```

fricas [B] time = 1.03, size = 229, normalized size = 1.89

$$-\frac{6\left(Bb^2c^2-2\,Abc^3\right)x^6+Ab^4+9\left(Bb^3c-2\,Ab^2c^2\right)x^4+2\left(Bb^4-2\,Ab^3c\right)x^2-6\left(\left(Bbc^3-2\,Ac^4\right)x^8+2\left(Bb^2c^2-2\right.\right.}{4\left(b^5c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + A*b^4 + 9*(B*b^3*c - 2*A*b^2*c^2)*x^4
 + 2*(B*b^4 - 2*A*b^3*c)*x^2 - 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 -
2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x^4)*log(c*x^2 + b) + 12*((B*b*c^3
 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x
^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

giac [A] time = 0.17, size = 132, normalized size = 1.09

$$-\frac{3\left(Bbc-2\,Ac^2\right)\log\left(|x|\right)}{b^5}+\frac{3\left(Bbc^2-2\,Ac^3\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^5c}-\frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3}{4\left(cx^4+bx^2\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*log(abs(x))/b^5 + 3/2*(B*b*c^2 - 2*A*c^3)*log(abs(c*x^
2 + b))/(b^5*c) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6 + 9*B*b^2*c*x^4 - 18*A*
b*c^2*x^4 + 2*B*b^3*x^2 - 4*A*b^2*c*x^2 + A*b^3)/((c*x^4 + b*x^2)^2*b^4)
```

maple [A] time = 0.06, size = 150, normalized size = 1.24

$$\frac{A\,c^2}{4\left(c\,x^2+b\right)^2b^3}-\frac{Bc}{4\left(c\,x^2+b\right)^2b^2}+\frac{3A\,c^2}{2\left(c\,x^2+b\right)b^4}+\frac{6A\,c^2\ln(x)}{b^5}-\frac{3A\,c^2\ln\left(c\,x^2+b\right)}{b^5}-\frac{Bc}{\left(c\,x^2+b\right)b^3}-\frac{3Bc\ln(x)}{b^4}+\frac{3Bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -3/b^5*c^2*ln(c*x^2+b)*A+3/2/b^4*c*ln(c*x^2+b)*B+3/2/b^4*c^2/(c*x^2+b)*A-1/
b^3*c/(c*x^2+b)*B+1/4/b^3*c^2/(c*x^2+b)^2*A-1/4/b^2*c/(c*x^2+b)^2*B-1/4*A/b
^3/x^4+3/2/b^4/x^2*A*c-1/2/b^3/x^2*B+6*c^2/b^5*ln(x)*A-3*c/b^4*ln(x)*B
```

maxima [A] time = 1.43, size = 137, normalized size = 1.13

$$-\frac{6\left(Bbc^2-2\,Ac^3\right)x^6+9\left(Bb^2c-2\,Abc^2\right)x^4+Ab^3+2\left(Bb^3-2\,Ab^2c\right)x^2}{4\left(b^4c^2x^8+2\,b^5cx^6+b^6x^4\right)}+\frac{3\left(Bbc-2\,Ac^2\right)\log\left(cx^2+b\right)}{2\,b^5}-\frac{3\left(Bb\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*b*c^2 - 2*A*c^3)*x^6 + 9*(B*b^2*c - 2*A*b*c^2)*x^4 + A*b^3 + 2*(
B*b^3 - 2*A*b^2*c)*x^2)/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4) + 3/2*(B*b*c
- 2*A*c^2)*log(c*x^2 + b)/b^5 - 3/2*(B*b*c - 2*A*c^2)*log(x^2)/b^5
```

mupad [B] time = 0.17, size = 131, normalized size = 1.08

$$\frac{\frac{x^2\,\left(2\,A\,c-B\,b\right)}{2\,b^2}-\frac{A}{4\,b}+\frac{3\,c^2\,x^6\,\left(2\,A\,c-B\,b\right)}{2\,b^4}+\frac{9\,c\,x^4\,\left(2\,A\,c-B\,b\right)}{4\,b^3}}{b^2\,x^4+2\,b\,c\,x^6+c^2\,x^8}-\frac{\ln\left(c\,x^2+b\right)\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{2\,b^5}+\frac{\ln(x)\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^2*(2*A*c - B*b))/(2*b^2) - A/(4*b) + (3*c^2*x^6*(2*A*c - B*b))/(2*b^4)
+ (9*c*x^4*(2*A*c - B*b))/(4*b^3))/(b^2*x^4 + c^2*x^8 + 2*b*c*x^6) - (log(b
 + c*x^2)*(6*A*c^2 - 3*B*b*c))/(2*b^5) + (log(x)*(6*A*c^2 - 3*B*b*c))/b^5
```

sympy [A] time = 1.25, size = 136, normalized size = 1.12

$$\frac{-Ab^3+x^6\left(12Ac^3-6Bbc^2\right)+x^4\left(18Abc^2-9Bb^2c\right)+x^2\left(4Ab^2c-2Bb^3\right)}{4b^6x^4+8b^5cx^6+4b^4c^2x^8}-\frac{3c\left(-2Ac+Bb\right)\log\left(x\right)}{b^5}+\frac{3c\left(-2Ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-A*b**3 + x**6*(12*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c)
+ x**2*(4*A*b**2*c - 2*B*b**3))/(4*b**6*x**4 + 8*b**5*c*x**6 + 4*b**4*c**2*
x**8) - 3*c*(-2*A*c + B*b)*log(x)/b**5 + 3*c*(-2*A*c + B*b)*log(b/c + x**2)
/(2*b**5)
```

3.87 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=140

$$\frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}+\frac{c^2x(11bB-15Ac)}{8b^5\left(b+cx^2\right)}+\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2x(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{bB-3Ac}{3b^4x^3}-\frac{A}{5b^3x^5}$$

```
[Out] -1/5*A/b^3/x^5+1/3*(3*A*c-B*b)/b^4/x^3+3*c*(-2*A*c+B*b)/b^5/x+1/4*c^2*(-A*c
+B*b)*x/b^4/(c*x^2+b)^2+1/8*c^2*(-15*A*c+11*B*b)*x/b^5/(c*x^2+b)+7/8*c^(3/2
)*(-9*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(11/2)
```

Rubi [A] time = 0.33, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1593, 456, 1805, 1802, 205}

$$\frac{c^2x(11bB-15Ac)}{8b^5\left(b+cx^2\right)}+\frac{c^2x(bB-Ac)}{4b^4\left(b+cx^2\right)^2}+\frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{bB-3Ac}{3b^4x^3}+\frac{3c(bB-2Ac)}{b^5x}-\frac{A}{5b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -A/(5*b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x) +
(c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*b^
5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8
*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^3}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} - \frac{1}{4}c^2\int \frac{-\frac{4A}{bc^2}-\frac{4(bB-Ac)x^2}{b^2c^2}+\frac{4(bB-Ac)x^4}{b^3c}-\frac{3(bB-Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)^2}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{c^2\int \frac{\frac{8A}{bc^2}+\frac{8(bB-2Ac)x^2}{b^2c^2}-\frac{8(2bB-3Ac)x^4}{b^3c}+\frac{(11bB-15Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)}\,dx}{8b}$$

$$= \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{c^2\int\left(\frac{8A}{b^2c^2x^6}+\frac{8(bB-3Ac)}{b^3c^2x^4}-\frac{24(bB-2Ac)}{b^4cx^2}+\frac{7(5bB-9Ac)}{b^4\left(b+cx^2\right)}\right)}{8b}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{\left(7c^2(5bB\right.}{}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{7c^{3/2}(5bB}{}$$

Mathematica [A] time = 0.08, size = 140, normalized size = 1.00

$$\frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}+\frac{c^2x(11bB-15Ac)}{8b^5\left(b+cx^2\right)}+\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2x(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{bB-3Ac}{3b^4x^3}-\frac{A}{5b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/5*A/(b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x)
+ (c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*
b^5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
(8*b^(11/2))
```

fricas [A] time = 0.97, size = 426, normalized size = 3.04

$$\left[\frac{210\left(5\,Bbc^3-9\,Ac^4\right)x^8+350\left(5\,Bb^2c^2-9\,Abc^3\right)x^6-48\,Ab^4+112\left(5\,Bb^3c-9\,Ab^2c^2\right)x^4-16\left(5\,Bb^4-9\,Ab^3c\right.}{240\left(b^5c^2x^9+2\,b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*b*c^3 - 9*A*c^4)*x^8 + 350*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
 48*A*b^4 + 112*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 16*(5*B*b^4 - 9*A*b^3*c)*x^
2 - 105*((5*B*b*c^3 - 9*A*c^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B
*b^3*c - 9*A*b^2*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), 1/120*(105*(5*B*b*c^3 -
9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 - 24*A*b^4 + 56*(5*B*b^3*c
 - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2 + 105*((5*B*b*c^3 - 9*A*c
^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B*b^3*c - 9*A*b^2*c^2)*x^5)*
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

giac [A] time = 0.16, size = 135, normalized size = 0.96

$$\frac{7\left(5\,Bbc^{2}-9\,Ac^{3}\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^{5}}+\frac{11\,Bbc^{3}x^{3}-15\,Ac^{4}x^{3}+13\,Bb^{2}c^{2}x-17\,Abc^{3}x}{8\,\left(cx^{2}+b\right)^{2}b^{5}}+\frac{45\,Bbcx^{4}-90\,Ac^{2}x^{4}-5\,Bb^{2}x^{2}}{15\,b^{5}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) + 1/8*(11*B
*b*c^3*x^3 - 15*A*c^4*x^3 + 13*B*b^2*c^2*x - 17*A*b*c^3*x)/((c*x^2 + b)^2*b
^5) + 1/15*(45*B*b*c*x^4 - 90*A*c^2*x^4 - 5*B*b^2*x^2 + 15*A*b*c*x^2 - 3*A*
b^2)/(b^5*x^5)
```

maple [A] time = 0.07, size = 177, normalized size = 1.26

$$-\frac{15A\,c^{4}x^{3}}{8\left(c\,x^{2}+b\right)^{2}b^{5}}+\frac{11B\,c^{3}x^{3}}{8\left(c\,x^{2}+b\right)^{2}b^{4}}-\frac{17A\,c^{3}x}{8\left(c\,x^{2}+b\right)^{2}b^{4}}+\frac{13B\,c^{2}x}{8\left(c\,x^{2}+b\right)^{2}b^{3}}-\frac{63A\,c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^{5}}+\frac{35B\,c^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/8/b^5*c^4/(c*x^2+b)^2*A*x^3+11/8/b^4*c^3/(c*x^2+b)^2*B*x^3-17/8/b^4*c^3
/(c*x^2+b)^2*A*x+13/8/b^3*c^2/(c*x^2+b)^2*B*x-63/8/b^5*c^3/(b*c)^(1/2)*arct
an(1/(b*c)^(1/2)*c*x)*A+35/8/b^4*c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)*
B-1/5*A/b^3/x^5+1/b^4/x^3*A*c-1/3/b^3/x^3*B-6*c^2/b^5/x*A+3*c/b^4/x*B
```

maxima [A] time = 3.02, size = 154, normalized size = 1.10

$$\frac{105\left(5\,Bbc^{3}-9\,Ac^{4}\right)x^{8}+175\left(5\,Bb^{2}c^{2}-9\,Abc^{3}\right)x^{6}-24\,Ab^{4}+56\left(5\,Bb^{3}c-9\,Ab^{2}c^{2}\right)x^{4}-8\left(5\,Bb^{4}-9\,Ab^{3}c\right)x^{2}}{120\left(b^{5}c^{2}x^{9}+2\,b^{6}cx^{7}+b^{7}x^{5}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*(5*B*b*c^3 - 9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
24*A*b^4 + 56*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2)/
(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) + 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*
x/sqrt(b*c))/(sqrt(b*c)*b^5)
```

mupad [B] time = 0.19, size = 135, normalized size = 0.96

$$-\frac{\frac{A}{5\,b}-\frac{x^{2}\,(9\,A\,c-5\,B\,b)}{15\,b^{2}}+\frac{35\,c^{2}\,x^{6}\,(9\,A\,c-5\,B\,b)}{24\,b^{4}}+\frac{7\,c^{3}\,x^{8}\,(9\,A\,c-5\,B\,b)}{8\,b^{5}}+\frac{7\,c\,x^{4}\,(9\,A\,c-5\,B\,b)}{15\,b^{3}}}{b^{2}\,x^{5}+2\,b\,c\,x^{7}+c^{2}\,x^{9}}-\frac{7\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(9\,A\,c-5\,B\,b)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (A/(5*b) - (x^2*(9*A*c - 5*B*b))/(15*b^2) + (35*c^2*x^6*(9*A*c - 5*B*b))/
(24*b^4) + (7*c^3*x^8*(9*A*c - 5*B*b))/(8*b^5) + (7*c*x^4*(9*A*c - 5*B*b))/
(15*b^3))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (7*c^(3/2)*atan((c^(1/2)*x)/b^(
1/2))*(9*A*c - 5*B*b))/(8*b^(11/2))
```

sympy [A] time = 0.92, size = 260, normalized size = 1.86

$$-\frac{7\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)\log\left(-\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}+\frac{7\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)\log\left(\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -7*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)*log(-7*b**6*sqrt(-c**3/b**11)*(-9*A*c
 + 5*B*b)/(-63*A*c**3 + 35*B*b*c**2) + x)/16 + 7*sqrt(-c**3/b**11)*(-9*A*c
+ 5*B*b)*log(7*b**6*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)/(-63*A*c**3 + 35*B*b
*c**2) + x)/16 + (-24*A*b**4 + x**8*(-945*A*c**4 + 525*B*b*c**3) + x**6*(-1
575*A*b*c**3 + 875*B*b**2*c**2) + x**4*(-504*A*b**2*c**2 + 280*B*b**3*c) +
x**2*(72*A*b**3*c - 40*B*b**4))/(120*b**7*x**5 + 240*b**6*c*x**7 + 120*b**5
*c**2*x**9)
```

3.88 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=148

$$-\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}+\frac{2c^2\log(x)(3bB-5Ac)}{b^6}+\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{3c(bB-2Ac)}{2b^5x^2}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{bB-3A}{4b^4x^4}$$

```
[Out] -1/6*A/b^3/x^6+1/4*(3*A*c-B*b)/b^4/x^4+3/2*c*(-2*A*c+B*b)/b^5/x^2+1/4*c^2*(
-A*c+B*b)/b^4/(c*x^2+b)^2+1/2*c^2*(-4*A*c+3*B*b)/b^5/(c*x^2+b)+2*c^2*(-5*A*
c+3*B*b)*ln(x)/b^6-c^2*(-5*A*c+3*B*b)*ln(c*x^2+b)/b^6
```

Rubi [A] time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 446, 77}

$$\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}+\frac{2c^2\log(x)(3bB-5Ac)}{b^6}+\frac{3c(bB-2Ac)}{2b^5x^2}-\frac{bB-3Ac}{4b^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -A/(6*b^3*x^6) - (b*B - 3*A*c)/(4*b^4*x^4) + (3*c*(b*B - 2*A*c))/(2*b^5*x^2
) + (c^2*(b*B - A*c))/(4*b^4*(b + c*x^2)^2) + (c^2*(3*b*B - 4*A*c))/(2*b^5*
(b + c*x^2)) + (2*c^2*(3*b*B - 5*A*c)*Log[x])/b^6 - (c^2*(3*b*B - 5*A*c)*Lo
g[b + c*x^2])/b^6
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^7\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{b^3x^4} + \frac{bB-3Ac}{b^4x^3} - \frac{3c(bB-2Ac)}{b^5x^2} + \frac{2c^2(3bB-5Ac)}{b^6x} - \frac{c^3(bB-Ac)}{b^4(b+cx)^3} - \frac{c^3}{}\right.\right.$$
$$= -\frac{A}{6b^3x^6} - \frac{bB-3Ac}{4b^4x^4} + \frac{3c(bB-2Ac)}{2b^5x^2} + \frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)} + \frac{2c^2(3bB-5}{b}$$

Mathematica [A] time = 0.14, size = 135, normalized size = 0.91

$$\frac{-\frac{2Ab^3}{x^6} + \frac{3b^2c^2(bB-Ac)}{\left(b+cx^2\right)^2} - \frac{3b^2(bB-3Ac)}{x^4} + \frac{6bc^2(3bB-4Ac)}{b+cx^2} + 12c^2(5Ac-3bB)\log\left(b+cx^2\right) + 24c^2\log(x)(3bB-5Ac) +}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*A*b^3)/x^6 - (3*b^2*(b*B - 3*A*c))/x^4 + (18*b*c*(b*B - 2*A*c))/x^2 +
(3*b^2*c^2*(b*B - A*c))/(b + c*x^2)^2 + (6*b*c^2*(3*b*B - 4*A*c))/(b + c*x^
2) + 24*c^2*(3*b*B - 5*A*c)*Log[x] + 12*c^2*(-3*b*B + 5*A*c)*Log[b + c*x^2]
)/(12*b^6)
```

fricas [A] time = 0.71, size = 267, normalized size = 1.80

$$\frac{12\left(3Bb^2c^3-5Abc^4\right)x^8 + 18\left(3Bb^3c^2-5Ab^2c^3\right)x^6 - 2Ab^5 + 4\left(3Bb^4c-5Ab^3c^2\right)x^4 - \left(3Bb^5-5Ab^4c\right)x^2 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + 18*(3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6
 - 2*A*b^5 + 4*(3*B*b^4*c - 5*A*b^3*c^2)*x^4 - (3*B*b^5 - 5*A*b^4*c)*x^2 -
12*((3*B*b*c^4 - 5*A*c^5)*x^10 + 2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3
*c^2 - 5*A*b^2*c^3)*x^6)*log(c*x^2 + b) + 24*((3*B*b*c^4 - 5*A*c^5)*x^10 +
2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6)*log(x))/
(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6)
```

giac [A] time = 0.17, size = 201, normalized size = 1.36

$$\frac{\left(3Bbc^2-5Ac^3\right)\log\left(x^2\right)}{b^6} - \frac{\left(3Bbc^3-5Ac^4\right)\log\left(\left|cx^2+b\right|\right)}{b^6c} + \frac{18Bbc^4x^4 - 30Ac^5x^4 + 42Bb^2c^3x^2 - 68Abc^4x^2}{4\left(cx^2+b\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6 - (3*B*b*c^3 - 5*A*c^4)*log(abs(c*x^2 +
b))/(b^6*c) + 1/4*(18*B*b*c^4*x^4 - 30*A*c^5*x^4 + 42*B*b^2*c^3*x^2 - 68*A*
b*c^4*x^2 + 25*B*b^3*c^2 - 39*A*b^2*c^3)/((c*x^2 + b)^2*b^6) - 1/12*(66*B*b
*c^2*x^6 - 110*A*c^3*x^6 - 18*B*b^2*c*x^4 + 36*A*b*c^2*x^4 + 3*B*b^3*x^2 -
9*A*b^2*c*x^2 + 2*A*b^3)/(b^6*x^6)
```

maple [A] time = 0.06, size = 180, normalized size = 1.22

$$-\frac{A\,c^{3}}{4\left(c\,x^{2}+b\right)^{2} b^{4}}+\frac{B\,c^{2}}{4\left(c\,x^{2}+b\right)^{2} b^{3}}-\frac{2A\,c^{3}}{\left(c\,x^{2}+b\right) b^{5}}-\frac{10A\,c^{3}\ln(x)}{b^{6}}+\frac{5A\,c^{3}\ln\left(c\,x^{2}+b\right)}{b^{6}}+\frac{3B\,c^{2}}{2\left(c\,x^{2}+b\right) b^{4}}+\frac{6B\,c^{2}\ln(x)}{b^{5}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^3,x)
```

```
[Out] 5/b^6*c^3*ln(c*x^2+b)*A-3/b^5*c^2*ln(c*x^2+b)*B-2/b^5*c^3/(c*x^2+b)*A+3/2/b
^4*c^2/(c*x^2+b)*B-1/4/b^4*c^3/(c*x^2+b)^2*A+1/4/b^3*c^2/(c*x^2+b)^2*B-1/6*
A/b^3/x^6+3/4/b^4/x^4*A*c-1/4/b^3/x^4*B-3*c^2/b^5/x^2*A+3/2*c/b^4/x^2*B-10*
c^3/b^6*ln(x)*A+6*c^2/b^5*ln(x)*B
```

maxima [A] time = 1.43, size = 170, normalized size = 1.15

$$\frac{12\left(3\,Bbc^{3}-5\,Ac^{4}\right)x^{8}+18\left(3\,Bb^{2}c^{2}-5\,Abc^{3}\right)x^{6}-2\,Ab^{4}+4\left(3\,Bb^{3}c-5\,Ab^{2}c^{2}\right)x^{4}-\left(3\,Bb^{4}-5\,Ab^{3}c\right)x^{2}}{12\left(b^{5}c^{2}x^{10}+2\,b^{6}cx^{8}+b^{7}x^{6}\right)}-\frac{\left(3\,Bbc^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*b*c^3 - 5*A*c^4)*x^8 + 18*(3*B*b^2*c^2 - 5*A*b*c^3)*x^6 - 2*A
*b^4 + 4*(3*B*b^3*c - 5*A*b^2*c^2)*x^4 - (3*B*b^4 - 5*A*b^3*c)*x^2)/(b^5*c^
2*x^10 + 2*b^6*c*x^8 + b^7*x^6) - (3*B*b*c^2 - 5*A*c^3)*log(c*x^2 + b)/b^6
+ (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6
```

mupad [B] time = 0.18, size = 155, normalized size = 1.05

$$\frac{\ln\left(c\,x^{2}+b\right)\left(5\,A\,c^{3}-3\,B\,b\,c^{2}\right)}{b^{6}}-\frac{\frac{A}{6\,b}-\frac{x^{2}\,(5\,A\,c-3\,B\,b)}{12\,b^{2}}+\frac{3\,c^{2}\,x^{6}\,(5\,A\,c-3\,B\,b)}{2\,b^{4}}+\frac{c^{3}\,x^{8}\,(5\,A\,c-3\,B\,b)}{b^{5}}+\frac{c\,x^{4}\,(5\,A\,c-3\,B\,b)}{3\,b^{3}}}{b^{2}\,x^{6}+2\,b\,c\,x^{8}+c^{2}\,x^{10}}-\frac{\ln(x)\,\left(10\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(b + c*x^2)*(5*A*c^3 - 3*B*b*c^2))/b^6 - (A/(6*b) - (x^2*(5*A*c - 3*B*b
))/(12*b^2) + (3*c^2*x^6*(5*A*c - 3*B*b))/(2*b^4) + (c^3*x^8*(5*A*c - 3*B*b
))/b^5 + (c*x^4*(5*A*c - 3*B*b))/(3*b^3))/(b^2*x^6 + c^2*x^10 + 2*b*c*x^8)
- (log(x)*(10*A*c^3 - 6*B*b*c^2))/b^6
```

sympy [A] time = 1.33, size = 165, normalized size = 1.11

$$\frac{-2Ab^{4}+x^{8}\left(-60Ac^{4}+36Bbc^{3}\right)+x^{6}\left(-90Abc^{3}+54Bb^{2}c^{2}\right)+x^{4}\left(-20Ab^{2}c^{2}+12Bb^{3}c\right)+x^{2}\left(5Ab^{3}c-3Bb^{4}\right)}{12b^{7}x^{6}+24b^{6}cx^{8}+12b^{5}c^{2}x^{10}}+\frac{2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**4 + x**8*(-60*A*c**4 + 36*B*b*c**3) + x**6*(-90*A*b*c**3 + 54*B*b*
*2*c**2) + x**4*(-20*A*b**2*c**2 + 12*B*b**3*c) + x**2*(5*A*b**3*c - 3*B*b*
*4))/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) + 2*c**2*(-5*A*c
+ 3*B*b)*log(x)/b**6 - c**2*(-5*A*c + 3*B*b)*log(b/c + x**2)/b**6
```

3.89 $\int x^7 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx$

Optimal. Leaf size=218

$$-\frac{7b^5(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}+\frac{7b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(3bB-4Ac)}{1024c^5}-\frac{7b^2\left(bx^2+cx^4\right)^{3/2}(3bB-4Ac)}{384c^4}$$

```
[Out] -7/384*b^2*(-4*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c^4+7/320*b*(-4*A*c+3*B*b)*x^
2*(c*x^4+b*x^2)^(3/2)/c^3-1/40*(-4*A*c+3*B*b)*x^4*(c*x^4+b*x^2)^(3/2)/c^2+1
/12*B*x^6*(c*x^4+b*x^2)^(3/2)/c-7/1024*b^5*(-4*A*c+3*B*b)*arctanh(x^2*c^(1/
2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+7/1024*b^3*(-4*A*c+3*B*b)*(2*c*x^2+b)*(c*x
^4+b*x^2)^(1/2)/c^5
```

Rubi [A] time = 0.38, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$-\frac{7b^2\left(bx^2+cx^4\right)^{3/2}(3bB-4Ac)}{384c^4}+\frac{7b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(3bB-4Ac)}{1024c^5}-\frac{7b^5(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (7*b^3*(3*b*B - 4*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^5) - (7*b
^2*(3*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(384*c^4) + (7*b*(3*b*B - 4*A*c)*
x^2*(b*x^2 + c*x^4)^(3/2))/(320*c^3) - ((3*b*B - 4*A*c)*x^4*(b*x^2 + c*x^4)
^(3/2))/(40*c^2) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c) - (7*b^5*(3*b*B - 4
*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^7\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int x^3(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$
$$=\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}+\frac{\left(3(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^3\sqrt{bx+cx^2}\,d\right.}{12c}$$
$$=-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2}+\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}+\frac{(7b(3bB-4Ac))\text{Subst}}{8}$$
$$=\frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2}+\frac{Bx^6\left(bx^2+\right.}{12c}$$
$$=-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4}{}$$
$$=\frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(}{}$$
$$=\frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(}{}$$
$$=\frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(}{}$$

Mathematica [A] time = 0.32, size = 193, normalized size = 0.89

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(-210b^4c\left(2A+Bx^2\right)+56b^3c^2x^2\left(5A+3Bx^2\right)-16b^2c^3x^4\left(14A+9Bx^2\right)+64bc^4x\right.\right.}{15360c^{11/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(315*b^5*B - 210*b^4*
c*(2*A + B*x^2) + 64*b*c^4*x^6*(3*A + 2*B*x^2) + 56*b^3*c^2*x^2*(5*A + 3*B*
x^2) + 256*c^5*x^8*(6*A + 5*B*x^2) - 16*b^2*c^3*x^4*(14*A + 9*B*x^2)) - 105
*b^(9/2)*(3*b*B - 4*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(15360*c^(11/2)*x*S
qrt[1 + (c*x^2)/b])
```

fricas [A] time = 1.48, size = 368, normalized size = 1.69

$$\left[-\frac{105\left(3Bb^6-4Ab^5c\right)\sqrt{c}\log\left(-2cx^2-b-2\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(1280Bc^6x^{10}+128\left(Bbc^5+12Ac^6\right)x^8+31\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6)*x^8 + 3
15*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56*(3*B*b^3
*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt(c*x^4 +
b*x^2))/c^6, 1/15360*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-c)/(c*x^2 + b)) + (1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6
)*x^8 + 315*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56
*(3*B*b^3*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt
(c*x^4 + b*x^2))/c^6]
```

giac [A] time = 0.23, size = 245, normalized size = 1.12

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10Bx^2\mathrm{sgn}(x)+\frac{Bbc^9\mathrm{sgn}(x)+12Ac^{10}\mathrm{sgn}(x)}{c^{10}}\right)x^2-\frac{3\left(3Bb^2c^8\mathrm{sgn}(x)-4Abc^9\mathrm{sgn}(x)\right)}{c^{10}}\right)x^2+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*x^2*sgn(x) + (B*b*c^9*sgn(x) + 12*A*c^10*sgn(x))/
c^10)*x^2 - 3*(3*B*b^2*c^8*sgn(x) - 4*A*b*c^9*sgn(x))/c^10)*x^2 + 7*(3*B*b^
3*c^7*sgn(x) - 4*A*b^2*c^8*sgn(x))/c^10)*x^2 - 35*(3*B*b^4*c^6*sgn(x) - 4*A
*b^3*c^7*sgn(x))/c^10)*x^2 + 105*(3*B*b^5*c^5*sgn(x) - 4*A*b^4*c^6*sgn(x))/
c^10)*sqrt(c*x^2 + b)*x + 7/1024*(3*B*b^6*sgn(x) - 4*A*b^5*c*sgn(x))*log(ab
s(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 7/2048*(3*B*b^6*log(abs(b)) - 4
*A*b^5*c*log(abs(b)))*sgn(x)/c^(11/2)
```

maple [A] time = 0.08, size = 290, normalized size = 1.33

$$\frac{\sqrt{cx^4+bx^2}\left(1280\left(cx^2+b\right)^{\frac{3}{2}}Bc^{\frac{9}{2}}x^9+1536\left(cx^2+b\right)^{\frac{3}{2}}Ac^{\frac{9}{2}}x^7-1152\left(cx^2+b\right)^{\frac{3}{2}}Bbc^{\frac{7}{2}}x^7-1344\left(cx^2+b\right)^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/15360*(c*x^4+b*x^2)^(1/2)*(1280*B*(c*x^2+b)^(3/2)*c^(9/2)*x^9+1536*A*(c*x
^2+b)^(3/2)*c^(9/2)*x^7-1152*B*(c*x^2+b)^(3/2)*c^(7/2)*x^7*b-1344*A*(c*x^2+
b)^(3/2)*c^(7/2)*x^5*b+1008*B*(c*x^2+b)^(3/2)*c^(5/2)*x^5*b^2+1120*A*(c*x^2
+b)^(3/2)*c^(5/2)*x^3*b^2-840*B*(c*x^2+b)^(3/2)*c^(3/2)*x^3*b^3-840*A*(c*x^
```

```
2+b)^(3/2)*c^(3/2)*x*b^3+630*B*(c*x^2+b)^(3/2)*c^(1/2)*x*b^4+420*A*(c*x^2+b
)^(1/2)*c^(3/2)*x*b^4-315*B*(c*x^2+b)^(1/2)*c^(1/2)*x*b^5+420*A*ln(c^(1/2)*
x+(c*x^2+b)^(1/2))*b^5*c-315*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^6)/x/(c*x^2+
b)^(1/2)/c^(11/2)
```

maxima [A] time = 1.62, size = 321, normalized size = 1.47

$$\frac{1}{7680}\left(\frac{768\left(cx^4+bx^2\right)^{\frac{3}{2}}x^4}{c}-\frac{420\sqrt{cx^4+bx^2}\,b^3x^2}{c^3}-\frac{672\left(cx^4+bx^2\right)^{\frac{3}{2}}bx^2}{c^2}+\frac{105\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\right.}{c^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7680*(768*(c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c
^3 - 672*(c*x^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt
(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c
*x^4 + b*x^2)^(3/2)*b^2/c^3)*A + 1/30720*(2560*(c*x^4 + b*x^2)^(3/2)*x^6/c
- 2304*(c*x^4 + b*x^2)^(3/2)*b*x^4/c^2 + 1260*sqrt(c*x^4 + b*x^2)*b^4*x^2/c
^4 + 2016*(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^3 - 315*b^6*log(2*c*x^2 + b + 2*s
qrt(c*x^4 + b*x^2)*sqrt(c))/c^(11/2) + 630*sqrt(c*x^4 + b*x^2)*b^5/c^5 - 16
80*(c*x^4 + b*x^2)^(3/2)*b^3/c^4)*B
```

mupad [B] time = 1.47, size = 289, normalized size = 1.33

$$\frac{A\,x^4\,\left(c\,x^4+b\,x^2\right)^{3/2}}{10\,c}+\frac{B\,x^6\,\left(c\,x^4+b\,x^2\right)^{3/2}}{12\,c}-\frac{3\,B\,b\left(\frac{7\,b\left(\frac{5\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,\left(c\,x^4+b\,x^2\right.}{4\,c}\right.}{10\,c}\right.}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c)
 - (3*B*b*((7*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2)^
(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^
2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(10*c) + (x^4*(b
*x^2 + c*x^4)^(3/2))/(5*c)))/(8*c) + (7*A*b*((5*b*((b^3*log(b + 2*c*x^2 + 2
*c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/
2))/(4*c)))/(20*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.90 $\int x^5 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx$

Optimal. Leaf size=181

$$\frac{b^4(7bB-10Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}-\frac{b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(7bB-10Ac)}{256c^4}+\frac{b\left(bx^2+cx^4\right)^{3/2}(7bB-10Ac)}{96c^3}-\frac{x}{}$$

```
[Out] 1/96*b*(-10*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/c^3-1/80*(-10*A*c+7*B*b)*x^2*(c*
x^4+b*x^2)^(3/2)/c^2+1/10*B*x^4*(c*x^4+b*x^2)^(3/2)/c+1/256*b^4*(-10*A*c+7*
B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/256*b^2*(-10*A*c+7*
B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

Rubi [A] time = 0.33, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$-\frac{b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(7bB-10Ac)}{256c^4}+\frac{b^4(7bB-10Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}+\frac{b\left(bx^2+cx^4\right)^{3/2}(7bB-10Ac)}{96c^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(b^2*(7*b*B - 10*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^4) + (b*(7
*b*B - 10*A*c)*(b*x^2 + c*x^4)^(3/2))/(96*c^3) - ((7*b*B - 10*A*c)*x^2*(b*x
^2 + c*x^4)^(3/2))/(80*c^2) + (B*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (b^4*(
7*b*B - 10*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
```

```
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^5\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$

$$=\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx\right.}{10c}$$

$$=-\frac{(7bB-10Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{80c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{10c}+\frac{(b(7bB-10Ac))\,\text{Subs}}{}$$

$$=\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}-\frac{(7bB-10Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{80c^2}+\frac{Bx^4\left(bx^2+cx\right.}{10c}$$

$$=-\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4}+\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}-\frac{(7b}{}$$

$$=-\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4}+\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}-\frac{(7b}{}$$

$$=-\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4}+\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}-\frac{(7b}{}$$

Mathematica [A] time = 0.30, size = 173, normalized size = 0.96

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(15b^{7/2}(7bB-10Ac)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(10b^3c\left(15A+7Bx^2\right)-4b^2c^2x^2\left(25A+14Bx^2\right.\right.\right.}{3840c^{9/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-105*b^4*B + 16*b*c^
3*x^4*(5*A + 3*B*x^2) + 96*c^4*x^6*(5*A + 4*B*x^2) + 10*b^3*c*(15*A + 7*B*x
```

```
^2) - 4*b^2*c^2*x^2*(25*A + 14*B*x^2)) + 15*b^(7/2)*(7*b*B - 10*A*c)*ArcSin
h[(Sqrt[c]*x)/Sqrt[b]]))/(3840*c^(9/2)*x*Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 0.90, size = 321, normalized size = 1.77

$$\left[-\frac{15\left(7\,Bb^5-10\,Ab^4c\right)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(384\,Bc^5x^8+48\left(Bbc^4+10\,Ac^5\right)x^6-105\,B\right.}{7680\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(7*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4
 + b*x^2)*sqrt(c)) - 2*(384*B*c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B
*b^4*c + 150*A*b^3*c^2 - 8*(7*B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2
 - 10*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^5, -1/3840*(15*(7*B*b^5 - 10*A
*b^4*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (384*B*
c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B*b^4*c + 150*A*b^3*c^2 - 8*(7*
B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2 - 10*A*b^2*c^3)*x^2)*sqrt(c*x
^4 + b*x^2))/c^5]
```

giac [A] time = 0.20, size = 211, normalized size = 1.17

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bx^2\mathrm{sgn}(x)+\frac{Bbc^7\mathrm{sgn}(x)+10\,Ac^8\mathrm{sgn}(x)}{c^8}\right)x^2-\frac{7\,Bb^2c^6\mathrm{sgn}(x)-10\,Abc^7\mathrm{sgn}(x)}{c^8}\right)x^2+\frac{5\left(7\,Bb\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*x^2*sgn(x) + (B*b*c^7*sgn(x) + 10*A*c^8*sgn(x))/c^8)*x
^2 - (7*B*b^2*c^6*sgn(x) - 10*A*b*c^7*sgn(x))/c^8)*x^2 + 5*(7*B*b^3*c^5*sgn
(x) - 10*A*b^2*c^6*sgn(x))/c^8)*x^2 - 15*(7*B*b^4*c^4*sgn(x) - 10*A*b^3*c^5
*sgn(x))/c^8)*sqrt(c*x^2 + b)*x - 1/256*(7*B*b^5*sgn(x) - 10*A*b^4*c*sgn(x)
)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/512*(7*B*b^5*log(abs(b
)) - 10*A*b^4*c*log(abs(b)))*sgn(x)/c^(9/2)
```

maple [A] time = 0.06, size = 248, normalized size = 1.37

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(384\left(c\,x^2+b\right)^{\frac{3}{2}}B\,c^{\frac{7}{2}}x^7+480\left(c\,x^2+b\right)^{\frac{3}{2}}A\,c^{\frac{7}{2}}x^5-336\left(c\,x^2+b\right)^{\frac{3}{2}}Bb\,c^{\frac{5}{2}}x^5-400\left(c\,x^2+b\right)^{\frac{3}{2}}Ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/3840*(c*x^4+b*x^2)^(1/2)*(384*B*(c*x^2+b)^(3/2)*c^(7/2)*x^7+480*A*(c*x^2+
b)^(3/2)*c^(7/2)*x^5-336*B*(c*x^2+b)^(3/2)*c^(5/2)*x^5*b-400*A*(c*x^2+b)^(3
/2)*c^(5/2)*x^3*b+280*B*(c*x^2+b)^(3/2)*c^(3/2)*x^3*b^2+300*A*(c*x^2+b)^(3/
2)*c^(3/2)*x*b^2-210*B*(c*x^2+b)^(3/2)*c^(1/2)*x*b^3-150*A*(c*x^2+b)^(1/2)*
c^(3/2)*x*b^3+105*B*(c*x^2+b)^(1/2)*c^(1/2)*x*b^4-150*A*ln(c^(1/2)*x+(c*x^2
+b)^(1/2))*b^4*c+105*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^5)/x/(c*x^2+b)^(1/2)
/c^(9/2)
```

maxima [A] time = 1.51, size = 273, normalized size = 1.51

$$\frac{1}{768}\left(\frac{60\,\sqrt{cx^4+bx^2}\,b^2x^2}{c^2}+\frac{96\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}{c}-\frac{15\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}\,b^3}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4 + b*x^2)^(3/2)*x^2/c
- 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt
(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b/c^2)*A + 1/7680*(768*(
c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^3 - 672*(c*x
^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^
2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c*x^4 + b*x^2)
^(3/2)*b^2/c^3)*B
```

mupad [B] time = 0.89, size = 233, normalized size = 1.29

$$\frac{A\,x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{8\,c}-\frac{5\,A\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}+\frac{B\,x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{10\,c}+\frac{7\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*A*b*((b^3*log(b + 2*c*x^2 + 2*c^(1
/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2
*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (B*x^4*(b*x^2 + c*x^4)^(3/2)
)/(10*c) + (7*B*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2
)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*
x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(20*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \sqrt{x^{2} \left(b + c x^{2}\right)} \left(A + B x^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.91 $\int x^3 \left(A+Bx^2\right) \sqrt{bx^2+cx^4} \, dx$

Optimal. Leaf size=125

$$-\frac{b^3(5bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(5bB-8Ac)}{128c^3}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bc\right.}{48c^2}$$

```
[Out] -1/48*(-6*B*c*x^2-8*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/c^2-1/128*b^3*(-8*A*c+5*
B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+1/128*b*(-8*A*c+5*B*b
)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A] time = 0.20, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 779, 612, 620, 206}

$$-\frac{b^3(5bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(5bB-8Ac)}{128c^3}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bc\right.}{48c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*(5*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - ((5*b*B -
 8*A*c - 6*B*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(48*c^2) - (b^3*(5*b*B - 8*A*c)*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
```

```
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^3\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\,\text{Subst}\left(\int x(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$

$$=-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{(b(5bB-8Ac))\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,d\right.}{32c^2}$$

$$=\frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}-$$

$$=\frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}-$$

$$=\frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}-$$

Mathematica [A] time = 0.23, size = 151, normalized size = 1.21

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(-2b^2c\left(12A+5Bx^2\right)+8bc^2x^2\left(2A+Bx^2\right)+16c^3x^4\left(4A+3Bx^2\right)+15b^3B\right)-3b^{5/}\right.}{384c^{7/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(15*b^3*B + 8*b*c^2*x
^2*(2*A + B*x^2) + 16*c^3*x^4*(4*A + 3*B*x^2) - 2*b^2*c*(12*A + 5*B*x^2)) -
 3*b^(5/2)*(5*b*B - 8*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(384*c^(7/2)*x*Sq
rt[1 + (c*x^2)/b])
```

fricas [A] time = 0.88, size = 272, normalized size = 2.18

$$\left[-\frac{3\left(5\,Bb^4-8\,Ab^3c\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(48\,Bc^4x^6+15\,Bb^3c-24\,Ab^2c^2+8\left(Bbc^3+8\,A\right.\right.}{768\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3
+ 8*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4,
 1/384*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c
)/(c*x^2 + b)) + (48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3 + 8
*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]
```

giac [A] time = 0.22, size = 177, normalized size = 1.42

$$\frac{1}{384}\left(2\left(4\left(6\,Bx^2\text{sgn}(x)+\frac{Bbc^5\text{sgn}(x)+8\,Ac^6\text{sgn}(x)}{c^6}\right)x^2-\frac{5\,Bb^2c^4\text{sgn}(x)-8\,Abc^5\text{sgn}(x)}{c^6}\right)x^2+\frac{3\left(5\,Bb^3c^3\text{sgn}(x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2*sgn(x) + (B*b*c^5*sgn(x) + 8*A*c^6*sgn(x))/c^6)*x^2 -
(5*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^6)*x^2 + 3*(5*B*b^3*c^3*sgn(x) -
8*A*b^2*c^4*sgn(x))/c^6)*sqrt(c*x^2 + b)*x + 1/128*(5*B*b^4*sgn(x) - 8*A*b^
3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 1/256*(5*B*b^4
*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(7/2)
```

maple [A] time = 0.05, size = 206, normalized size = 1.65

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(48\left(c\,x^2+b\right)^{\frac{3}{2}} B\,c^{\frac{5}{2}}x^5+64\left(c\,x^2+b\right)^{\frac{3}{2}} A\,c^{\frac{5}{2}}x^3-40\left(c\,x^2+b\right)^{\frac{3}{2}} Bb\,c^{\frac{3}{2}}x^3+24A\,b^3c\ln\left(\sqrt{c}\,x+\sqrt{c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(1/2)*(48*B*(c*x^2+b)^(3/2)*c^(5/2)*x^5+64*A*(c*x^2+b)^
(3/2)*c^(5/2)*x^3-40*B*(c*x^2+b)^(3/2)*c^(3/2)*x^3*b-48*A*(c*x^2+b)^(3/2)*c
^(3/2)*x*b+30*B*(c*x^2+b)^(3/2)*c^(1/2)*x*b^2+24*A*(c*x^2+b)^(1/2)*c^(3/2)*
x*b^2-15*B*(c*x^2+b)^(1/2)*c^(1/2)*x*b^3+24*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))
*b^3*c-15*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^4)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

maxima [B] time = 1.49, size = 225, normalized size = 1.80

$$-\frac{1}{96}\left(\frac{12\sqrt{cx^4+bx^2}\,bx^2}{c}-\frac{3\,b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}+\frac{6\sqrt{cx^4+bx^2}\,b^2}{c^2}-\frac{16\left(cx^4+bx^2\right)^{\frac{3}{2}}}{c}\right)A+\frac{}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 + b
*x^2)^(3/2)/c)*A + 1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4 +
b*x^2)^(3/2)*x^2/c - 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
)/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b/c^2
)*B
```

mupad [B] time = 0.74, size = 177, normalized size = 1.42

$$\frac{B\,x^2\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c}-\frac{5\,B\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}+\frac{A\,b^3\,\ln\left(b+2\,c\,x^2+2\right.}{32\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (B*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*B*b*((b^3*log(b + 2*c*x^2 + 2*c^(1
/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2
*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (A*b^3*log(b + 2*c*x^2 + 2*c
^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) + (A*(b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.92 $\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=107

$$\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(bB-2Ac)}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] 1/6*B*(c*x^4+b*x^2)^(3/2)/c+1/16*b^2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^
4+b*x^2)^(1/2))/c^(5/2)-1/16*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
^2
```

Rubi [A] time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2034, 640, 612, 620, 206}

$$\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(bB-2Ac)}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (B*(b*x^2 + c
*x^4)^(3/2))/(6*c) + (b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/(16*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$
$$=\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{4c}$$
$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{\left(b^2(bB-2Ac)\right)\text{Subst}}{32}$$
$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{\left(b^2(bB-2Ac)\right)\text{Subst}}{1}$$
$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^2(bB-2Ac)\tanh^{-1}}{16c^{5/2}}$$

Mathematica [A] time = 0.20, size = 129, normalized size = 1.21

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^{3/2}(bB-2Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)+\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(2bc\left(3A+Bx^2\right)+4c^2x^2\left(3A+2Bx^2\right)-3b^2B\right)\right)}{48c^{5/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-3*b^2*B + 2*b*c*(3*
A + B*x^2) + 4*c^2*x^2*(3*A + 2*B*x^2)) + 3*b^(3/2)*(b*B - 2*A*c)*ArcSinh[(
Sqrt[c]*x)/Sqrt[b]]))/(48*c^(5/2)*x*Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 0.98, size = 223, normalized size = 2.08

$$\left[-\frac{3\left(Bb^3-2\,Ab^2c\right)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(8\,Bc^3x^4-3\,Bb^2c+6\,Abc^2+2\left(Bbc^2+6\,Ac^3\right)x^2\right)}{96\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 2*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 - 3*B*b^2*c + 6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^
3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*(B*b^3 - 2*A*b^2*c)*sqrt(-c)*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*B*c^3*x^4 - 3*B*b^2*c +
6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

giac [A] time = 0.19, size = 140, normalized size = 1.31

$$\frac{1}{48}\left(2\left(4\,Bx^2\text{sgn}(x)+\frac{Bbc^3\text{sgn}(x)+6\,Ac^4\text{sgn}(x)}{c^4}\right)x^2-\frac{3\left(Bb^2c^2\text{sgn}(x)-2\,Abc^3\text{sgn}(x)\right)}{c^4}\right)\sqrt{cx^2+b}\,x-\frac{\left(Bb^3\text{sgn}(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2*sgn(x) + (B*b*c^3*sgn(x) + 6*A*c^4*sgn(x))/c^4)*x^2 - 3*(B
*b^2*c^2*sgn(x) - 2*A*b*c^3*sgn(x))/c^4)*sqrt(c*x^2 + b)*x - 1/16*(B*b^3*sg
n(x) - 2*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1
/32*(B*b^3*log(abs(b)) - 2*A*b^2*c*log(abs(b)))*sgn(x)/c^(5/2)
```

maple [A] time = 0.05, size = 164, normalized size = 1.53

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(8\left(c\,x^{2}+b\right)^{\frac{3}{2}}B\,c^{\frac{3}{2}}x^{3}-6A\,b^{2}c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)+3B\,b^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)-6\sqrt{c\,x^{2}+}\right.}{48\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*B*c^(3/2)*(c*x^2+b)^(3/2)*x^3+12*A*c^(3/2)*(c*x
^2+b)^(3/2)*x-6*B*c^(1/2)*(c*x^2+b)^(3/2)*x*b-6*A*c^(3/2)*(c*x^2+b)^(1/2)*x
*b+3*B*c^(1/2)*(c*x^2+b)^(1/2)*x*b^2-6*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*
c+3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

maxima [A] time = 1.42, size = 177, normalized size = 1.65

$$\frac{1}{16}\left(4\sqrt{cx^{4}+bx^{2}}\,x^{2}-\frac{b^{2}\log\left(2\,cx^{2}+b+2\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{cx^{4}+bx^{2}}\,b}{c}\right)A-\frac{1}{96}\left(\frac{12\sqrt{cx^{4}+bx^{2}}\,bx^{2}}{c}-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^
2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*A - 1/96*(12*sqrt(c*x^4 +
b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(
5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 + b*x^2)^(3/2)/c)*B
```

mupad [B] time = 0.75, size = 140, normalized size = 1.31

$$\frac{A\left(\frac{b}{4\,c}+\frac{x^{2}}{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{2}+\frac{B\,b^{3}\ln\left(b+2\,c\,x^{2}+2\sqrt{c}\,|x|\sqrt{c\,x^{2}+b}\right)}{32\,c^{5/2}}-\frac{A\,b^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}}\right)}{16\,c^{3/2}}+\frac{B\sqrt{c\,x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 + (B*b^3*log(b + 2*c*x^2 + 2*
c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) - (A*b^2*log((b/2 + c*x^2)/
c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2)) + (B*(b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^{2}\left(b+cx^{2}\right)}\left(A+Bx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.93 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=100

$$-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}-\frac{\sqrt{bx^2+cx^4}\,(bB-4Ac)}{8c}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}$$

```
[Out] 1/4*B*(c*x^4+b*x^2)^(3/2)/c/x^2-1/8*b*(-4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x
^4+b*x^2)^(1/2))/c^(3/2)-1/8*(-4*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.20, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2034, 794, 664, 620, 206}

$$-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}-\frac{\sqrt{bx^2+cx^4}\,(bB-4Ac)}{8c}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{4cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] -((b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(8*c) + (B*(b*x^2 + c*x^4)^(3/2))/(4*c
*x^2) - (b*(b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(
3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
```

```
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)$$
$$= \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} + \frac{\left(bB-Ac+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)}{4c}$$
$$= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c} + \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{(b(bB-4Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}$$
$$= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c} + \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{(b(bB-4Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,d$$
$$= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c} + \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Mathematica [A] time = 0.16, size = 91, normalized size = 0.91

$$\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{c}\left(4Ac+bB+2Bcx^2\right)-\frac{\sqrt{b}(bB-4Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{x\sqrt{\frac{cx^2}{b}+1}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*(b*B + 4*A*c + 2*B*c*x^2) - (Sqrt[b]*(b*B -
 4*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(x*Sqrt[1 + (c*x^2)/b])))/(8*c^(3/2))
```

fricas [A] time = 1.06, size = 172, normalized size = 1.72

$$\left[-\frac{(Bb^2-4\,Abc)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(2\,Bc^2x^2+Bbc+4\,Ac^2\right)\sqrt{cx^4+bx^2}}{16\,c^2},\ \frac{(Bb^2-4\,Ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*(2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/8*
((B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)
) + (2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

giac [A] time = 0.18, size = 103, normalized size = 1.03

$$\frac{1}{8}\left(2\,Bx^2\text{sgn}(x)+\frac{Bbc\text{sgn}(x)+4\,Ac^2\text{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}\,x+\frac{\left(Bb^2\text{sgn}(x)-4\,Abc\text{sgn}(x)\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right.\right.}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2*sgn(x) + (B*b*c*sgn(x) + 4*A*c^2*sgn(x))/c^2)*sqrt(c*x^2 + b)*
x + 1/8*(B*b^2*sgn(x) - 4*A*b*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b
)))/c^(3/2) - 1/16*(B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(3/2)
```

maple [A] time = 0.05, size = 124, normalized size = 1.24

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(4Abc\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)-B\,b^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)+4\sqrt{c\,x^2+b}\;A\,c^{\frac{3}{2}}x-\sqrt{c\,x^2+b}\;Bb\sqrt{c}\;x}{8\sqrt{c\,x^2+b}\;c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(2*B*c^(1/2)*(c*x^2+b)^(3/2)*x+4*A*c^(3/2)*(c*x^2+b
)^(1/2)*x-B*c^(1/2)*(c*x^2+b)^(1/2)*x*b+4*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b
*c-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2)/c^(3/2)/(c*x^2+b)^(1/2)/x
```

maxima [A] time = 1.45, size = 128, normalized size = 1.28

$$\frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{cx^4+bx^2}\right)A+\frac{1}{16}\left(4\,\sqrt{cx^4+bx^2}\,x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+b}\right.}{c^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 2*sqrt(c*
x^4 + b*x^2))*A + 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b + 2
*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*B
```

mupad [B] time = 0.61, size = 117, normalized size = 1.17

$$\frac{A\,\sqrt{c\,x^4+b\,x^2}}{2}+\frac{B\,\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}}{2}+\frac{A\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}}-\frac{B\,b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x,x)
```

```
[Out] (A*(b*x^2 + c*x^4)^(1/2))/2 + (B*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2
 + (A*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(4*c^(1/2)) - (
B*b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x, x)
```

3.94 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$\frac{\sqrt{bx^2+cx^4}\,(2Ac+bB)}{2b}+\frac{(2Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4}$$

```
[Out] -A*(c*x^4+b*x^2)^(3/2)/b/x^4+1/2*(2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x
^2)^(1/2))/c^(1/2)+1/2*(2*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b
```

Rubi [A] time = 0.21, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 792, 664, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}\,(2Ac+bB)}{2b}+\frac{(2Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] ((b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - (A*(b*x^2 + c*x^4)^(3/2))/(b*x^
4) + ((b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&& !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{\left(-2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right.}{b}$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{1}{4}(bB+2Ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\right.$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{1}{2}(bB+2Ac)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,\right.$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A\left(bx^2+cx^4\right)^{3/2}}{bx^4} + \frac{(bB+2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.15, size = 78, normalized size = 0.80

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\frac{x(2Ac+bB)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}\,\sqrt{\frac{cx^2}{b}+1}}-2A+Bx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*A + B*x^2 + ((b*B + 2*A*c)*x*ArcSinh[(Sqrt[c]*x)
/Sqrt[b]])/(Sqrt[b]*Sqrt[c]*Sqrt[1 + (c*x^2)/b])))/(2*x^2)
```

fricas [A] time = 0.98, size = 161, normalized size = 1.66

$$\left[\frac{(Bb+2\,Ac)\sqrt{c}\,x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(Bcx^2-2\,Ac\right)}{4\,cx^2}, -\frac{(Bb+2\,Ac)\sqrt{-c}\,x^2\,\text{arctan}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((B*b + 2*A*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2), -1/2*((B*b + 2*A
*c)*sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*
x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2)]
```

giac [A] time = 0.27, size = 92, normalized size = 0.95

$$\frac{1}{2}\sqrt{cx^2+b}\,Bx\mathrm{sgn}(x)+\frac{2\,Ab\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b}-\frac{\left(Bb\sqrt{c}\,\mathrm{sgn}(x)+2\,Ac^{\frac{3}{2}}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*x*sgn(x) + 2*A*b*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + b))^2 - b) - 1/4*(B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)*sgn(x))*log((sqrt(
c)*x - sqrt(c*x^2 + b))^2)/c
```

maple [A] time = 0.06, size = 130, normalized size = 1.34

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(2Abcx\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+B\,b^2x\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+2\sqrt{c\,x^2+b}\,A\,c^{\frac{3}{2}}x^2+\sqrt{c\,x^2+b}\,\right.}{2\sqrt{c\,x^2+b}\,b\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(1/2)*(2*A*c^(3/2)*(c*x^2+b)^(1/2)*x^2+B*c^(1/2)*(c*x^2+b
)^(1/2)*x^2*b-2*A*c^(1/2)*(c*x^2+b)^(3/2)+2*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))
*x*b*c+B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x*b^2)/x^2/(c*x^2+b)^(1/2)/b/c^(1/2)
```

maxima [A] time = 1.48, size = 105, normalized size = 1.08

$$\frac{1}{2}\left(\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)A+\frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x^
4 + b*x^2)/x^2)*A + 1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
/sqrt(c) + 2*sqrt(c*x^4 + b*x^2))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**3, x)
```

3.95 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=80

$$-\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}-\frac{B\sqrt{bx^2+cx^4}}{x^2}+B\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/3*A*(c*x^4+b*x^2)^(3/2)/b/x^6+B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))
*c^(1/2)-B*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.20, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 792, 662, 620, 206}

$$-\frac{A\left(bx^2+cx^4\right)^{3/2}}{3bx^6}-\frac{B\sqrt{bx^2+cx^4}}{x^2}+B\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] -((B*Sqrt[b*x^2 + c*x^4])/x^2) - (A*(b*x^2 + c*x^4)^(3/2))/(3*b*x^6) + B*Sq
rt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{B\sqrt{bx^2+cx^4}}{x^2} - \frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + \frac{1}{2}(Bc)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{B\sqrt{bx^2+cx^4}}{x^2} - \frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + (Bc)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{B\sqrt{bx^2+cx^4}}{x^2} - \frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + B\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.13, size = 86, normalized size = 1.08

$$\frac{\sqrt{x^2(b+cx^2)}\left(-A(b+cx^2)+\frac{3\sqrt{b}B\sqrt{c}x^3\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{\frac{cx^2}{b}+1}}-3bBx^2\right)}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-3*b*B*x^2 - A*(b + c*x^2) + (3*Sqrt[b]*B*Sqrt[c]*x
^3*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/Sqrt[1 + (c*x^2)/b]))/(3*b*x^4)
```

fricas [A] time = 0.98, size = 160, normalized size = 2.00

$$\left[\frac{3\,Bb\sqrt{c}\,x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\left((3\,Bb+Ac)x^2+Ab\right)}{6\,bx^4}, -\frac{3\,Bb\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*b*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) -
 2*sqrt(c*x^4 + b*x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4), -1/3*(3*B*b*sqrt
(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*
x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4)]
```

giac [B] time = 0.50, size = 163, normalized size = 2.04

$$-\frac{1}{2}B\sqrt{c}\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\text{sgn}(x)+\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}\,\text{sgn}(x)+3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}\right.}{3\left(\left(\sqrt{c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b*sqrt(c)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*A*c^(3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c)*sgn
(x) + 3*B*b^3*sqrt(c)*sgn(x) + A*b^2*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + b))^2 - b)^3
```

maple [A] time = 0.06, size = 109, normalized size = 1.36

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3Bbc\,x^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-3\sqrt{c\,x^2+b}\,B\,c^{\frac{3}{2}}x^4+3\left(c\,x^2+b\right)^{\frac{3}{2}}B\sqrt{c}\,x^2+\left(c\,x^2+b\right)^{\frac{3}{2}}A\sqrt{c}\right)}{3\sqrt{c\,x^2+b}\,b\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)*(-3*B*c^(3/2)*(c*x^2+b)^(1/2)*x^4+3*B*c^(1/2)*(c*x
^2+b)^(3/2)*x^2-3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x^3*b*c+(c*x^2+b)^(3/2)*A
*c^(1/2))/x^4/(c*x^2+b)^(1/2)/b/c^(1/2)
```

maxima [A] time = 1.42, size = 96, normalized size = 1.20

$$\frac{1}{2}\left(\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)B-\frac{1}{3}\,A\left(\frac{\sqrt{cx^4+bx^2}\,c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x^
4 + b*x^2)/x^2)*B - 1/3*A*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b*x
^2)/x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**5, x)
```

3.96 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=61

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(5bB-2Ac)}{15b^2x^6}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

```
[Out] -1/5*A*(c*x^4+b*x^2)^(3/2)/b/x^8-1/15*(-2*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/b^
2/x^6
```

Rubi [A] time = 0.16, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 650}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(5bB-2Ac)}{15b^2x^6}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^8) - ((5*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/
2))/(15*b^2*x^6)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8} + \frac{\left(-4(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx, x, x^2\right)}{5b}$$
$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{5bx^8} - \frac{(5bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.72

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(3Ab-2Acx^2+5bBx^2\right)}{15b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

```
[Out] -1/15*((x^2*(b + c*x^2))^(3/2)*(3*A*b + 5*b*B*x^2 - 2*A*c*x^2))/(b^2*x^8)
```

fricas [A] time = 0.98, size = 59, normalized size = 0.97

$$-\frac{\left(\left(5\,Bbc-2\,Ac^2\right)x^4+3\,Ab^2+\left(5\,Bb^2+Abc\right)x^2\right)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*((5*B*b*c - 2*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + A*b*c)*x^2)*sqrt(c*x^
4 + b*x^2)/(b^2*x^6)
```

giac [B] time = 0.92, size = 250, normalized size = 4.10

$$\frac{2\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^8Bc^{\frac{3}{2}}\text{sgn}(x)-30\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6Bbc^{\frac{3}{2}}\text{sgn}(x)+30\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6Ac^{\frac{5}{2}}\text{sgn}(x)+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(3/2)*sgn(x) - 30*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*B*b*c^(3/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*A*c^(5/2)*sgn(x) + 20*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(3/2)*sgn(x)
 + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(5/2)*sgn(x) - 10*(sqrt(c)*x -
sqrt(c*x^2 + b))^2*B*b^3*c^(3/2)*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^
2*A*b^2*c^(5/2)*sgn(x) + 5*B*b^4*c^(3/2)*sgn(x) - 2*A*b^3*c^(5/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

maple [A] time = 0.05, size = 48, normalized size = 0.79

$$-\frac{\left(c\,x^2+b\right)\left(-2Ac\,x^2+5Bb\,x^2+3Ab\right)\sqrt{c\,x^4+b\,x^2}}{15b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x)
```

```
[Out] -1/15*(c*x^2+b)*(-2*A*c*x^2+5*B*b*x^2+3*A*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

maxima [B] time = 1.50, size = 111, normalized size = 1.82

$$-\frac{1}{3}B\left(\frac{\sqrt{cx^4+bx^2}\,c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right)+\frac{1}{15}A\left(\frac{2\sqrt{cx^4+bx^2}\,c^2}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}\,c}{bx^4}-\frac{3\sqrt{cx^4+bx^2}}{x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*B*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b*x^2)/x^4) + 1/15*A*(
2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4) - 3*sqr
t(c*x^4 + b*x^2)/x^6)
```

mupad [B] time = 0.47, size = 113, normalized size = 1.85

$$\frac{\left(A\,c^2+B\,b\,c\right)\,\sqrt{c\,x^4+b\,x^2}}{5\,b^2\,x^2}-\frac{\left(5\,B\,b^2+A\,c\,b\right)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^4}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6}-\frac{c\,\left(A\,c+8\,B\,b\right)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^7,x)
```

```
[Out] ((A*c^2 + B*b*c)*(b*x^2 + c*x^4)^(1/2))/(5*b^2*x^2) - ((5*B*b^2 + A*b*c)*(b
*x^2 + c*x^4)^(1/2))/(15*b^2*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (c*
(A*c + 8*B*b)*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**7, x)
```

3.97 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=96

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{105b^3x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{35b^2x^8}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(3/2)/b/x^10-1/35*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b
^2/x^8+2/105*c*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^6
```

Rubi [A] time = 0.21, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2034, 792, 658, 650}

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{105b^3x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}(7bB-4Ac)}{35b^2x^8}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(7*b*x^10) - ((7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(35*b^2*x^8) + (2*c*(7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^
6)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
```

```
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^9}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx,x,x^2\right)$$

$$=-\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}+\frac{\left(-5(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx,x,\right.}{7b}$$

$$=-\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}-\frac{(7bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{(c(7bB-4Ac))\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{35b^2}$$

$$=-\frac{A\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}-\frac{(7bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}+\frac{2c(7bB-4Ac)\left(bx^2+cx^4\right)}{105b^3x^6}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.69

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(A\left(-15b^2+12bcx^2-8c^2x^4\right)+7bBx^2\left(2cx^2-3b\right)\right)}{105b^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(7*b*B*x^2*(-3*b + 2*c*x^2) + A*(-15*b^2 + 12*b*c*
x^2 - 8*c^2*x^4)))/(105*b^3*x^10)
```

fricas [A] time = 1.02, size = 85, normalized size = 0.89

$$\frac{\left(2\left(7\,Bbc^2-4\,Ac^3\right)x^6-\left(7\,Bb^2c-4\,Abc^2\right)x^4-15\,Ab^3-3\left(7\,Bb^3+Ab^2c\right)x^2\right)\sqrt{cx^4+bx^2}}{105\,b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(7*B*b*c^2 - 4*A*c^3)*x^6 - (7*B*b^2*c - 4*A*b*c^2)*x^4 - 15*A*b^3
 - 3*(7*B*b^3 + A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^8)
```

giac [B] time = 1.36, size = 310, normalized size = 3.23

$$4\left(105\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{5}{2}}\text{sgn}(x)-175\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{5}{2}}\text{sgn}(x)+280\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}Ac^{\frac{7}{2}}\text{sg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(5/2)*sgn(x) - 175*(sqrt(c)
*x - sqrt(c*x^2 + b))^8*B*b*c^(5/2)*sgn(x) + 280*(sqrt(c)*x - sqrt(c*x^2 +
b))^8*A*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(5/2)*s
gn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b*c^(7/2)*sgn(x) - 42*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(5/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2
```

```
+ b))^4*A*b^2*c^(7/2)*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^4*c^(
5/2)*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^3*c^(7/2)*sgn(x) - 7*B
*b^5*c^(5/2)*sgn(x) + 4*A*b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b)
)^2 - b)^7
```

maple [A] time = 0.04, size = 70, normalized size = 0.73

$$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-14Bbc\,x^{4}-12Abc\,x^{2}+21B\,b^{2}x^{2}+15b^{2}A\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105b^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x)
```

```
[Out] -1/105*(c*x^2+b)*(8*A*c^2*x^4-14*B*b*c*x^4-12*A*b*c*x^2+21*B*b^2*x^2+15*A*b
^2)*(c*x^4+b*x^2)^(1/2)/b^3/x^8
```

maxima [A] time = 1.48, size = 161, normalized size = 1.68

$$\frac{1}{15}B\left(\frac{2\sqrt{cx^{4}+bx^{2}}\,c^{2}}{b^{2}x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c}{bx^{4}}-\frac{3\sqrt{cx^{4}+bx^{2}}}{x^{6}}\right)-\frac{1}{105}A\left(\frac{8\sqrt{cx^{4}+bx^{2}}\,c^{3}}{b^{3}x^{2}}-\frac{4\sqrt{cx^{4}+bx^{2}}\,c^{2}}{b^{2}x^{4}}+\frac{3\sqrt{cx^{4}+b}}{bx^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/15*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4)
 - 3*sqrt(c*x^4 + b*x^2)/x^6) - 1/105*A*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2
) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6) +
 15*sqrt(c*x^4 + b*x^2)/x^8)
```

mupad [B] time = 0.68, size = 160, normalized size = 1.67

$$\frac{4\,A\,c^{2}\,\sqrt{c\,x^{4}+b\,x^{2}}}{105\,b^{2}\,x^{4}}-\frac{B\,\sqrt{c\,x^{4}+b\,x^{2}}}{5\,x^{6}}-\frac{A\,c\,\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b\,x^{6}}-\frac{B\,c\,\sqrt{c\,x^{4}+b\,x^{2}}}{15\,b\,x^{4}}-\frac{A\,\sqrt{c\,x^{4}+b\,x^{2}}}{7\,x^{8}}-\frac{8\,A\,c^{3}\,\sqrt{c\,x^{4}+b\,x^{2}}}{105\,b^{3}\,x^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^9,x)
```

```
[Out] (4*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (B*(b*x^2 + c*x^4)^(1/2))/(
5*x^6) - (A*c*(b*x^2 + c*x^4)^(1/2))/(35*b*x^6) - (B*c*(b*x^2 + c*x^4)^(1/2
))/(15*b*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*A*c^3*(b*x^2 + c*x^4
)^(1/2))/(105*b^3*x^2) + (2*B*c^2*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\left(b+cx^{2}\right)}\left(A+Bx^{2}\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**9, x)
```

3.98 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=133

$$-\frac{8c^2(bx^2+cx^4)^{3/2}(3bB-2Ac)}{315b^4x^6}+\frac{4c(bx^2+cx^4)^{3/2}(3bB-2Ac)}{105b^3x^8}-\frac{(bx^2+cx^4)^{3/2}(3bB-2Ac)}{21b^2x^{10}}-\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(3/2)/b/x^12-1/21*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b
^2/x^10+4/105*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^8-8/315*c^2*(-2*A*
c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^6
```

Rubi [A] time = 0.24, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{8c^2(bx^2+cx^4)^{3/2}(3bB-2Ac)}{315b^4x^6}+\frac{4c(bx^2+cx^4)^{3/2}(3bB-2Ac)}{105b^3x^8}-\frac{(bx^2+cx^4)^{3/2}(3bB-2Ac)}{21b^2x^{10}}-\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(9*b*x^12) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3
/2))/(21*b^2*x^10) + (4*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x
^8) - (8*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*b^4*x^6)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
```

```
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx, x, x^2\right)$$
$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^5}\,dx, x, x^2\right)}{9b}$$
$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{(2c(3bB-2Ac))\operatorname{Subst}\left(\int \frac{\sqrt{bx}}{x}\right.}{21b^2}$$
$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}$$
$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.66

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(A\left(35b^3-30b^2cx^2+24bc^2x^4-16c^3x^6\right)+3bBx^2\left(15b^2-12bcx^2+8c^2x^4\right)\right)}{315b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

```
[Out] -1/315*((x^2*(b + c*x^2))^(3/2)*(3*b*B*x^2*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^4
) + A*(35*b^3 - 30*b^2*c*x^2 + 24*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^12)
```

fricas [A] time = 1.00, size = 109, normalized size = 0.82

$$-\frac{\left(8\left(3\,Bbc^3-2\,Ac^4\right)x^8-4\left(3\,Bb^2c^2-2\,Abc^3\right)x^6+35\,Ab^4+3\left(3\,Bb^3c-2\,Ab^2c^2\right)x^4+5\left(9\,Bb^4+Ab^3c\right)x^2\right)\sqrt{cx^4}}{315\,b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*b*c^3 - 2*A*c^4)*x^8 - 4*(3*B*b^2*c^2 - 2*A*b*c^3)*x^6 + 35*
A*b^4 + 3*(3*B*b^3*c - 2*A*b^2*c^2)*x^4 + 5*(9*B*b^4 + A*b^3*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^4*x^10)
```

giac [B] time = 1.97, size = 370, normalized size = 2.78

$$16\left(210\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{7}{2}}\operatorname{sgn}(x)-315\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{7}{2}}\operatorname{sgn}(x)+630\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}Ac^{\frac{9}{2}}\operatorname{sg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(7/2)*sgn(x) - 315*(sqrt(c
)*x - sqrt(c*x^2 + b))^10*B*b*c^(7/2)*sgn(x) + 630*(sqrt(c)*x - sqrt(c*x^2
+ b))^10*A*c^(9/2)*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(7/2
)*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(9/2)*sgn(x) - 42*(sqr
t(c)*x - sqrt(c*x^2 + b))^6*B*b^3*c^(7/2)*sgn(x) + 168*(sqrt(c)*x - sqrt(c*
x^2 + b))^6*A*b^2*c^(9/2)*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^
4*c^(7/2)*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(9/2)*sgn(x)
- 27*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(7/2)*sgn(x) + 18*(sqrt(c)*x -
 sqrt(c*x^2 + b))^2*A*b^4*c^(9/2)*sgn(x) + 3*B*b^6*c^(7/2)*sgn(x) - 2*A*b^5
*c^(9/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

maple [A] time = 0.05, size = 94, normalized size = 0.71

$$-\frac{\left(c\,x^{2}+b\right)\left(-16A\,c^{3}x^{6}+24Bb\,c^{2}x^{6}+24Ab\,c^{2}x^{4}-36B\,b^{2}c\,x^{4}-30A\,b^{2}c\,x^{2}+45B\,b^{3}x^{2}+35A\,b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{315b^{4}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x)
```

```
[Out] -1/315*(c*x^2+b)*(-16*A*c^3*x^6+24*B*b*c^2*x^6+24*A*b*c^2*x^4-36*B*b^2*c*x^
4-30*A*b^2*c*x^2+45*B*b^3*x^2+35*A*b^3)*(c*x^4+b*x^2)^(1/2)/x^10/b^4
```

maxima [A] time = 1.49, size = 209, normalized size = 1.57

$$-\frac{1}{105}B\left(\frac{8\sqrt{cx^{4}+bx^{2}}\,c^{3}}{b^{3}x^{2}}-\frac{4\sqrt{cx^{4}+bx^{2}}\,c^{2}}{b^{2}x^{4}}+\frac{3\sqrt{cx^{4}+bx^{2}}\,c}{bx^{6}}+\frac{15\sqrt{cx^{4}+bx^{2}}}{x^{8}}\right)+\frac{1}{315}A\left(\frac{16\sqrt{cx^{4}+bx^{2}}\,c^{4}}{b^{4}x^{2}}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/105*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(
b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6) + 15*sqrt(c*x^4 + b*x^2)/x^8) +
1/315*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/(
b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/(b
*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10)
```

mupad [B] time = 1.04, size = 210, normalized size = 1.58

$$\frac{2\,A\,c^{2}\,\sqrt{c\,x^{4}+b\,x^{2}}}{105\,b^{2}\,x^{6}}-\frac{B\,\sqrt{c\,x^{4}+b\,x^{2}}}{7\,x^{8}}-\frac{A\,c\,\sqrt{c\,x^{4}+b\,x^{2}}}{63\,b\,x^{8}}-\frac{B\,c\,\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b\,x^{6}}-\frac{A\,\sqrt{c\,x^{4}+b\,x^{2}}}{9\,x^{10}}-\frac{8\,A\,c^{3}\,\sqrt{c\,x^{4}+b\,x^{2}}}{315\,b^{3}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^11,x)
```

```
[Out] (2*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (B*(b*x^2 + c*x^4)^(1/2))/(
7*x^8) - (A*c*(b*x^2 + c*x^4)^(1/2))/(63*b*x^8) - (B*c*(b*x^2 + c*x^4)^(1/2
))/(35*b*x^6) - (A*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (8*A*c^3*(b*x^2 + c*x^
4)^(1/2))/(315*b^3*x^4) + (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2) +
(4*B*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (8*B*c^3*(b*x^2 + c*x^4)^(1
/2))/(105*b^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\left(b+cx^{2}\right)}\left(A+Bx^{2}\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**11, x)
```

3.99 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal. Leaf size=170

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{3465b^5x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{1155b^4x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{231b^3x^{10}}-\frac{\left(bx^2+cx^4\right)^{3}}{99}$$

```
[Out] -1/11*A*(c*x^4+b*x^2)^(3/2)/b/x^14-1/99*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)
/b^2/x^12+2/231*c*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^10-8/1155*c^2*(
-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^8+16/3465*c^3*(-8*A*c+11*B*b)*(c*x
^4+b*x^2)^(3/2)/b^5/x^6
```

Rubi [A] time = 0.30, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2034, 792, 658, 650}

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{3465b^5x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{1155b^4x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}(11bB-8Ac)}{231b^3x^{10}}-\frac{\left(bx^2+cx^4\right)^{3}}{99}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(3/2))/(11*b*x^14) - ((11*b*B - 8*A*c)*(b*x^2 + c*x^4)^
(3/2))/(99*b^2*x^12) + (2*c*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(231*b^
3*x^10) - (8*c^2*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(1155*b^4*x^8) + (
16*c^3*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{\left(-7(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^6}\,dx, x\right.}{11b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} - \frac{(c(11bB-8Ac))\,\text{Subst}\left(\int\right.}{33b^2}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)\left(bx^2+cx\right.}{231b^3x^{10}}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)\left(bx^2+cx\right.}{231b^3x^{10}}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)\left(bx^2+cx\right.}{231b^3x^{10}}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.55

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(x^2\left(\frac{cx^2}{b}+1\right)\left(35b^3-30b^2cx^2+24bc^2x^4-16c^3x^6\right)(8Ac-11bB)-315Ab^3\left(b+cx^2\right)\right)}{3465b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-315*A*b^3*(b + c*x^2) + (-11*b*B + 8*A*c)*x^2*(1 +
 (c*x^2)/b)*(35*b^3 - 30*b^2*c*x^2 + 24*b*c^2*x^4 - 16*c^3*x^6)))/(3465*b^4
*x^12)
```

fricas [A] time = 1.34, size = 133, normalized size = 0.78

$$\frac{\left(16\left(11\,Bbc^4-8\,Ac^5\right)x^{10}-8\left(11\,Bb^2c^3-8\,Abc^4\right)x^8+6\left(11\,Bb^3c^2-8\,Ab^2c^3\right)x^6-315\,Ab^5-5\left(11\,Bb^4c-8\,Ab^3\right.\right.}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/3465*(16*(11*B*b*c^4 - 8*A*c^5)*x^10 - 8*(11*B*b^2*c^3 - 8*A*b*c^4)*x^8 +
 6*(11*B*b^3*c^2 - 8*A*b^2*c^3)*x^6 - 315*A*b^5 - 5*(11*B*b^4*c - 8*A*b^3*c
^2)*x^4 - 35*(11*B*b^5 + A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

giac [B] time = 2.97, size = 430, normalized size = 2.53

$$32\left(3465\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{9}{2}}\text{sgn}(x)-4851\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{9}{2}}\text{sgn}(x)+11088\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(9/2)*sgn(x) - 4851*(sqr
t(c)*x - sqrt(c*x^2 + b))^12*B*b*c^(9/2)*sgn(x) + 11088*(sqrt(c)*x - sqrt(c
*x^2 + b))^12*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^
2*c^(9/2)*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(11/2)*sgn(x
) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(9/2)*sgn(x) + 2640*(sqrt(c
)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(11/2)*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x
^2 + b))^6*B*b^4*c^(9/2)*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^
3*c^(11/2)*sgn(x) - 605*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(9/2)*sgn(x
) + 440*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(11/2)*sgn(x) + 121*(sqrt(c
)*x - sqrt(c*x^2 + b))^2*B*b^6*c^(9/2)*sgn(x) - 88*(sqrt(c)*x - sqrt(c*x^2
+ b))^2*A*b^5*c^(11/2)*sgn(x) - 11*B*b^7*c^(9/2)*sgn(x) + 8*A*b^6*c^(11/2)*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

maple [A] time = 0.05, size = 118, normalized size = 0.69

$$-\frac{\left(c x^2+b\right)\left(128 A c^4 x^8-176 B b c^3 x^8-192 A b c^3 x^6+264 B b^2 c^2 x^6+240 A b^2 c^2 x^4-330 B b^3 c x^4-280 A b^3 c x^2+3\right.}{3465 b^5 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x)
```

```
[Out] -1/3465*(c*x^2+b)*(128*A*c^4*x^8-176*B*b*c^3*x^8-192*A*b*c^3*x^6+264*B*b^2*
c^2*x^6+240*A*b^2*c^2*x^4-330*B*b^3*c*x^4-280*A*b^3*c*x^2+385*B*b^4*x^2+315
*A*b^4)*(c*x^4+b*x^2)^(1/2)/b^5/x^12
```

maxima [A] time = 1.57, size = 257, normalized size = 1.51

$$\frac{1}{315} B\left(\frac{16 \sqrt{cx^4+bx^2}\, c^4}{b^4x^2}-\frac{8 \sqrt{cx^4+bx^2}\, c^3}{b^3x^4}+\frac{6 \sqrt{cx^4+bx^2}\, c^2}{b^2x^6}-\frac{5 \sqrt{cx^4+bx^2}\, c}{bx^8}-\frac{35 \sqrt{cx^4+bx^2}}{x^{10}}\right)-\frac{1}{3465} A\left(128\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/315*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/(
b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/(b
*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10) - 1/3465*A*(128*sqrt(c*x^4 + b*x^2)*c^
5/(b^5*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^4) + 48*sqrt(c*x^4 + b*x^2)
*c^3/(b^3*x^6) - 40*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^8) + 35*sqrt(c*x^4 + b*x
^2)*c/(b*x^10) + 315*sqrt(c*x^4 + b*x^2)/x^12)
```

mupad [B] time = 1.41, size = 260, normalized size = 1.53

$$\frac{8\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{693\,b^2\,x^8}-\frac{B\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{99\,b\,x^{10}}-\frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{16\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^13,x)
```

```
[Out] (8*A*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (B*(b*x^2 + c*x^4)^(1/2))/(
9*x^10) - (A*c*(b*x^2 + c*x^4)^(1/2))/(99*b*x^10) - (B*c*(b*x^2 + c*x^4)^(1
/2))/(63*b*x^8) - (A*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (16*A*c^3*(b*x^2 +
c*x^4)^(1/2))/(1155*b^3*x^6) + (64*A*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b^4*x
^4) - (128*A*c^5*(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2) + (2*B*c^2*(b*x^2 +
c*x^4)^(1/2))/(105*b^2*x^6) - (8*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^4)
 + (16*B*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)} \left(A + Bx^2\right)}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**13, x)
```

3.100 $\int x^4 \left(A + Bx^2\right) \sqrt{bx^2 + cx^4}\, dx$

Optimal. Leaf size=131

$$-\frac{8b^2\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{315c^4x^3}+\frac{4b\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{105c^3x}-\frac{x\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{21c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}$$

```
[Out] -8/315*b^2*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^4/x^3+4/105*b*(-3*A*c+2*B*b
)*(c*x^4+b*x^2)^(3/2)/c^3/x-1/21*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2)^(3/2)/c^2+1
/9*B*x^3*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A] time = 0.22, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2039, 2016, 2000}

$$-\frac{8b^2\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{315c^4x^3}-\frac{x\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{21c^2}+\frac{4b\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{105c^3x}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-8*b^2*(2*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*c^4*x^3) + (4*b*(2*b*B
- 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x) - ((2*b*B - 3*A*c)*x*(b*x^2 + c
*x^4)^(3/2))/(21*c^2) + (B*x^3*(b*x^2 + c*x^4)^(3/2))/(9*c)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^4\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c} - \frac{(6bB-9Ac)\int x^4\sqrt{bx^2+cx^4}\,dx}{9c}$$
$$= -\frac{(2bB-3Ac)x\left(bx^2+cx^4\right)^{3/2}}{21c^2} + \frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c} + \frac{(4b(2bB-3Ac))\int x^2}{21c^2}$$
$$= \frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x} - \frac{(2bB-3Ac)x\left(bx^2+cx^4\right)^{3/2}}{21c^2} + \frac{Bx^3\left(bx^2+c\right.}{9c}$$
$$= -\frac{8b^2(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{315c^4x^3} + \frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x} - \frac{(2bB-3}{}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.63

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(24b^2c\left(A+Bx^2\right)-6bc^2x^2\left(6A+5Bx^2\right)+5c^3x^4\left(9A+7Bx^2\right)-16b^3B\right)}{315c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-16*b^3*B + 24*b^2*c*(A + B*x^2) - 6*b*c^2*x^2*(6
*A + 5*B*x^2) + 5*c^3*x^4*(9*A + 7*B*x^2)))/(315*c^4*x^3)
```

fricas [A] time = 1.39, size = 106, normalized size = 0.81

$$\frac{\left(35\,Bc^4x^8+5\left(Bbc^3+9\,Ac^4\right)x^6-16\,Bb^4+24\,Ab^3c-3\left(2\,Bb^2c^2-3\,Abc^3\right)x^4+4\left(2\,Bb^3c-3\,Ab^2c^2\right)x^2\right)\sqrt{cx^4+}}{315\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*c^4*x^8 + 5*(B*b*c^3 + 9*A*c^4)*x^6 - 16*B*b^4 + 24*A*b^3*c - 3
*(2*B*b^2*c^2 - 3*A*b*c^3)*x^4 + 4*(2*B*b^3*c - 3*A*b^2*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^4*x)
```

giac [A] time = 0.17, size = 140, normalized size = 1.07

$$\frac{8\left(2\,Bb^{\frac{9}{2}}-3\,Ab^{\frac{7}{2}}c\right)\mathrm{sgn}(x)}{315\,c^4}+\frac{35\left(cx^2+b\right)^{\frac{9}{2}}B\mathrm{sgn}(x)-135\left(cx^2+b\right)^{\frac{7}{2}}Bb\mathrm{sgn}(x)+189\left(cx^2+b\right)^{\frac{5}{2}}Bb^2\mathrm{sgn}(x)-10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*(2*B*b^(9/2) - 3*A*b^(7/2)*c)*sgn(x)/c^4 + 1/315*(35*(c*x^2 + b)^(9/2
)*B*sgn(x) - 135*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 189*(c*x^2 + b)^(5/2)*B*b^2
*sgn(x) - 105*(c*x^2 + b)^(3/2)*B*b^3*sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*sgn
(x) - 126*(c*x^2 + b)^(5/2)*A*b*c*sgn(x) + 105*(c*x^2 + b)^(3/2)*A*b^2*c*sg
n(x))/c^4
```

maple [A] time = 0.05, size = 91, normalized size = 0.69

$$\frac{\left(c\,x^2+b\right)\left(35B\,c^3x^6+45A\,c^3x^4-30Bb\,c^2x^4-36Ab\,c^2x^2+24B\,b^2c\,x^2+24A\,b^2c-16B\,b^3\right)\sqrt{c\,x^4+b\,x^2}}{315c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/315*(c*x^2+b)*(35*B*c^3*x^6+45*A*c^3*x^4-30*B*b*c^2*x^4-36*A*b*c^2*x^2+24
*B*b^2*c*x^2+24*A*b^2*c-16*B*b^3)*(c*x^4+b*x^2)^(1/2)/c^4/x
```

maxima [A] time = 1.42, size = 106, normalized size = 0.81

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}\,A}{105\,c^3}+\frac{\left(35\,c^4x^8+5\,bc^3x^6-6\,b^2c^2x^4+8\,b^3cx^2-16\,b^4\right)\sqrt{cx^2+b}\,B}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*A/c^
3 + 1/315*(35*c^4*x^8 + 5*b*c^3*x^6 - 6*b^2*c^2*x^4 + 8*b^3*c*x^2 - 16*b^4)
*sqrt(c*x^2 + b)*B/c^4
```

mupad [B] time = 0.25, size = 103, normalized size = 0.79

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^8}{9}-\frac{16\,B\,b^4-24\,A\,b^3\,c}{315\,c^4}+\frac{x^6\,\left(45\,A\,c^4+5\,B\,b\,c^3\right)}{315\,c^4}-\frac{4\,b^2\,x^2\,\left(3\,A\,c-2\,B\,b\right)}{315\,c^3}+\frac{b\,x^4\,\left(3\,A\,c-2\,B\,b\right)}{105\,c^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^8)/9 - (16*B*b^4 - 24*A*b^3*c)/(315*c^4) + (x^
6*(45*A*c^4 + 5*B*b*c^3))/(315*c^4) - (4*b^2*x^2*(3*A*c - 2*B*b))/(315*c^3)
 + (b*x^4*(3*A*c - 2*B*b))/(105*c^2)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.101 $\int x^2 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx$

Optimal. Leaf size=94

$$\frac{2b\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{105c^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{35c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}$$

```
[Out] 2/105*b*(-7*A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/c^3/x^3-1/35*(-7*A*c+4*B*b)*(c*x
^4+b*x^2)^(3/2)/c^2/x+1/7*B*x*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A] time = 0.17, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2039, 2016, 2000}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{35c^2x}+\frac{2b\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{105c^3x^3}+\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - ((4*b*B - 7*A*c
)*(b*x^2 + c*x^4)^(3/2))/(35*c^2*x) + (B*x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^2\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}-\frac{(4bB-7Ac)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c}$$

$$=-\frac{(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}+\frac{(2b(4bB-7Ac))\int\sqrt{bx^2}}{35c^2}$$

$$=\frac{2b(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{Bx\left(bx^2+cx^4\right)}{7c}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.68

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(-2bc\left(7A+6Bx^2\right)+3c^2x^2\left(7A+5Bx^2\right)+8b^2B\right)}{105c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(8*b^2*B + 3*c^2*x^2*(7*A + 5*B*x^2) - 2*b*c*(7*A
+ 6*B*x^2)))/(105*c^3*x^3)
```

fricas [A] time = 1.04, size = 82, normalized size = 0.87

$$\frac{\left(15\,Bc^3x^6+3\left(Bbc^2+7\,Ac^3\right)x^4+8\,Bb^3-14\,Ab^2c-\left(4\,Bb^2c-7\,Abc^2\right)x^2\right)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*c^3*x^6 + 3*(B*b*c^2 + 7*A*c^3)*x^4 + 8*B*b^3 - 14*A*b^2*c - (4
*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^3*x)
```

giac [A] time = 0.21, size = 105, normalized size = 1.12

$$-\frac{2\left(4\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{105\,c^3}+\frac{15\left(cx^2+b\right)^{\frac{7}{2}}B\mathrm{sgn}(x)-42\left(cx^2+b\right)^{\frac{5}{2}}Bb\mathrm{sgn}(x)+35\left(cx^2+b\right)^{\frac{3}{2}}Bb^2\mathrm{sgn}(x)+21\left(cx^2\right.}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(4*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/
2)*B*sgn(x) - 42*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*B*b^2*
sgn(x) + 21*(c*x^2 + b)^(5/2)*A*c*sgn(x) - 35*(c*x^2 + b)^(3/2)*A*b*c*sgn(x
))/c^3
```

maple [A] time = 0.05, size = 67, normalized size = 0.71

$$-\frac{\left(c\,x^2+b\right)\left(-15B\,c^2x^4-21A\,c^2x^2+12Bbc\,x^2+14Abc-8B\,b^2\right)\sqrt{c\,x^4+b\,x^2}}{105c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/105*(c*x^2+b)*(-15*B*c^2*x^4-21*A*c^2*x^2+12*B*b*c*x^2+14*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(1/2)/c^3/x
```

maxima [A] time = 1.54, size = 83, normalized size = 0.88

$$\frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^2+b}\,A}{15\,c^2}+\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}\,B}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)*A/c^2 + 1/105*(15*c^3*x^
6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*B/c^3
```

mupad [B] time = 0.19, size = 83, normalized size = 0.88

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^6}{7}+\frac{8\,B\,b^3-14\,A\,b^2\,c}{105\,c^3}+\frac{x^4\,\left(21\,A\,c^3+3\,B\,b\,c^2\right)}{105\,c^3}+\frac{b\,x^2\,\left(7\,A\,c-4\,B\,b\right)}{105\,c^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^6)/7 + (8*B*b^3 - 14*A*b^2*c)/(105*c^3) + (x^4
*(21*A*c^3 + 3*B*b*c^2))/(105*c^3) + (b*x^2*(7*A*c - 4*B*b))/(105*c^2)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.102 $\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=61

$$\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{\left(bx^2+cx^4\right)^{3/2}(2bB-5Ac)}{15c^2x^3}$$

```
[Out] -1/15*(-5*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3+1/5*B*(c*x^4+b*x^2)^(3/2)/
c/x
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1145, 2000}

$$\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{\left(bx^2+cx^4\right)^{3/2}(2bB-5Ac)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((2*b*B - 5*A*c)*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (B*(b*x^2 + c*x^4)^
(3/2))/(5*c*x)
```

Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx &= \frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{(2bB-5Ac)\int \sqrt{bx^2+cx^4}\,dx}{5c}\\ &= -\frac{(2bB-5Ac)\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(5Ac-2bB+3Bcx^2\right)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-2*b*B + 5*A*c + 3*B*c*x^2))/(15*c^2*x^3)
```

fricas [A] time = 0.85, size = 57, normalized size = 0.93

$$\frac{\left(3\,Bc^2x^4 - 2\,Bb^2 + 5\,Abc + \left(Bbc + 5\,Ac^2\right)x^2\right)\sqrt{cx^4 + bx^2}}{15\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^4 - 2*B*b^2 + 5*A*b*c + (B*b*c + 5*A*c^2)*x^2)*sqrt(c*x^4 +
 b*x^2)/(c^2*x)
```

giac [A] time = 0.16, size = 72, normalized size = 1.18

$$\frac{\left(2\,Bb^{\frac{5}{2}} - 5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\,c^2} + \frac{3\left(cx^2 + b\right)^{\frac{5}{2}}B\mathrm{sgn}(x) - 5\left(cx^2 + b\right)^{\frac{3}{2}}Bb\mathrm{sgn}(x) + 5\left(cx^2 + b\right)^{\frac{3}{2}}Ac\mathrm{sgn}(x)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*B
*sgn(x) - 5*(c*x^2 + b)^(3/2)*B*b*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c*sgn(x))/
c^2
```

maple [A] time = 0.05, size = 45, normalized size = 0.74

$$\frac{\left(c\,x^2 + b\right)\left(3Bc\,x^2 + 5Ac - 2bB\right)\sqrt{c\,x^4 + b\,x^2}}{15c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/15*(c*x^2+b)*(3*B*c*x^2+5*A*c-2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

maxima [A] time = 1.46, size = 51, normalized size = 0.84

$$\frac{\left(cx^2 + b\right)^{\frac{3}{2}}A}{3\,c} + \frac{\left(3\,c^2x^4 + bcx^2 - 2\,b^2\right)\sqrt{cx^2 + b}\,B}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2 + b)^(3/2)*A/c + 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 +
 b)*B/c^2
```

mupad [B] time = 0.15, size = 60, normalized size = 0.98

$$\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(\frac{B\,x^4}{5} - \frac{2\,B\,b^2 - 5\,A\,b\,c}{15\,c^2} + \frac{x^2\,\left(5\,A\,c^2 + B\,b\,c\right)}{15\,c^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^4)/5 - (2*B*b^2 - 5*A*b*c)/(15*c^2) + (x^2*(5*
A*c^2 + B*b*c))/(15*c^2)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 \left(b + cx^2\right)} \left(A + Bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.103 $\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=78

$$\frac{A\sqrt{bx^2+cx^4}}{x}-A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{B\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

```
[Out] 1/3*B*(c*x^4+b*x^2)^(3/2)/c/x^3-A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^
(1/2)+A*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2021, 2008, 206}

$$\frac{A\sqrt{bx^2+cx^4}}{x}-A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{B\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

```
[Out] (A*Sqrt[b*x^2 + c*x^4])/x + (B*(b*x^2 + c*x^4)^(3/2))/(3*c*x^3) - A*Sqrt[b]
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} + A\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$$
$$= \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} + (Ab)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} - (Ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} - A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Mathematica [A] time = 0.09, size = 84, normalized size = 1.08

$$\frac{x\left((b+cx^2)(3Ac+bB+Bcx^2) - 3A\sqrt{b}\,c\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{3c\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

```
[Out] (x*((b + c*x^2)*(b*B + 3*A*c + B*c*x^2) - 3*A*Sqrt[b]*c*Sqrt[b + c*x^2]*Arc
Tanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.74, size = 159, normalized size = 2.04

$$\left[\frac{3A\sqrt{b}\,cx\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\sqrt{cx^4+bx^2}\left(Bcx^2+Bb+3Ac\right)}{6cx}, \frac{3A\sqrt{-b}\,cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) + \sqrt{\vphantom{x}}}{3cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x), 1/3*(3*A*sqrt(
-b)*c*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x)]
```

giac [A] time = 0.20, size = 116, normalized size = 1.49

$$\frac{Ab\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}} - \frac{\left(3Abc\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + B\sqrt{-b}\,b^{\frac{3}{2}} + 3A\sqrt{-b}\sqrt{b}\,c\right)\text{sgn}(x)}{3\sqrt{-b}\,c} + \frac{(cx^2+b)^{\frac{3}{2}}Bc^2\text{sgn}(x) + 3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] A*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/3*(3*A*b*c*arctan(
sqrt(b)/sqrt(-b)) + B*sqrt(-b)*b^(3/2) + 3*A*sqrt(-b)*sqrt(b)*c)*sgn(x)/(sq
rt(-b)*c) + 1/3*((c*x^2 + b)^(3/2)*B*c^2*sgn(x) + 3*sqrt(c*x^2 + b)*A*c^3*s
gn(x))/c^3
```

maple [A] time = 0.05, size = 85, normalized size = 1.09

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(3A\sqrt{b}\,c\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^{2}+b}\,Ac-\left(c\,x^{2}+b\right)^{\frac{3}{2}}B\right)}{3\sqrt{c\,x^{2}+b}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)*(3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(1/2)*c
-B*(c*x^2+b)^(3/2)-3*A*(c*x^2+b)^(1/2)*c)/x/(c*x^2+b)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$A\int\frac{\sqrt{cx^{2}+b}}{x}\,dx+\frac{\left(cx^{2}+b\right)^{\frac{3}{2}}B}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] A*integrate(sqrt(c*x^2 + b)/x, x) + 1/3*(c*x^2 + b)^(3/2)*B/c
```

mupad [B] time = 0.50, size = 99, normalized size = 1.27

$$\frac{A\,\sqrt{c\,x^{4}+b\,x^{2}}}{x}+\frac{B\,\left(c\,x^{2}+b\right)\,\sqrt{c\,x^{4}+b\,x^{2}}}{3\,c\,x}+\frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c\,x^{4}+b\,x^{2}}\,1\mathrm{i}}{\sqrt{c}\,x^{2}\,\sqrt{\frac{b}{c\,x^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^2,x)
```

```
[Out] (A*(b*x^2 + c*x^4)^(1/2))/x + (B*(b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*c*x)
 + (A*b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 + c*x^4)^(1/2)*1i)/(c^(
1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\left(b+cx^{2}\right)}\,\left(A+Bx^{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**2, x)
```

3.104 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{bx^2+cx^4}\,(Ac+2bB)}{2bx}-\frac{(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}$$

```
[Out] -1/2*A*(c*x^4+b*x^2)^(3/2)/b/x^5-1/2*(A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b
*x^2)^(1/2))/b^(1/2)+1/2*(A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

Rubi [A] time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2021, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}\,(Ac+2bB)}{2bx}-\frac{(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{2bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] ((2*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(2*b*x) - (A*(b*x^2 + c*x^4)^(3/2))/(2*
b*x^5) - ((2*b*B + A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{(-2bB-Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx}{2b}$$
$$= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{1}{2}(-2bB-Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{1}{2}(2bB+Ac)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.$$
$$= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{(2bB+Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 0.94

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\,(A-2Bx^2)\sqrt{b+cx^2}+x^2(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{2\sqrt{b}\,x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] -1/2*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*(A - 2*B*x^2)*Sqrt[b + c*x^2] + (2*b*B
 + A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(Sqrt[b]*x^3*Sqrt[b + c*x^2]
)
```

fricas [A] time = 1.14, size = 169, normalized size = 1.69

$$\left[\frac{(2\,Bb+Ac)\sqrt{b}\,x^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,Bbx^2-Ab\right)}{4\,bx^3},\ \frac{(2\,Bb+Ac)\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{cx}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*b + A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)
*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3), 1/2*((2*
B*b + A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))
+ sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3)]
```

giac [A] time = 0.22, size = 76, normalized size = 0.76

$$\frac{2\,\sqrt{cx^2+b}\,Bc\,\mathrm{sgn}(x)+\frac{\left(2\,Bbc\,\mathrm{sgn}(x)+Ac^2\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{\sqrt{cx^2+b}\,Ac\,\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + b)*B*c*sgn(x) + (2*B*b*c*sgn(x) + A*c^2*sgn(x))*arctan(
sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x^2 + b)*A*c*sgn(x)/x^2)/c
```

maple [A] time = 0.05, size = 135, normalized size = 1.35

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(A\sqrt{b}\,c\,x^{2}\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)+2B\,b^{\frac{3}{2}}x^{2}\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^{2}+b}\,A c\,x^{2}-2\sqrt{c\,x^{2}+b}\,B b\,x^{2}}{2\sqrt{c\,x^{2}+b}\,b\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(A*b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2
*c+2*B*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2-A*(c*x^2+b)^(1/2)*x^
2*c-2*B*(c*x^2+b)^(1/2)*x^2*b+A*(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2 + A\right)\,\sqrt{c\,x^4 + b\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}\,\left(A + Bx^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**4, x)
```

3.105 $\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=103

$$-\frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}\,(4bB-Ac)}{8bx^3}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}$$

```
[Out] -1/4*A*(c*x^4+b*x^2)^(3/2)/b/x^7-1/8*c*(-A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^
4+b*x^2)^(1/2))/b^(3/2)-1/8*(-A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2020, 2008, 206}

$$-\frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}\,(4bB-Ac)}{8bx^3}-\frac{A\left(bx^2+cx^4\right)^{3/2}}{4bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -((4*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) - (A*(b*x^2 + c*x^4)^(3/2))/
(4*b*x^7) - (c*(4*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b
^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{(-4bB+Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx}{4b}$$
$$= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} + \frac{(c(4bB-Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b}$$
$$= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{(c(4bB-Ac))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.}{8b}$$
$$= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}$$

Mathematica [A] time = 0.12, size = 95, normalized size = 0.92

$$-\frac{(b+cx^2)(2Ab+Acx^2+4bBx^2)+cx^4\sqrt{\frac{cx^2}{b}+1}\,(4bB-Ac)\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{8bx^3\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -1/8*((b + c*x^2)*(2*A*b + 4*b*B*x^2 + A*c*x^2) + c*(4*b*B - A*c)*x^4*Sqrt[
1 + (c*x^2)/b]*ArcTanh[Sqrt[1 + (c*x^2)/b]])/(b*x^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.01, size = 198, normalized size = 1.92

$$\left[-\frac{(4\,Bbc-Ac^2)\sqrt{b}\,x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,(2\,Ab^2+(4\,Bb^2+Abc)x^2)}{16\,b^2x^5}, \frac{(4\,Bbc-Ac^2)\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 +
b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b*c)*x
^2))/(b^2*x^5), 1/8*((4*B*b*c - A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b
*c)*x^2))/(b^2*x^5)]
```

giac [A] time = 0.30, size = 132, normalized size = 1.28

$$\frac{\frac{(4\,Bbc^2\text{sgn}(x)-Ac^3\text{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b} - \frac{4\,(cx^2+b)^{\frac{3}{2}}Bbc^2\text{sgn}(x)-4\,\sqrt{cx^2+b}\,Bb^2c^2\text{sgn}(x)+(cx^2+b)^{\frac{3}{2}}Ac^3\text{sgn}(x)+\sqrt{cx^2+b}\,Abc^3\text{sgn}(x)}{bc^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*b*c^2*sgn(x) - A*c^3*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sq
rt(-b)*b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn(x) - 4*sqrt(c*x^2 + b)*B*b^2*c
^2*sgn(x) + (c*x^2 + b)^(3/2)*A*c^3*sgn(x) + sqrt(c*x^2 + b)*A*b*c^3*sgn(x)
)/(b*c^2*x^4))/c
```

maple [A] time = 0.06, size = 174, normalized size = 1.69

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(A\sqrt{b}\,c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-4B\,b^{\frac{3}{2}}c\,x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^2+b}\,A\,c^2x^4+4\sqrt{c\,x^2+b}\,B\right.}{8\sqrt{c\,x^2+b}\,b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(A*b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*
c^2-4*B*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*c-A*(c*x^2+b)^(1/2)
*x^4*c^2+4*B*(c*x^2+b)^(1/2)*x^4*b*c+A*(c*x^2+b)^(3/2)*x^2*c-4*B*(c*x^2+b)^
(3/2)*x^2*b-2*A*(c*x^2+b)^(3/2)*b)/x^5/(c*x^2+b)^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**6, x)
```

3.106 $\int x^5 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=223

$$-\frac{b^6(9bB-14Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}}+\frac{b^4\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(9bB-14Ac)}{2048c^5}-\frac{b^2\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(9bB-}{768c^4}$$

```
[Out] -1/768*b^2*(-14*A*c+9*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^4+1/240*b*(-14
*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/c^3-1/168*(-14*A*c+9*B*b)*x^2*(c*x^4+b*x^2)
^(5/2)/c^2+1/14*B*x^4*(c*x^4+b*x^2)^(5/2)/c-1/2048*b^6*(-14*A*c+9*B*b)*arct
anh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+1/2048*b^4*(-14*A*c+9*B*b)*(2
*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^5
```

Rubi [A] time = 0.40, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2034, 794, 670, 640, 612, 620, 206}

$$\frac{b^4\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(9bB-14Ac)}{2048c^5}-\frac{b^2\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(9bB-14Ac)}{768c^4}-\frac{b^6(9bB-14Ac)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{bx^2}}\right.}{2048c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (b^4*(9*b*B - 14*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(2048*c^5) - (b^2*
(9*b*B - 14*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(768*c^4) + (b*(9*b*B
 - 14*A*c)*(b*x^2 + c*x^4)^(5/2))/(240*c^3) - ((9*b*B - 14*A*c)*x^2*(b*x^2
+ c*x^4)^(5/2))/(168*c^2) + (B*x^4*(b*x^2 + c*x^4)^(5/2))/(14*c) - (b^6*(9*
b*B - 14*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2048*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^5\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx=\frac{1}{2}\text{Subst}\left(\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}dx,x,x^2\right)$$

$$=\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c}+\frac{\left(2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2\left(bx+c\right.\right.}{14c}$$

$$=-\frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c}+\frac{(b(9bB-14Ac))}{}$$

$$=\frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3}-\frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2}+\frac{Bx^4\left(bx\right.}{}$$

$$=-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}+\frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5}}{240c^3}$$

$$=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2\right.}{768c^4}$$

$$=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2\right.}{768c^4}$$

$$=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2\right.}{768c^4}$$

Mathematica [A] time = 0.36, size = 215, normalized size = 0.96

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(-210b^5c\left(7A+3Bx^2\right)+28b^4c^2x^2\left(35A+18Bx^2\right)-16b^3c^3x^4\left(49A+27Bx^2\right)+\right.\right.}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(945*b^6*B - 210*b^5*
c*(7*A + 3*B*x^2) + 96*b^2*c^4*x^6*(7*A + 4*B*x^2) + 2560*c^6*x^10*(7*A + 6
*B*x^2) + 28*b^4*c^2*x^2*(35*A + 18*B*x^2) - 16*b^3*c^3*x^4*(49*A + 27*B*x^
2) + 256*b*c^5*x^8*(91*A + 75*B*x^2)) - 105*b^(11/2)*(9*b*B - 14*A*c)*ArcSi
nh[(Sqrt[c]*x)/Sqrt[b]]))/(215040*c^(11/2)*x*Sqrt[1 + (c*x^2)/b])
```

fricas [**A**] time = 1.57, size = 418, normalized size = 1.87

$$\left[-\frac{105\left(9\,Bb^{7}-14\,Ab^{6}c\right)\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\left(15360\,Bc^{7}x^{12}+1280\left(15\,Bbc^{6}+14\,Ac^{7}\right)x^{10}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(9*B*b^7 - 14*A*b^6*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*
x^4 + b*x^2)*sqrt(c)) - 2*(15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*
x^10 + 128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 -
 48*(9*B*b^3*c^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4
- 70*(9*B*b^5*c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6, 1/215040*(
105*(9*B*b^7 - 14*A*b^6*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*
x^2 + b)) + (15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*x^10 + 128*(3*
B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 - 48*(9*B*b^3*c
^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4 - 70*(9*B*b^5*
c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6]
```

giac [**A**] time = 0.34, size = 280, normalized size = 1.26

$$\frac{1}{215040}\left(2\left(4\left(2\left(8\left(10\left(12\,Bcx^{2}\mathrm{sgn}(x)+\frac{15\,Bbc^{12}\mathrm{sgn}(x)+14\,Ac^{13}\mathrm{sgn}(x)}{c^{12}}\right)x^{2}+\frac{3\,Bb^{2}c^{11}\mathrm{sgn}(x)+182\,Abc^{12}\mathrm{sgn}(}{c^{12}}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/215040*(2*(4*(2*(8*(10*(12*B*c*x^2*sgn(x) + (15*B*b*c^12*sgn(x) + 14*A*c^
13*sgn(x))/c^12)*x^2 + (3*B*b^2*c^11*sgn(x) + 182*A*b*c^12*sgn(x))/c^12)*x^
2 - 3*(9*B*b^3*c^10*sgn(x) - 14*A*b^2*c^11*sgn(x))/c^12)*x^2 + 7*(9*B*b^4*c
^9*sgn(x) - 14*A*b^3*c^10*sgn(x))/c^12)*x^2 - 35*(9*B*b^5*c^8*sgn(x) - 14*A
*b^4*c^9*sgn(x))/c^12)*x^2 + 105*(9*B*b^6*c^7*sgn(x) - 14*A*b^5*c^8*sgn(x))
/c^12)*sqrt(c*x^2 + b)*x + 1/2048*(9*B*b^7*sgn(x) - 14*A*b^6*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 1/4096*(9*B*b^7*log(abs(b)) -
 14*A*b^6*c*log(abs(b)))*sgn(x)/c^(11/2)
```

maple [**A**] time = 0.08, size = 328, normalized size = 1.47

$$\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(15360\left(cx^{2}+b\right)^{\frac{5}{2}}Bc^{\frac{9}{2}}x^{9}+17920\left(cx^{2}+b\right)^{\frac{5}{2}}Ac^{\frac{9}{2}}x^{7}-11520\left(cx^{2}+b\right)^{\frac{5}{2}}Bbc^{\frac{7}{2}}x^{7}-12544\left(cx^{2}+b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/215040*(c*x^4+b*x^2)^(3/2)*(15360*B*(c*x^2+b)^(5/2)*c^(9/2)*x^9+17920*A*(
c*x^2+b)^(5/2)*c^(9/2)*x^7-11520*B*(c*x^2+b)^(5/2)*c^(7/2)*x^7*b-12544*A*(c
```

```
*x^2+b)^(5/2)*c^(7/2)*x^5*b+8064*B*(c*x^2+b)^(5/2)*c^(5/2)*x^5*b^2+7840*A*(
c*x^2+b)^(5/2)*c^(5/2)*x^3*b^2-5040*B*(c*x^2+b)^(5/2)*c^(3/2)*x^3*b^3-3920*
A*(c*x^2+b)^(5/2)*c^(3/2)*x*b^3+2520*B*(c*x^2+b)^(5/2)*c^(1/2)*x*b^4+980*A*
(c*x^2+b)^(3/2)*c^(3/2)*x*b^4-630*B*(c*x^2+b)^(3/2)*c^(1/2)*x*b^5+1470*A*(c
*x^2+b)^(1/2)*c^(3/2)*x*b^5-945*B*(c*x^2+b)^(1/2)*c^(1/2)*x*b^6+1470*A*ln(c
^(1/2)*x+(c*x^2+b)^(1/2))*b^6*c-945*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^7)/x^
3/(c*x^2+b)^(3/2)/c^(11/2)
```

maxima [A] time = 1.58, size = 363, normalized size = 1.63

$$-\frac{1}{30720}\left(\frac{420\sqrt{cx^4+bx^2}\,b^4x^2}{c^3}-\frac{1120\left(cx^4+bx^2\right)^{\frac{3}{2}}b^2x^2}{c^2}-\frac{2560\left(cx^4+bx^2\right)^{\frac{5}{2}}x^2}{c}-\frac{105\,b^6\log\left(2\,cx^2+b+2\,\sqrt{c}\right.}{c^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1120*(c*x^4 + b*x^2)^(3/2)*
b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c - 105*b^6*log(2*c*x^2 + b +
2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sqrt(c*x^4 + b*x^2)*b^5/c^4 -
560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^4 + b*x^2)^(5/2)*b/c^2)*A + 1
/143360*(10240*(c*x^4 + b*x^2)^(5/2)*x^4/c + 1260*sqrt(c*x^4 + b*x^2)*b^5*x
^2/c^4 - 3360*(c*x^4 + b*x^2)^(3/2)*b^3*x^2/c^3 - 7680*(c*x^4 + b*x^2)^(5/2
)*b*x^2/c^2 - 315*b^7*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(1
1/2) + 630*sqrt(c*x^4 + b*x^2)*b^6/c^5 - 1680*(c*x^4 + b*x^2)^(3/2)*b^4/c^4
 + 5376*(c*x^4 + b*x^2)^(5/2)*b^2/c^3)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.107 $\int x^3 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^5(7bB-12Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}-\frac{b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(7bB-12Ac)}{1024c^4}+\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(7bB-1}{384c^3}$$

```
[Out] 1/384*b*(-12*A*c+7*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^3-1/120*(-10*B*c*
x^2-12*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/c^2+1/1024*b^5*(-12*A*c+7*B*b)*arctan
h(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/1024*b^3*(-12*A*c+7*B*b)*(2*c*
x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

Rubi [A] time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 779, 612, 620, 206}

$$-\frac{b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(7bB-12Ac)}{1024c^4}+\frac{b^5(7bB-12Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}+\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(7bB-}{384c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(b^3*(7*b*B - 12*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^4) + (b*(
7*b*B - 12*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(384*c^3) - ((7*b*B -
12*A*c - 10*B*c*x^2)*(b*x^2 + c*x^4)^(5/2))/(120*c^2) + (b^5*(7*b*B - 12*A*
c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^3 \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(bx+cx^2\right)^{3/2} dx, x, x^2\right)$$

$$= -\frac{\left(7bB-12Ac-10Bcx^2\right)\left(bx^2+cx^4\right)^{5/2}}{120c^2} + \frac{(b(7bB-12Ac))\,\text{Subst}\left(\int \left(bx\right.\right.}{48c^2}$$

$$= \frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3} - \frac{\left(7bB-12Ac-10Bcx^2\right)\left(bx^2+\right.}{120c^2}$$

$$= -\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4} + \frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx\right.}{384c^3}$$

$$= -\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4} + \frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx\right.}{384c^3}$$

$$= -\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4} + \frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx\right.}{384c^3}$$

Mathematica [A] time = 0.32, size = 193, normalized size = 1.16

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(15b^{9/2}(7bB-12Ac)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(10b^4c\left(18A+7Bx^2\right)-8b^3c^2x^2\left(15A+7Bx\right.\right.\right.}{15360c^{9/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-105*b^5*B + 48*b^2*
c^3*x^4*(2*A + B*x^2) + 256*c^5*x^8*(6*A + 5*B*x^2) - 8*b^3*c^2*x^2*(15*A +
 7*B*x^2) + 10*b^4*c*(18*A + 7*B*x^2) + 64*b*c^4*x^6*(33*A + 26*B*x^2)) + 1
5*b^(9/2)*(7*b*B - 12*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(15360*c^(9/2)*x*
Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 1.29, size = 369, normalized size = 2.21

$$\left[-\frac{15\left(7\,Bb^6-12\,Ab^5c\right)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(1280\,Bc^6x^{10}+128\left(13\,Bbc^5+12\,Ac^6\right)x^8-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 12*A*c^6)*x^8
- 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^6 - 8*(7*B*b^
3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^2)*sqrt(c*x^4
 + b*x^2))/c^5, -1/15360*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*
```

```
x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 1
2*A*c^6)*x^8 - 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^
6 - 8*(7*B*b^3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^
2)*sqrt(c*x^4 + b*x^2))/c^5]
```

giac [A] time = 0.23, size = 246, normalized size = 1.47

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bcx^2\mathrm{sgn}(x)+\frac{13\,Bbc^{10}\mathrm{sgn}(x)+12\,Ac^{11}\mathrm{sgn}(x)}{c^{10}}\right)x^2+\frac{3\left(Bb^2c^9\mathrm{sgn}(x)+44\,Abc^{10}\mathrm{sgn}(x)\right)}{c^{10}}\right)x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*c*x^2*sgn(x) + (13*B*b*c^10*sgn(x) + 12*A*c^11*sg
n(x))/c^10)*x^2 + 3*(B*b^2*c^9*sgn(x) + 44*A*b*c^10*sgn(x))/c^10)*x^2 - (7*
B*b^3*c^8*sgn(x) - 12*A*b^2*c^9*sgn(x))/c^10)*x^2 + 5*(7*B*b^4*c^7*sgn(x) -
 12*A*b^3*c^8*sgn(x))/c^10)*x^2 - 15*(7*B*b^5*c^6*sgn(x) - 12*A*b^4*c^7*sgn
(x))/c^10)*sqrt(c*x^2 + b)*x - 1/1024*(7*B*b^6*sgn(x) - 12*A*b^5*c*sgn(x))*
log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/2048*(7*B*b^6*log(abs(b)
) - 12*A*b^5*c*log(abs(b)))*sgn(x)/c^(9/2)
```

maple [A] time = 0.07, size = 286, normalized size = 1.71

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(1280\left(c\,x^2+b\right)^{\frac{5}{2}}B\,c^{\frac{7}{2}}x^7+1536\left(c\,x^2+b\right)^{\frac{5}{2}}A\,c^{\frac{7}{2}}x^5-896\left(c\,x^2+b\right)^{\frac{5}{2}}Bb\,c^{\frac{5}{2}}x^5-180A\,b^5c\ln\left(\sqrt{c}\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/15360*(c*x^4+b*x^2)^(3/2)*(1280*B*(c*x^2+b)^(5/2)*c^(7/2)*x^7+1536*A*(c*x
^2+b)^(5/2)*c^(7/2)*x^5-896*B*(c*x^2+b)^(5/2)*c^(5/2)*x^5*b-960*A*(c*x^2+b)
^(5/2)*c^(5/2)*x^3*b+560*B*(c*x^2+b)^(5/2)*c^(3/2)*x^3*b^2+480*A*(c*x^2+b)^
(5/2)*c^(3/2)*x*b^2-280*B*(c*x^2+b)^(5/2)*c^(1/2)*x*b^3-120*(c*x^2+b)^(3/2)
*A*b^3*c^(3/2)*x+70*(c*x^2+b)^(3/2)*B*b^4*c^(1/2)*x-180*(c*x^2+b)^(1/2)*A*b
^4*c^(3/2)*x+105*(c*x^2+b)^(1/2)*B*b^5*c^(1/2)*x-180*A*b^5*c*ln(c^(1/2)*x+(
c*x^2+b)^(1/2))+105*B*b^6*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2
)/c^(9/2)
```

maxima [B] time = 1.56, size = 315, normalized size = 1.89

$$\frac{1}{2560}\left(\frac{60\sqrt{cx^4+bx^2}\,b^3x^2}{c^2}-\frac{160\left(cx^4+bx^2\right)^{\frac{3}{2}}bx^2}{c}-\frac{15\,b^5\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\sqrt{cx^4+bx^2}\,b^4}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2560*(60*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x^
2/c - 15*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*
sqrt(c*x^4 + b*x^2)*b^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*x^4
 + b*x^2)^(5/2)/c)*A - 1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1120*
(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c - 105*
b^6*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sqrt(c*x
^4 + b*x^2)*b^5/c^4 - 560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^4 + b*x
^2)^(5/2)*b/c^2)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(x^{2} \left(b + cx^{2}\right)\right)^{\frac{3}{2}} \left(A + Bx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.108 $\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$-\frac{3b^4(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(bB-2Ac)}{256c^3}-\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(bB-2Ac)}{32c^2}$$

```
[Out] -1/32*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^2+1/10*B*(c*x^4+b*x^2)
^(5/2)/c-3/256*b^4*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^
(7/2)+3/256*b^2*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2034, 640, 612, 620, 206}

$$\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(bB-2Ac)}{256c^3}-\frac{3b^4(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(bB-2Ac)}{32c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^2*(b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - ((b*B -
 2*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(32*c^2) + (B*(b*x^2 + c*x^4)^
(5/2))/(10*c) - (3*b^4*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/(256*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
```

```
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx=\frac{1}{2}\text{Subst}\left(\int(A+Bx)\left(bx+cx^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}dx,x,x^2\right)}{4c}$$
$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{\left(3b^2(bB-2A\right.}{}$$
$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right.}{32c^2}$$
$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right.}{32c^2}$$
$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right.}{32c^2}$$

Mathematica [A] time = 0.27, size = 171, normalized size = 1.16

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(-10b^3c\left(3A+Bx^2\right)+4b^2c^2x^2\left(5A+2Bx^2\right)+16bc^3x^4\left(15A+11Bx^2\right)+32c^4x^6\right.\right.}{1280c^{7/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(15*b^4*B - 10*b^3*c*
(3*A + B*x^2) + 4*b^2*c^2*x^2*(5*A + 2*B*x^2) + 32*c^4*x^6*(5*A + 4*B*x^2)
+ 16*b*c^3*x^4*(15*A + 11*B*x^2)) - 15*b^(7/2)*(b*B - 2*A*c)*ArcSinh[(Sqrt[
c]*x)/Sqrt[b]]))/(1280*c^(7/2)*x*Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 1.12, size = 316, normalized size = 2.14

$$\left[-\frac{15\left(Bb^5-2\,Ab^4c\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(128\,Bc^5x^8+16\left(11\,Bbc^4+10\,Ac^5\right)x^6+15\,Bb\right.}{2560\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(B*b^5 - 2*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(128*B*c^5*x^8 + 16*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*
b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 + 30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A
*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*(B*b^5 - 2*A*b^4*c)*sqr
t(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (128*B*c^5*x^8 + 1
6*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 +
30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
4]
```

giac [A] time = 0.22, size = 207, normalized size = 1.40

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bcx^2\mathrm{sgn}(x)+\frac{11\,Bbc^8\mathrm{sgn}(x)+10\,Ac^9\mathrm{sgn}(x)}{c^8}\right)x^2+\frac{Bb^2c^7\mathrm{sgn}(x)+30\,Abc^8\mathrm{sgn}(x)}{c^8}\right)x^2-\frac{5\left(Bb^3c^6\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*c*x^2*sgn(x) + (11*B*b*c^8*sgn(x) + 10*A*c^9*sgn(x))/c
^8)*x^2 + (B*b^2*c^7*sgn(x) + 30*A*b*c^8*sgn(x))/c^8)*x^2 - 5*(B*b^3*c^6*sg
n(x) - 2*A*b^2*c^7*sgn(x))/c^8)*x^2 + 15*(B*b^4*c^5*sgn(x) - 2*A*b^3*c^6*sg
n(x))/c^8)*sqrt(c*x^2 + b)*x + 3/256*(B*b^5*sgn(x) - 2*A*b^4*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 3/512*(B*b^5*log(abs(b)) - 2*A
*b^4*c*log(abs(b)))*sgn(x)/c^(7/2)
```

maple [A] time = 0.06, size = 244, normalized size = 1.65

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(128\left(c\,x^2+b\right)^{\frac{5}{2}}B\,c^{\frac{5}{2}}x^5+30A\,b^4c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-15B\,b^5\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+30\sqrt{c\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(128*B*(c*x^2+b)^(5/2)*c^(5/2)*x^5+160*A*(c*x^2+
b)^(5/2)*c^(5/2)*x^3-80*B*(c*x^2+b)^(5/2)*c^(3/2)*x^3*b-80*A*(c*x^2+b)^(5/2
)*c^(3/2)*x*b+40*B*(c*x^2+b)^(5/2)*c^(1/2)*x*b^2+20*(c*x^2+b)^(3/2)*A*b^2*c
^(3/2)*x-10*(c*x^2+b)^(3/2)*B*b^3*c^(1/2)*x+30*(c*x^2+b)^(1/2)*A*b^3*c^(3/2
)*x-15*(c*x^2+b)^(1/2)*B*b^4*c^(1/2)*x+30*A*b^4*c*ln(c^(1/2)*x+(c*x^2+b)^(1
/2))-15*B*b^5*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

maxima [B] time = 1.51, size = 267, normalized size = 1.80

$$\frac{1}{256}\left(32\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2-\frac{12\sqrt{cx^4+bx^2}\,b^2x^2}{c}+\frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\,\sqrt{cx^4+bx^2}\,b^3}{c^2}+\frac{16\,(\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3*
b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4
 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*A + 1/2560*(60*sqrt(c*x^4
 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x^2/c - 15*b^5*log(2*c*
x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b
^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*x^4 + b*x^2)^(5/2)/c)*B
```

mupad [B] time = 1.01, size = 236, normalized size = 1.59

$$\frac{B\left(c\,x^4+b\,x^2\right)^{5/2}}{10\,c}+\frac{A\left(c\,x^4+b\,x^2\right)^{3/2}\left(c\,x^2+\frac{b}{2}\right)}{8\,c}-\frac{3\,A\,b^2\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}-\frac{B\,b\left(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (B*(b*x^2 + c*x^4)^(5/2))/(10*c) + (A*(b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))/
(8*c) - (3*A*b^2*((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2) - (b^2*log((b/2 +
 c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(32*c) - (B*b*((x^2
*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(
4*c) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))
))/(16*c) + (b*(b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.109 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x} dx$

Optimal. Leaf size=144

$$\frac{b^3(3bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}(3bB-8Ac)}{128c^2}-\frac{(bx^2+cx^4)^{3/2}(3bB-8Ac)}{48c}+\frac{B(bx^2+c}{8cx^2}$$

```
[Out] -1/48*(-8*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c+1/8*B*(c*x^4+b*x^2)^(5/2)/c/x^2+
1/128*b^3*(-8*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-1
/128*b*(-8*A*c+3*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 794, 664, 612, 620, 206}

$$\frac{b^3(3bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}(3bB-8Ac)}{128c^2}-\frac{(bx^2+cx^4)^{3/2}(3bB-8Ac)}{48c}+\frac{B(bx^2+c}{8cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] -(b*(3*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) - ((3*b*B
- 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(48*c) + (B*(b*x^2 + c*x^4)^(5/2))/(8*c*x^2
) + (b^3*(3*b*B - 8*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c
^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
```

```
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{B\left(bx^2+cx^4\right)^{5/2}}{8cx^2} + \frac{\left(bB-Ac+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)}{8c}$$

$$= -\frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{8cx^2} - \frac{(b(3bB-8Ac))\,\text{Subst}\left(\int \right.}{32c}$$

$$= -\frac{b(3bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(b\right.}{}$$

$$= -\frac{b(3bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(b\right.}{}$$

$$= -\frac{b(3bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(b\right.}{}$$

Mathematica [A] time = 0.24, size = 151, normalized size = 1.05

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^{5/2}(3bB-8Ac)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(6b^2c\left(4A+Bx^2\right)+8bc^2x^2\left(14A+9Bx^2\right)+1\right.\right.}{384c^{5/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-9*b^3*B + 6*b^2*c*(
4*A + B*x^2) + 16*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c^2*x^2*(14*A + 9*B*x^2)) +
 3*b^(5/2)*(3*b*B - 8*A*c)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(384*c^(5/2)*x*Sq
rt[1 + (c*x^2)/b])
```

fricas [A] time = 0.89, size = 275, normalized size = 1.91

$$\left[-\frac{3\left(3\,Bb^4-8\,Ab^3c\right)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(48\,Bc^4x^6-9\,Bb^3c+24\,Ab^2c^2+8\left(9\,Bbc^3+\right.\right.}{768\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3, -1/384*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt
(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3]
```

giac [A] time = 0.20, size = 178, normalized size = 1.24

$$\frac{1}{384}\left(2\left(4\left(6\,Bcx^2\mathrm{sgn}(x)+\frac{9\,Bbc^6\mathrm{sgn}(x)+8\,Ac^7\mathrm{sgn}(x)}{c^6}\right)x^2+\frac{3\,Bb^2c^5\mathrm{sgn}(x)+56\,Abc^6\mathrm{sgn}(x)}{c^6}\right)x^2-\frac{3\left(3\,Bb^3c^4\mathrm{sg}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*c*x^2*sgn(x) + (9*B*b*c^6*sgn(x) + 8*A*c^7*sgn(x))/c^6)*x^
2 + (3*B*b^2*c^5*sgn(x) + 56*A*b*c^6*sgn(x))/c^6)*x^2 - 3*(3*B*b^3*c^4*sgn(
x) - 8*A*b^2*c^5*sgn(x))/c^6)*sqrt(c*x^2 + b)*x - 1/128*(3*B*b^4*sgn(x) - 8
*A*b^3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1/256*(3*
B*b^4*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(5/2)
```

maple [A] time = 0.06, size = 202, normalized size = 1.40

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-24A\,b^3c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+9B\,b^4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-24\sqrt{c\,x^2+b}\,A\,b^2c^{\frac{3}{2}}x+9\sqrt{c\,x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(3/2)*(48*B*c^(3/2)*(c*x^2+b)^(5/2)*x^3+64*A*c^(3/2)*(c
*x^2+b)^(5/2)*x-24*B*c^(1/2)*(c*x^2+b)^(5/2)*x*b-16*(c*x^2+b)^(3/2)*A*b*c^(
3/2)*x+6*(c*x^2+b)^(3/2)*B*b^2*c^(1/2)*x-24*(c*x^2+b)^(1/2)*A*b^2*c^(3/2)*x
+9*(c*x^2+b)^(1/2)*B*b^3*c^(1/2)*x-24*A*b^3*c*ln(c^(1/2)*x+(c*x^2+b)^(1/2))
+9*B*b^4*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2)/c^(5/2)
```

maxima [A] time = 1.49, size = 216, normalized size = 1.50

$$\frac{1}{96}\left(12\,\sqrt{cx^4+bx^2}\,bx^2-\frac{3\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}+16\left(cx^4+bx^2\right)^{\frac{3}{2}}+\frac{6\,\sqrt{cx^4+bx^2}\,b^2}{c}\right)A+\frac{1}{256}\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c))/c^(3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^2)
*b^2/c)*A + 1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)*b^
2*x^2/c + 3*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) -
6*sqrt(c*x^4 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x, x)
```

3.110 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=137

$$-\frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b}+\frac{(b+2cx^2)\sqrt{bx^2+cx^4}(bB-6Ac)}{16c}+\frac{A(bx^2+cx^4)}{bx^4}$$

```
[Out] 1/6*(-6*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b+A*(c*x^4+b*x^2)^(5/2)/b/x^4-1/16*b^2
*(-6*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/16*(-6*A*c
+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.30, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2034, 792, 664, 612, 620, 206}

$$-\frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b}+\frac{(b+2cx^2)\sqrt{bx^2+cx^4}(bB-6Ac)}{16c}+\frac{A(bx^2+cx^4)}{bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] ((b*B - 6*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + ((b*B - 6*A*c)*(
b*x^2 + c*x^4)^(3/2))/(6*b) + (A*(b*x^2 + c*x^4)^(5/2))/(b*x^4) - (b^2*(b*B
 - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx,x,x^2\right)$$

$$= \frac{A\left(bx^2+cx^4\right)^{5/2}}{bx^4} - \frac{\left(-2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx,\right.}{b}$$

$$= \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+cx^4\right)^{5/2}}{bx^4} - \frac{1}{4}(-bB+6Ac)\,\text{Subst}\left(\int \sqrt{\right.$$

$$= \frac{(bB-6Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+\right.}{bx^4}$$

$$= \frac{(bB-6Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+\right.}{bx^4}$$

$$= \frac{(bB-6Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+\right.}{bx^4}$$

Mathematica [A] time = 0.13, size = 130, normalized size = 0.95

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(2bc\left(15A+7Bx^2\right)+4c^2x^2\left(3A+2Bx^2\right)+3b^2B\right)-3b^{3/2}(bB-6Ac)\sinh^{-1}\left(\frac{\sqrt{c}}{\sqrt{b}}\right.\right.}{48c^{3/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(3*b^2*B + 4*c^2*x^2*
(3*A + 2*B*x^2) + 2*b*c*(15*A + 7*B*x^2)) - 3*b^(3/2)*(b*B - 6*A*c)*ArcSinh
[(Sqrt[c]*x)/Sqrt[b]]))/(48*c^(3/2)*x*Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 1.28, size = 224, normalized size = 1.64

$$\left[-\frac{3\left(Bb^3-6\,Ab^2c\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(8\,Bc^3x^4+3\,Bb^2c+30\,Abc^2+2\left(7\,Bbc^2+6\,A\right.\right.}{96\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 3*B*b^2*c + 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A
*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*(B*b^3 - 6*A*b^2*c)*sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 3*B*b^2*c
+ 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

giac [A] time = 0.21, size = 142, normalized size = 1.04

$$\frac{1}{48}\left(2\left(4\,Bcx^2\mathrm{sgn}(x)+\frac{7\,Bbc^4\mathrm{sgn}(x)+6\,Ac^5\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\left(Bb^2c^3\mathrm{sgn}(x)+10\,Abc^4\mathrm{sgn}(x)\right)}{c^4}\right)\sqrt{cx^2+b}\,x+\frac{\left(Bb^3\mathrm{sg}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*c*x^2*sgn(x) + (7*B*b*c^4*sgn(x) + 6*A*c^5*sgn(x))/c^4)*x^2 +
3*(B*b^2*c^3*sgn(x) + 10*A*b*c^4*sgn(x))/c^4)*sqrt(c*x^2 + b)*x + 1/16*(B*b
^3*sgn(x) - 6*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(3/2
) - 1/32*(B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(3/2)
```

maple [A] time = 0.05, size = 162, normalized size = 1.18

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(18A\,b^2c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-3B\,b^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+18\sqrt{c\,x^2+b}\,\,Abc^{\frac{3}{2}}x-3\sqrt{c\,x^2+b}\right.}{48\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(8*B*c^(1/2)*(c*x^2+b)^(5/2)*x+12*(c*x^2+b)^(3/2)*
A*c^(3/2)*x-2*(c*x^2+b)^(3/2)*B*b*c^(1/2)*x+18*(c*x^2+b)^(1/2)*A*b*c^(3/2)*
x-3*(c*x^2+b)^(1/2)*B*b^2*c^(1/2)*x+18*A*b^2*c*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
)-3*B*b^3*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

maxima [A] time = 1.43, size = 168, normalized size = 1.23

$$\frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{\sqrt{c}}+6\,\sqrt{cx^4+bx^2}\,b+\frac{4\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^2}\right)A+\frac{1}{96}\left(12\,\sqrt{cx^4+bx^2}\,bx^2-\frac{3\,b^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*sq
rt(c*x^4 + b*x^2)*b + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*A + 1/96*(12*sqrt(c*x^4
+ b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(
3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**3, x)
```

3.111 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=128

$$\frac{(bx^2+cx^4)^{3/2}(4Ac+bB)}{4bx^2}+\frac{3}{8}\sqrt{bx^2+cx^4}\,(4Ac+bB)+\frac{3b(4Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}-\frac{A(bx^2+cx^4)^{5/2}}{bx^6}$$

```
[Out] 1/4*(4*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^2-A*(c*x^4+b*x^2)^(5/2)/b/x^6+3/8*b
*(4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)+3/8*(4*A*c+B*
b)*(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2034, 792, 664, 620, 206}

$$\frac{(bx^2+cx^4)^{3/2}(4Ac+bB)}{4bx^2}+\frac{3}{8}\sqrt{bx^2+cx^4}\,(4Ac+bB)+\frac{3b(4Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}-\frac{A(bx^2+cx^4)^{5/2}}{bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (3*(b*B + 4*A*c)*Sqrt[b*x^2 + c*x^4])/8 + ((b*B + 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(4*b*x^2) - (A*(b*x^2 + c*x^4)^(5/2))/(b*x^6) + (3*b*(b*B + 4*A*c)*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5}dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^3}dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{\left(-3(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^2} d\right.}{b}$$

$$= \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{8}(3(bB+4Ac))\text{Subst}\left(\int \right.$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} +$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} +$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} +$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.75

$$\frac{\sqrt{x^2(b+cx^2)}\left(\frac{3\sqrt{b}\,x(4Ac+bB)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{b}+1}} - 8Ab + 4Acx^2 + 5bBx^2 + 2Bcx^4\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-8*A*b + 5*b*B*x^2 + 4*A*c*x^2 + 2*B*c*x^4 + (3*Sqr
t[b]*(b*B + 4*A*c)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[c]*Sqrt[1 + (c*x^2
)/b])))/(8*x^2)
```

fricas [A] time = 1.27, size = 209, normalized size = 1.63

$$\left[\frac{3(Bb^2+4Abc)\sqrt{c}\,x^2\log\left(-2cx^2-b-2\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(2Bc^2x^4-8Abc+\left(5Bbc+4Ac^2\right)x^2\right)\sqrt{cx^4+}}{16cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*b^2 + 4*A*b*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*
x^2)*sqrt(c)) + 2*(2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4*A*c^2)*x^2)*sqrt(c*
```

```
x^4 + b*x^2))/(c*x^2), -1/8*(3*(B*b^2 + 4*A*b*c)*sqrt(-c)*x^2*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4
*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x^2)]
```

giac [A] time = 0.35, size = 126, normalized size = 0.98

$$\frac{2\,Ab^2\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b}+\frac{1}{8}\left(2\,Bcx^2\mathrm{sgn}(x)+\frac{5\,Bbc^2\mathrm{sgn}(x)+4\,Ac^3\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}\,x-\frac{3\left(Bb^2\sqrt{c}\,\mathrm{sgn}(x)+4\,Abc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 2*A*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b) + 1/8*(2*B*c*x
^2*sgn(x) + (5*B*b*c^2*sgn(x) + 4*A*c^3*sgn(x))/c^2)*sqrt(c*x^2 + b)*x - 3/
16*(B*b^2*sqrt(c)*sgn(x) + 4*A*b*c^(3/2)*sgn(x))*log((sqrt(c)*x - sqrt(c*x^
2 + b))^2)/c
```

maple [A] time = 0.06, size = 174, normalized size = 1.36

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(12A\,b^2cx\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)+3B\,b^3x\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)+12\sqrt{c\,x^2+b}\;Ab\,c^{\frac{3}{2}}x^2+3\sqrt{c\,x^2}\right.}{8\left(c\,x^2+b\right)^{\frac{3}{2}}b\sqrt{c}\;x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(3/2)*(8*A*c^(3/2)*(c*x^2+b)^(3/2)*x^2+12*A*c^(3/2)*(c*x^
2+b)^(1/2)*x^2*b+2*B*c^(1/2)*(c*x^2+b)^(3/2)*x^2*b-8*A*c^(1/2)*(c*x^2+b)^(5
/2)+3*B*c^(1/2)*(c*x^2+b)^(1/2)*x^2*b^2+12*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*
x*b^2*c+3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x*b^3)/x^4/(c*x^2+b)^(3/2)/b/c^(1
/2)
```

maxima [A] time = 1.49, size = 148, normalized size = 1.16

$$\frac{1}{4}\left(3\,b\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{6\,\sqrt{cx^4+bx^2}\,b}{x^2}+\frac{2\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^4}\right)A+\frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{\right.}}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 6*sqrt(
c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*A + 1/16*(3*b^2*log(2*c
*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*sqrt(c*x^4 + b*x^2)*b
 + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**5, x)
```

3.112 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=136

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(2Ac+3bB)}{3bx^4}+\frac{c\sqrt{bx^2+cx^4}\,(2Ac+3bB)}{2b}+\frac{1}{2}\sqrt{c}\,(2Ac+3bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{A\left(bx^2+cx^4\right)^{5}}{3bx^8}$$

```
[Out] -1/3*(2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^4-1/3*A*(c*x^4+b*x^2)^(5/2)/b/x^
8+1/2*(2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(1/2)+1/2*c*
(2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b
```

Rubi [A] time = 0.27, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2034, 792, 662, 664, 620, 206}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(2Ac+3bB)}{3bx^4}+\frac{c\sqrt{bx^2+cx^4}\,(2Ac+3bB)}{2b}+\frac{1}{2}\sqrt{c}\,(2Ac+3bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{A\left(bx^2+cx^4\right)^{5}}{3bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

```
[Out] (c*(3*b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - ((3*b*B + 2*A*c)*(b*x^2 + c
*x^4)^(3/2))/(3*b*x^4) - (A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^8) + (Sqrt[c]*(3*
b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
```

```
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8} + \frac{\left(-4(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^3}\,d\right.}{3b}$$

$$= -\frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8} + \frac{\left(c\left(-4(-bB+Ac)+\frac{5}{2}(-\right.\right.}{}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{3bx^8}$$

Mathematica [C] time = 0.05, size = 98, normalized size = 0.72

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(bx^2(2Ac+3bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{b}\right)+A\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{3bx^4\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

```
[Out] -1/3*(Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(3*b*B
 + 2*A*c)*x^2*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/b)]))/(b*x^4*Sqr
t[1 + (c*x^2)/b])
```

fricas [A] time = 1.10, size = 189, normalized size = 1.39

$$\left[\frac{3\,(3\,Bb+2\,Ac)\sqrt{c}\,x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(3\,Bcx^4-2\,(3\,Bb+4\,Ac)x^2-2\,Ab\right)\sqrt{cx^4+bx^2}}{12\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*b + 2*A*c)*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^
2)*sqrt(c)) + 2*(3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*x^4 + b*
x^2))/x^4, -1/6*(3*(3*B*b + 2*A*c)*sqrt(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-c)/(c*x^2 + b)) - (3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*
x^4 + b*x^2))/x^4]
```

giac [A] time = 0.53, size = 225, normalized size = 1.65

$$\frac{1}{2}\sqrt{cx^2+b}\,Bcx\mathrm{sgn}(x)-\frac{1}{4}\left(3\,Bb\sqrt{c}\,\mathrm{sgn}(x)+2\,Ac^{\frac{3}{2}}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)+\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4 Bb\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*c*x*sgn(x) - 1/4*(3*B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + b))^2) + 2/3*(3*(sqrt(c)*x - sqrt(c*x
^2 + b))^4*B*b^2*sqrt(c)*sgn(x) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(
3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*sqrt(c)*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(3/2)*sgn(x) + 3*B*b^4*sqrt(c)*sgn(x)
 + 4*A*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

maple [A] time = 0.06, size = 219, normalized size = 1.61

$$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-6A\,b^2c^2x^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-9B\,b^3c\,x^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-6\sqrt{c\,x^2+b}\,Ab\,c^{\frac{5}{2}}x^4-9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x)
```

```
[Out] -1/6*(c*x^4+b*x^2)^(3/2)*(-4*A*(c*x^2+b)^(3/2)*c^(5/2)*x^4-6*B*(c*x^2+b)^(3
/2)*c^(3/2)*x^4*b+4*A*(c*x^2+b)^(5/2)*c^(3/2)*x^2-6*A*(c*x^2+b)^(1/2)*c^(5/
2)*x^4*b+6*B*(c*x^2+b)^(5/2)*c^(1/2)*x^2*b-9*B*(c*x^2+b)^(1/2)*c^(3/2)*x^4*
b^2-6*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x^3*b^2*c^2-9*B*ln(c^(1/2)*x+(c*x^2+b
)^(1/2))*x^3*b^3*c+2*A*(c*x^2+b)^(5/2)*c^(1/2)*b)/x^6/(c*x^2+b)^(3/2)/b^2/c
^(1/2)
```

maxima [A] time = 1.50, size = 167, normalized size = 1.23

$$\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{7\,\sqrt{cx^4+bx^2}\,c}{x^2}-\frac{\sqrt{cx^4+bx^2}\,b}{x^4}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^6}\right)A+\frac{1}{4}\left(3\,b\sqrt{c}\,\log\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c*
x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^6)
*A + 1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 6*
sqrt(c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**7, x)
```

3.113 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9} dx$

Optimal. Leaf size=104

$$-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B(bx^2+cx^4)^{3/2}}{3x^6}$$

```
[Out] -1/3*B*(c*x^4+b*x^2)^(3/2)/x^6-1/5*A*(c*x^4+b*x^2)^(5/2)/b/x^10+B*c^(3/2)*a
rctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))-B*c*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.25, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 792, 662, 620, 206}

$$-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{B(bx^2+cx^4)^{3/2}}{3x^6}-\frac{Bc\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

```
[Out] -((B*c*Sqrt[b*x^2 + c*x^4])/x^2) - (B*(b*x^2 + c*x^4)^(3/2))/(3*x^6) - (A*(
b*x^2 + c*x^4)^(5/2))/(5*b*x^10) + B*c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x
^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}+\frac{1}{2}B\,\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{B\left(bx^2+cx^4\right)^{3/2}}{3x^6}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}+\frac{1}{2}(Bc)\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B\left(bx^2+cx^4\right)^{3/2}}{3x^6}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}+\frac{1}{2}\left(Bc^2\right)\text{Subst}\Bigg(\int\\
&= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B\left(bx^2+cx^4\right)^{3/2}}{3x^6}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}+\left(Bc^2\right)\text{Subst}\Bigg(\int \frac{}{1}\\
&= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B\left(bx^2+cx^4\right)^{3/2}}{3x^6}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\Bigg(\frac{}{\sqrt{bx}}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 94, normalized size = 0.90

$$
-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3A\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}+5b^2Bx^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{b}\right)\right)}{15bx^6\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

```
[Out] -1/15*(Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b^2
*B*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)/b)]))/(b*x^6*Sqrt[1 +
(c*x^2)/b])
```

fricas [A] time = 0.76, size = 207, normalized size = 1.99

$$
\left[\frac{15\,Bbc^{\frac{3}{2}}x^6\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(\left(20\,Bbc+3\,Ac^2\right)x^4+3\,Ab^2+\left(5\,Bb^2+6\,Abc\right)x^2\right)\sqrt{cx^4+}}{30\,bx^6}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*b*c^(3/2)*x^6*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + 6*A*b*c)*x^2)*sqrt(c*
x^4 + b*x^2))/(b*x^6), -1/15*(15*B*b*sqrt(-c)*c*x^6*arctan(sqrt(c*x^4 + b*x
```

```
^2)*sqrt(-c)/(c*x^2 + b)) + ((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2
+ 6*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^6)]
```

giac [B] time = 1.02, size = 254, normalized size = 2.44

$$-\frac{1}{2} B c^{\frac{3}{2}} \log\left(\left(\sqrt{c}\, x - \sqrt{cx^2 + b}\right)^2\right) \operatorname{sgn}(x) + \frac{2\left(30\left(\sqrt{c}\, x - \sqrt{cx^2+b}\right)^8 Bbc^{\frac{3}{2}} \operatorname{sgn}(x) + 15\left(\sqrt{c}\, x - \sqrt{cx^2+b}\right)^8 Ac^{\frac{5}{2}} \operatorname{sg}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/2*B*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/15*(30*(sqrt
(c)*x - sqrt(c*x^2 + b))^8*B*b*c^(3/2)*sgn(x) + 15*(sqrt(c)*x - sqrt(c*x^2
+ b))^8*A*c^(5/2)*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3/2)
*sgn(x) + 110*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(3/2)*sgn(x) + 30*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + b))^2*B*b^4*c^(3/2)*sgn(x) + 20*B*b^5*c^(3/2)*sgn(x) + 3*A*b^4*c^(5/2
)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

maple [A] time = 0.06, size = 153, normalized size = 1.47

$$-\frac{\left(c x^4 + b x^2\right)^{\frac{3}{2}} \left(-15 B b^2 c^2 x^5 \ln\left(\sqrt{c}\, x + \sqrt{c x^2 + b}\right) - 15\sqrt{c x^2 + b}\, Bb c^{\frac{5}{2}} x^6 - 10\left(c x^2 + b\right)^{\frac{3}{2}} B c^{\frac{5}{2}} x^6 + 10\left(c x^2 + b\cdots\right.\right.}{15\left(c x^2 + b\right)^{\frac{3}{2}} b^2 \sqrt{c}\, x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/15*(c*x^4+b*x^2)^(3/2)*(-10*B*(c*x^2+b)^(3/2)*c^(5/2)*x^6+10*B*(c*x^2+b)
^(5/2)*c^(3/2)*x^4-15*B*(c*x^2+b)^(1/2)*c^(5/2)*x^6*b-15*B*ln(c^(1/2)*x+(c*
x^2+b)^(1/2))*x^5*b^2*c^2+5*(c*x^2+b)^(5/2)*B*b*c^(1/2)*x^2+3*(c*x^2+b)^(5/
2)*A*b*c^(1/2))/x^8/(c*x^2+b)^(3/2)/b^2/c^(1/2)
```

maxima [B] time = 1.38, size = 177, normalized size = 1.70

$$\frac{1}{6}\left(3 c^{\frac{3}{2}} \log\left(2 cx^2 + b + 2\sqrt{cx^4 + bx^2}\,\sqrt{c}\right) - \frac{7\sqrt{cx^4 + bx^2}\, c}{x^2} - \frac{\sqrt{cx^4 + bx^2}\, b}{x^4} - \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^6}\right) B - \frac{1}{10} A\left(\frac{2\sqrt{cx^4 + b\cdots}}{bx^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c*
x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^6)
*B - 1/10*A*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x^4
- 3*sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B x^2 + A\right) \left(c x^4 + b x^2\right)^{3/2}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**9, x)
```

3.114 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=61

$$-\frac{(bx^2+cx^4)^{5/2}(7bB-2Ac)}{35b^2x^{10}}-\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(5/2)/b/x^12-1/35*(-2*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/b
^2/x^10
```

Rubi [A] time = 0.17, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 650}

$$-\frac{(bx^2+cx^4)^{5/2}(7bB-2Ac)}{35b^2x^{10}}-\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(7*b*x^12) - ((7*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5
/2))/(35*b^2*x^10)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^5}\,d\right.}{7b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} - \frac{(7bB-2Ac)(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.72

$$-\frac{(x^2(b+cx^2))^{5/2}(5Ab-2Acx^2+7bBx^2)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
[Out] -1/35*((x^2*(b + c*x^2))^(5/2)*(5*A*b + 7*b*B*x^2 - 2*A*c*x^2))/(b^2*x^12)
```

fricas [A] time = 1.07, size = 82, normalized size = 1.34

$$-\frac{\left((7\,Bbc^2-2\,Ac^3)x^6+(14\,Bb^2c+Abc^2)x^4+5\,Ab^3+(7\,Bb^3+8\,Ab^2c)x^2\right)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
[Out] -1/35*((7*B*b*c^2 - 2*A*c^3)*x^6 + (14*B*b^2*c + A*b*c^2)*x^4 + 5*A*b^3 + (
7*B*b^3 + 8*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^8)
```

giac [B] time = 1.89, size = 370, normalized size = 6.07

$$\frac{2\left(35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{5}{2}}\text{sgn}(x)-70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{5}{2}}\text{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}Ac^{\frac{7}{2}}\text{sgn}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(5/2)*sgn(x) - 70*(sqrt(c)*x
- sqrt(c*x^2 + b))^10*B*b*c^(5/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))
^10*A*c^(7/2)*sgn(x) + 105*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(5/2)*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(7/2)*sgn(x) - 140*(sqrt(c)
*x - sqrt(c*x^2 + b))^6*B*b^3*c^(5/2)*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2
+ b))^6*A*b^2*c^(7/2)*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^4*c^(
5/2)*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(7/2)*sgn(x) - 14*
(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(5/2)*sgn(x) + 14*(sqrt(c)*x - sqrt
(c*x^2 + b))^2*A*b^4*c^(7/2)*sgn(x) + 7*B*b^6*c^(5/2)*sgn(x) - 2*A*b^5*c^(7
/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

maple [A] time = 0.05, size = 48, normalized size = 0.79

$$-\frac{(c\,x^2+b)(-2Ac\,x^2+7Bb\,x^2+5Ab)(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x)
```

```
[Out] -1/35*(c*x^2+b)*(-2*A*c*x^2+7*B*b*x^2+5*A*b)*(c*x^4+b*x^2)^(3/2)/b^2/x^10
```

maxima [B] time = 1.56, size = 193, normalized size = 3.16

$$-\frac{1}{10}B\left(\frac{2\sqrt{cx^4+bx^2}\,c^2}{bx^2}-\frac{\sqrt{cx^4+bx^2}\,c}{x^4}-\frac{3\sqrt{cx^4+bx^2}\,b}{x^6}+\frac{5\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^8}\right)+\frac{1}{140}A\left(\frac{8\sqrt{cx^4+bx^2}\,c^3}{b^2x^2}-\frac{4\sqrt{cx^4+}}{bx^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x^4 - 3*
sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8) + 1/140*A*(8*sqrt(
c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b*x^4) + 3*sqrt(c
*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(c*x^4 + b*x^2)^(3/
2)/x^10)
```

mupad [B] time = 1.03, size = 156, normalized size = 2.56

$$\frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}-\frac{8\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6}-\frac{2\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{5\,x^4}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4}-\frac{A\,b\,\sqrt{c\,x^4+b\,x}}{7\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*A*c*(b*x^2 + c*x^4)^(1/2)
)/(35*x^6) - (B*b*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (2*B*c*(b*x^2 + c*x^4)^(
1/2))/(5*x^4) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (A*b*(b*x^2 + c*
x^4)^(1/2))/(7*x^8) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**11, x)
```

3.115 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}} \, dx$

Optimal. Leaf size=96

$$\frac{2c\left(bx^2+cx^4\right)^{5/2}(9bB-4Ac)}{315b^3x^{10}}-\frac{\left(bx^2+cx^4\right)^{5/2}(9bB-4Ac)}{63b^2x^{12}}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(5/2)/b/x^14-1/63*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/b
^2/x^12+2/315*c*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^10
```

Rubi [A] time = 0.23, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{2c\left(bx^2+cx^4\right)^{5/2}(9bB-4Ac)}{315b^3x^{10}}-\frac{\left(bx^2+cx^4\right)^{5/2}(9bB-4Ac)}{63b^2x^{12}}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(9*b*x^14) - ((9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5
/2))/(63*b^2*x^12) + (2*c*(9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x
^10)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
```

```
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^7}\,dx, x, x^2\right)$$
$$= -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{\left(-7(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^6}\,dx, x\right.}{9b}$$
$$= -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}} - \frac{(c(9bB-4Ac))\,\text{Subst}\left(\int \frac{(bx}{}\right.}{63b^2}$$
$$= -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}} + \frac{2c(9bB-4Ac)(bx^2+cx^4)^{5/}}{315b^3x^{10}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.69

$$\frac{(x^2(b+cx^2))^{5/2}\left(A(-35b^2+20bcx^2-8c^2x^4)+9bBx^2(2cx^2-5b)\right)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(9*b*B*x^2*(-5*b + 2*c*x^2) + A*(-35*b^2 + 20*b*c*
x^2 - 8*c^2*x^4)))/(315*b^3*x^14)
```

fricas [A] time = 1.09, size = 109, normalized size = 1.14

$$\frac{\left(2\left(9\,Bbc^3-4\,Ac^4\right)x^8-\left(9\,Bb^2c^2-4\,Abc^3\right)x^6-35\,Ab^4-3\left(24\,Bb^3c+Ab^2c^2\right)x^4-5\left(9\,Bb^4+10\,Ab^3c\right)x^2\right)\sqrt{cx^4+}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(9*B*b*c^3 - 4*A*c^4)*x^8 - (9*B*b^2*c^2 - 4*A*b*c^3)*x^6 - 35*A*b
^4 - 3*(24*B*b^3*c + A*b^2*c^2)*x^4 - 5*(9*B*b^4 + 10*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^3*x^10)
```

giac [B] time = 2.86, size = 430, normalized size = 4.48

$$\frac{4\left(315\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{7}{2}}\text{sgn}(x)-315\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{7}{2}}\text{sgn}(x)+840\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}Ac^{\frac{9}{2}}\text{sgr}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(7/2)*sgn(x) - 315*(sqrt(c)
*x - sqrt(c*x^2 + b))^12*B*b*c^(7/2)*sgn(x) + 840*(sqrt(c)*x - sqrt(c*x^2 +
 b))^12*A*c^(9/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^(7/
```

```
2)*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(9/2)*sgn(x) - 819*
(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(7/2)*sgn(x) + 1764*(sqrt(c)*x - sq
rt(c*x^2 + b))^8*A*b^2*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*B*b^4*c^(7/2)*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^3*c^(9/2)*s
gn(x) - 9*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(7/2)*sgn(x) + 144*(sqrt(
c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(9/2)*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2
 + b))^2*B*b^6*c^(7/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^5*c^
(9/2)*sgn(x) - 9*B*b^7*c^(7/2)*sgn(x) + 4*A*b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + b))^2 - b)^9
```

maple [A] time = 0.05, size = 70, normalized size = 0.73

$$-\frac{\left(c\,x^2+b\right)\left(8A\,c^2x^4-18Bbc\,x^4-20Abc\,x^2+45B\,b^2x^2+35b^2A\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{315b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x)
```

```
[Out] -1/315*(c*x^2+b)*(8*A*c^2*x^4-18*B*b*c*x^4-20*A*b*c*x^2+45*B*b^2*x^2+35*A*b
^2)*(c*x^4+b*x^2)^(3/2)/b^3/x^12
```

maxima [B] time = 1.53, size = 241, normalized size = 2.51

$$\frac{1}{140}B\left(\frac{8\sqrt{cx^4+bx^2}\,c^3}{b^2x^2}-\frac{4\sqrt{cx^4+bx^2}\,c^2}{bx^4}+\frac{3\sqrt{cx^4+bx^2}\,c}{x^6}+\frac{15\sqrt{cx^4+bx^2}\,b}{x^8}-\frac{35\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{10}}\right)-\frac{1}{630}A\Bigg(16$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/140*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b
*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(c*
x^4 + b*x^2)^(3/2)/x^10) - 1/630*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) -
8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) - 5
*sqrt(c*x^4 + b*x^2)*c/x^8 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^4 + b
*x^2)^(3/2)/x^12)
```

mupad [B] time = 1.44, size = 206, normalized size = 2.15

$$\frac{4\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4}-\frac{10\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{8\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6}-\frac{A\,b\,\sqrt{c\,x^4}}{9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x)
```

```
[Out] (4*A*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*A*c*(b*x^2 + c*x^4)^(1/
2))/(63*x^8) - (B*b*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*B*c*(b*x^2 + c*x^4)
^(1/2))/(35*x^6) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (A*b*(b*x^2
+ c*x^4)^(1/2))/(9*x^10) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2) -
(B*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/
(35*b^2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**13,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**13, x)
```

3.116 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=133

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{3465b^4x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{693b^3x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{99b^2x^{14}}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

```
[Out] -1/11*A*(c*x^4+b*x^2)^(5/2)/b/x^16-1/99*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)
/b^2/x^14+4/693*c*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^12-8/3465*c^2*(
-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^10
```

Rubi [A] time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{3465b^4x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{693b^3x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}(11bB-6Ac)}{99b^2x^{14}}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(11*b*x^16) - ((11*b*B - 6*A*c)*(b*x^2 + c*x^4)^
(5/2))/(99*b^2*x^14) + (4*c*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*b^
3*x^12) - (8*c^2*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*b^4*x^10)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
```

```
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} + \frac{\left(-8(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^7}\,dx, x\right.}{11b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{99b^2x^{14}} - \frac{(2c(11bB-6Ac))\,\text{Subst}\left(\int\right.}{99b^2}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)\left(bx^2+cx^4\right.}{693b^3x^{12}}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\left(bx^2+cx^4\right)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)\left(bx^2+cx^4\right.}{693b^3x^{12}}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.67

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(3A\left(105b^3-70b^2cx^2+40bc^2x^4-16c^3x^6\right)+11bBx^2\left(35b^2-20bcx^2+8c^2x^4\right)\right)}{3465b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
[Out] -1/3465*((x^2*(b + c*x^2))^(5/2)*(11*b*B*x^2*(35*b^2 - 20*b*c*x^2 + 8*c^2*x
^4) + 3*A*(105*b^3 - 70*b^2*c*x^2 + 40*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^16)
```

fricas [A] time = 1.34, size = 134, normalized size = 1.01

$$-\frac{\left(8\left(11\,Bbc^4-6\,Ac^5\right)x^{10}-4\left(11\,Bb^2c^3-6\,Abc^4\right)x^8+3\left(11\,Bb^3c^2-6\,Ab^2c^3\right)x^6+315\,Ab^5+5\left(110\,Bb^4c+3\,Ab^3c\right.\right.}{3465\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
[Out] -1/3465*(8*(11*B*b*c^4 - 6*A*c^5)*x^10 - 4*(11*B*b^2*c^3 - 6*A*b*c^4)*x^8 +
 3*(11*B*b^3*c^2 - 6*A*b^2*c^3)*x^6 + 315*A*b^5 + 5*(110*B*b^4*c + 3*A*b^3*
c^2)*x^4 + 35*(11*B*b^5 + 12*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

giac [B] time = 3.23, size = 490, normalized size = 3.68

$$\frac{16\left(2310\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{16}Bc^{\frac{9}{2}}\text{sgn}(x)-1155\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}Bbc^{\frac{9}{2}}\text{sgn}(x)+6930\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}Ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + b))^16*B*c^(9/2)*sgn(x) - 1155*(sqr
t(c)*x - sqrt(c*x^2 + b))^14*B*b*c^(9/2)*sgn(x) + 6930*(sqrt(c)*x - sqrt(c*
x^2 + b))^14*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^2
*c^(9/2)*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b*c^(11/2)*sgn(x
) - 4851*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^3*c^(9/2)*sgn(x) + 15246*(sqr
t(c)*x - sqrt(c*x^2 + b))^10*A*b^2*c^(11/2)*sgn(x) + 2475*(sqrt(c)*x - sqrt
(c*x^2 + b))^8*B*b^4*c^(9/2)*sgn(x) + 4950*(sqrt(c)*x - sqrt(c*x^2 + b))^8*
A*b^3*c^(11/2)*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^5*c^(9/2)*s
gn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^4*c^(11/2)*sgn(x) + 605*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*B*b^6*c^(9/2)*sgn(x) - 330*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*A*b^5*c^(11/2)*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*
b^7*c^(9/2)*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^6*c^(11/2)*sgn(
x) + 11*B*b^8*c^(9/2)*sgn(x) - 6*A*b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + b))^2 - b)^11
```

maple [A] time = 0.05, size = 94, normalized size = 0.71

$$-\frac{\left(c\,x^2+b\right)\left(-48A\,c^3x^6+88Bb\,c^2x^6+120Ab\,c^2x^4-220B\,b^2c\,x^4-210A\,b^2c\,x^2+385B\,b^3x^2+315A\,b^3\right)\left(c\,x^4+\right.}{3465b^4x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x)
```

```
[Out] -1/3465*(c*x^2+b)*(-48*A*c^3*x^6+88*B*b*c^2*x^6+120*A*b*c^2*x^4-220*B*b^2*c
*x^4-210*A*b^2*c*x^2+385*B*b^3*x^2+315*A*b^3)*(c*x^4+b*x^2)^(3/2)/x^14/b^4
```

maxima [B] time = 1.58, size = 289, normalized size = 2.17

$$-\frac{1}{630}B\left(\frac{16\sqrt{cx^4+bx^2}\,c^4}{b^3x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^3}{b^2x^4}+\frac{6\sqrt{cx^4+bx^2}\,c^2}{bx^6}-\frac{5\sqrt{cx^4+bx^2}\,c}{x^8}-\frac{35\sqrt{cx^4+bx^2}\,b}{x^{10}}+\frac{105\left(cx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/630*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/
(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/x^8
 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^4 + b*x^2)^(3/2)/x^12) + 1/9240
*A*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/(b^3
*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^2)*c^2/(
b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b/x^12 - 1
155*(c*x^4 + b*x^2)^(3/2)/x^14)
```

mupad [B] time = 1.92, size = 256, normalized size = 1.92

$$\frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6}-\frac{4\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{10\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8}-\frac{A\,b\,\sqrt{c\,x^4}}{11\,x^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*A*c*(b*x^2 + c*x^4)^(1/2
))/(33*x^10) - (B*b*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (10*B*c*(b*x^2 + c*x^
4)^(1/2))/(63*x^8) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (A*b*(b*x^
2 + c*x^4)^(1/2))/(11*x^12) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4
) + (16*A*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2) - (B*c^2*(b*x^2 + c*x^4
)^(1/2))/(105*b*x^6) + (4*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (8*B
*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \left(A + Bx^2\right)}{x^{15}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**15, x)
```

3.117 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=170

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}(13bB-8Ac)}{15015b^5x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}(13bB-8Ac)}{3003b^4x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}(13bB-8Ac)}{429b^3x^{14}}-\frac{\left(bx^2+cx\right.}{}$$

```
[Out] -1/13*A*(c*x^4+b*x^2)^(5/2)/b/x^18-1/143*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2
)/b^2/x^16+2/429*c*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^14-8/3003*c^2*
(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^12+16/15015*c^3*(-8*A*c+13*B*b)*(
c*x^4+b*x^2)^(5/2)/b^5/x^10
```

Rubi [A] time = 0.32, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}(13bB-8Ac)}{15015b^5x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}(13bB-8Ac)}{3003b^4x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}(13bB-8Ac)}{429b^3x^{14}}-\frac{\left(bx^2+cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(13*b*x^18) - ((13*b*B - 8*A*c)*(b*x^2 + c*x^4)^
(5/2))/(143*b^2*x^16) + (2*c*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b
^3*x^14) - (8*c^2*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*b^4*x^12) +
 (16*c^3*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^10)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{17}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{\left(-9(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^8}\,dx, x\right.}{13b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{(3c(13bB-8Ac))\,\text{Subst}\left(\int\right.}{143b^2}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\left(bx^2+cx^4\right.}{429b^3x^{14}}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\left(bx^2+cx^4\right.}{429b^3x^{14}}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\left(bx^2+cx^4\right.}{429b^3x^{14}}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.56

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\left(\frac{cx^3}{b}+x\right)^2\left(105b^3-70b^2cx^2+40bc^2x^4-16c^3x^6\right)(8Ac-13bB)-1155Ab^2\left(b+cx^2\right)^2\right)}{15015b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-1155*A*b^2*(b + c*x^2)^2 + (-13*b*B + 8*A*c)*(x +
(c*x^3)/b)^2*(105*b^3 - 70*b^2*c*x^2 + 40*b*c^2*x^4 - 16*c^3*x^6)))/(15015*
b^3*x^14)
```

fricas [A] time = 1.24, size = 157, normalized size = 0.92

$$\frac{\left(16\left(13\,Bbc^5-8\,Ac^6\right)x^{12}-8\left(13\,Bb^2c^4-8\,Abc^5\right)x^{10}+6\left(13\,Bb^3c^3-8\,Ab^2c^4\right)x^8-1155\,Ab^6-5\left(13\,Bb^4c^2-8\,Ab^3\right.\right.}{15015\,b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] 1/15015*(16*(13*B*b*c^5 - 8*A*c^6)*x^12 - 8*(13*B*b^2*c^4 - 8*A*b*c^5)*x^10
 + 6*(13*B*b^3*c^3 - 8*A*b^2*c^4)*x^8 - 1155*A*b^6 - 5*(13*B*b^4*c^2 - 8*A*
b^3*c^3)*x^6 - 35*(52*B*b^5*c + A*b^4*c^2)*x^4 - 105*(13*B*b^6 + 14*A*b^5*c
)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

giac [B] time = 4.41, size = 550, normalized size = 3.24

$$32\left(15015\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{18}Bc^{\frac{11}{2}}\operatorname{sgn}(x)-3003\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{16}Bbc^{\frac{11}{2}}\operatorname{sgn}(x)+48048\left(\sqrt{c}\,x-\sqrt{cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + b))^18*B*c^(11/2)*sgn(x) - 3003*(
sqrt(c)*x - sqrt(c*x^2 + b))^16*B*b*c^(11/2)*sgn(x) + 48048*(sqrt(c)*x - sq
rt(c*x^2 + b))^16*A*c^(13/2)*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + b))^14
*B*b^2*c^(11/2)*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b*c^(13/2
)*sgn(x) - 28314*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^3*c^(11/2)*sgn(x) + 1
09824*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^2*c^(13/2)*sgn(x) + 13728*(sqrt(
c)*x - sqrt(c*x^2 + b))^10*B*b^4*c^(11/2)*sgn(x) + 37752*(sqrt(c)*x - sqrt(
c*x^2 + b))^10*A*b^3*c^(13/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8
*B*b^5*c^(11/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^4*c^(13/2
)*sgn(x) + 3718*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^6*c^(11/2)*sgn(x) - 228
8*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^5*c^(13/2)*sgn(x) - 1014*(sqrt(c)*x -
 sqrt(c*x^2 + b))^4*B*b^7*c^(11/2)*sgn(x) + 624*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*A*b^6*c^(13/2)*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^8*c^(1
1/2)*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^7*c^(13/2)*sgn(x) - 1
3*B*b^9*c^(11/2)*sgn(x) + 8*A*b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + b))^2 - b)^13
```

maple [A] time = 0.04, size = 118, normalized size = 0.69

$$-\frac{\left(c\,x^2+b\right)\left(128A\,c^4x^8-208Bb\,c^3x^8-320Ab\,c^3x^6+520B\,b^2c^2x^6+560A\,b^2c^2x^4-910B\,b^3c\,x^4-840A\,b^3c\,x^2\right.}{15015b^5x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x)
```

```
[Out] -1/15015*(c*x^2+b)*(128*A*c^4*x^8-208*B*b*c^3*x^8-320*A*b*c^3*x^6+520*B*b^2
*c^2*x^6+560*A*b^2*c^2*x^4-910*B*b^3*c*x^4-840*A*b^3*c*x^2+1365*B*b^4*x^2+1
155*A*b^4)*(c*x^4+b*x^2)^(3/2)/b^5/x^16
```

maxima [B] time = 1.62, size = 337, normalized size = 1.98

$$\frac{1}{9240}B\left(\frac{128\sqrt{cx^4+bx^2}\,c^5}{b^4x^2}-\frac{64\sqrt{cx^4+bx^2}\,c^4}{b^3x^4}+\frac{48\sqrt{cx^4+bx^2}\,c^3}{b^2x^6}-\frac{40\sqrt{cx^4+bx^2}\,c^2}{bx^8}+\frac{35\sqrt{cx^4+bx^2}\,c}{x^{10}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] 1/9240*B*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^
4/(b^3*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^2)
*c^2/(b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b/x^
12 - 1155*(c*x^4 + b*x^2)^(3/2)/x^14) - 1/30030*A*(256*sqrt(c*x^4 + b*x^2)*
c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^4) + 96*sqrt(c*x^4 + b*x
^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^8) + 70*sqrt(c*x^4 +
b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4 + b*x^2)*c/x^12 - 693*sqrt(c*x^4 + b*x^
2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3/2)/x^16)
```

mupad [B] time = 2.50, size = 306, normalized size = 1.80

$$\frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8}-\frac{14\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{4\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}}-\frac{A\,b\,\sqrt{c\,x^4+b}}{13\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x)
```

```
[Out] (8*A*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*A*c*(b*x^2 + c*x^4)^(1
/2))/(143*x^12) - (B*b*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (4*B*c*(b*x^2 + c
*x^4)^(1/2))/(33*x^10) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (A*b*
(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(5005*b
^3*x^6) + (64*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*A*c^6*(b*
x^2 + c*x^4)^(1/2))/(15015*b^5*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*
x^8) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (8*B*c^4*(b*x^2 + c*
x^4)^(1/2))/(1155*b^3*x^4) + (16*B*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**17, x)
```

3.118 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{19}}\,dx$

Optimal. Leaf size=207

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{45045b^6x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{9009b^5x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{1287b^4x^{14}}+\frac{8c\left(bx\right.}{}$$

```
[Out] -1/15*A*(c*x^4+b*x^2)^(5/2)/b/x^20-1/39*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/
b^2/x^18+8/429*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^16-16/1287*c^2*(-
2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^14+64/9009*c^3*(-2*A*c+3*B*b)*(c*x^4
+b*x^2)^(5/2)/b^5/x^12-128/45045*c^4*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^6
/x^10
```

Rubi [A] time = 0.35, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{45045b^6x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{9009b^5x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}(3bB-2Ac)}{1287b^4x^{14}}+\frac{8c\left(bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] -(A*(b*x^2 + c*x^4)^(5/2))/(15*b*x^20) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(
5/2))/(39*b^2*x^18) + (8*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b^3*
x^16) - (16*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(1287*b^4*x^14) + (6
4*c^3*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(9009*b^5*x^12) - (128*c^4*(3*
b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(45045*b^6*x^10)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{10}}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} + \frac{\left(-10(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^9}\,dx,\right.}{15b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} - \frac{(4c(3bB-2Ac))\operatorname{Subst}\left(\int \right.}{39b^2}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/}}{429b^3x^{16}}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/}}{429b^3x^{16}}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/}}{429b^3x^{16}}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/}}{429b^3x^{16}}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.43

$$\frac{\left(x^2(b+cx^2)\right)^{5/2}\left(-3003Ab^5-x^2\left(1155b^4-840b^3cx^2+560b^2c^2x^4-320bc^3x^6+128c^4x^8\right)(3bB-2Ac)\right)}{45045b^6x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-3003*A*b^5 - (3*b*B - 2*A*c)*x^2*(1155*b^4 - 840
*b^3*c*x^2 + 560*b^2*c^2*x^4 - 320*b*c^3*x^6 + 128*c^4*x^8)))/(45045*b^6*x^
20)
```

fricas [A] time = 1.87, size = 181, normalized size = 0.87

$$-\frac{\left(128\left(3\,Bbc^6-2\,Ac^7\right)x^{14}-64\left(3\,Bb^2c^5-2\,Abc^6\right)x^{12}+48\left(3\,Bb^3c^4-2\,Ab^2c^5\right)x^{10}-40\left(3\,Bb^4c^3-2\,Ab^3c^4\right)x^8+\right.}{45045\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/45045*(128*(3*B*b*c^6 - 2*A*c^7)*x^14 - 64*(3*B*b^2*c^5 - 2*A*b*c^6)*x^1
2 + 48*(3*B*b^3*c^4 - 2*A*b^2*c^5)*x^10 - 40*(3*B*b^4*c^3 - 2*A*b^3*c^4)*x^
```

```
8 + 3003*A*b^7 + 35*(3*B*b^5*c^2 - 2*A*b^4*c^3)*x^6 + 63*(70*B*b^6*c + A*b^
5*c^2)*x^4 + 231*(15*B*b^7 + 16*A*b^6*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^6*x^16
)
```

giac [B] time = 5.43, size = 582, normalized size = 2.81

$$\frac{256\left(18018\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{20}Bc^{\frac{13}{2}}\operatorname{sgn}(x)+60060\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{18}Ac^{\frac{15}{2}}\operatorname{sgn}(x)-12870\left(\sqrt{c}\,x-\sqrt{cx^2+\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] 256/45045*(18018*(sqrt(c)*x - sqrt(c*x^2 + b))^20*B*c^(13/2)*sgn(x) + 60060
*(sqrt(c)*x - sqrt(c*x^2 + b))^18*A*c^(15/2)*sgn(x) - 12870*(sqrt(c)*x - sq
rt(c*x^2 + b))^16*B*b^2*c^(13/2)*sgn(x) + 128700*(sqrt(c)*x - sqrt(c*x^2 +
b))^16*A*b*c^(15/2)*sgn(x) - 32175*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*b^3*c
^(13/2)*sgn(x) + 141570*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b^2*c^(15/2)*sgn
(x) + 15015*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^4*c^(13/2)*sgn(x) + 50050*
(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^3*c^(15/2)*sgn(x) + 9009*(sqrt(c)*x -
sqrt(c*x^2 + b))^10*B*b^5*c^(13/2)*sgn(x) + 6006*(sqrt(c)*x - sqrt(c*x^2 +
b))^10*A*b^4*c^(15/2)*sgn(x) + 4095*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^6*c
^(13/2)*sgn(x) - 2730*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^5*c^(15/2)*sgn(x)
 - 1365*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^7*c^(13/2)*sgn(x) + 910*(sqrt(c
)*x - sqrt(c*x^2 + b))^6*A*b^6*c^(15/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^
2 + b))^4*B*b^8*c^(13/2)*sgn(x) - 210*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^7
*c^(15/2)*sgn(x) - 45*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^9*c^(13/2)*sgn(x)
 + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^8*c^(15/2)*sgn(x) + 3*B*b^10*c^(1
3/2)*sgn(x) - 2*A*b^9*c^(15/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)^15
```

maple [A] time = 0.05, size = 142, normalized size = 0.69

$$-\frac{\left(c\,x^2+b\right)\left(-256A\,c^5x^{10}+384Bb\,c^4x^{10}+640Ab\,c^4x^8-960B\,b^2c^3x^8-1120A\,b^2c^3x^6+1680B\,b^3c^2x^6+1680\cdots\right.}{45045b^6x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x)
```

```
[Out] -1/45045*(c*x^2+b)*(-256*A*c^5*x^10+384*B*b*c^4*x^10+640*A*b*c^4*x^8-960*B*
b^2*c^3*x^8-1120*A*b^2*c^3*x^6+1680*B*b^3*c^2*x^6+1680*A*b^3*c^2*x^4-2520*B
*b^4*c*x^4-2310*A*b^4*c*x^2+3465*B*b^5*x^2+3003*A*b^5)*(c*x^4+b*x^2)^(3/2)/
x^18/b^6
```

maxima [B] time = 1.65, size = 385, normalized size = 1.86

$$-\frac{1}{30030}B\left(\frac{256\sqrt{cx^4+bx^2}\,c^6}{b^5x^2}-\frac{128\sqrt{cx^4+bx^2}\,c^5}{b^4x^4}+\frac{96\sqrt{cx^4+bx^2}\,c^4}{b^3x^6}-\frac{80\sqrt{cx^4+bx^2}\,c^3}{b^2x^8}+\frac{70\sqrt{cx^4+bx^2}\,c^2}{bx^{10}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] -1/30030*B*(256*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2)
*c^5/(b^4*x^4) + 96*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b*x
^2)*c^3/(b^2*x^8) + 70*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4 + b
```

```
*x^2)*c/x^12 - 693*sqrt(c*x^4 + b*x^2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3/2)/
x^16) + 1/180180*A*(1024*sqrt(c*x^4 + b*x^2)*c^7/(b^6*x^2) - 512*sqrt(c*x^4
 + b*x^2)*c^6/(b^5*x^4) + 384*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^6) - 320*sqrt(
c*x^4 + b*x^2)*c^4/(b^3*x^8) + 280*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^10) - 252
*sqrt(c*x^4 + b*x^2)*c^2/(b*x^12) + 231*sqrt(c*x^4 + b*x^2)*c/x^14 + 3003*s
qrt(c*x^4 + b*x^2)*b/x^16 - 15015*(c*x^4 + b*x^2)^(3/2)/x^18)
```

mupad [B] time = 3.21, size = 356, normalized size = 1.72

$$\frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1287\,b^2\,x^{10}}-\frac{16\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{195\,x^{14}}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}}-\frac{14\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{715\,b\,x^{12}}-\frac{A\,b\,\sqrt{c\,x^4+}}{15\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(1287*b^2*x^10) - (16*A*c*(b*x^2 + c*x^4)^(
1/2))/(195*x^14) - (B*b*(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (14*B*c*(b*x^2 +
 c*x^4)^(1/2))/(143*x^12) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(715*b*x^12) - (A
*b*(b*x^2 + c*x^4)^(1/2))/(15*x^16) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(900
9*b^3*x^8) + (32*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^6) - (128*A*c^6*
(b*x^2 + c*x^4)^(1/2))/(45045*b^5*x^4) + (256*A*c^7*(b*x^2 + c*x^4)^(1/2))/
(45045*b^6*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) + (8*B*c^3*(b*
x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (16*B*c^4*(b*x^2 + c*x^4)^(1/2))/(5005
*b^3*x^6) + (64*B*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*B*c^6*(
b*x^2 + c*x^4)^(1/2))/(15015*b^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**19,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**19, x)
```

3.119 $\int x^4 \left(A+Bx^2\right) \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=168

$$\frac{16b^3 \left(bx^2+cx^4\right)^{5/2} (8bB-13Ac)}{15015c^5x^5}-\frac{8b^2 \left(bx^2+cx^4\right)^{5/2} (8bB-13Ac)}{3003c^4x^3}+\frac{2b \left(bx^2+cx^4\right)^{5/2} (8bB-13Ac)}{429c^3x}-\frac{x \left(bx^2+\right.}{}$$

```
[Out] 16/15015*b^3*(-13*A*c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^5/x^5-8/3003*b^2*(-13*A*
c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^3+2/429*b*(-13*A*c+8*B*b)*(c*x^4+b*x^2)^
(5/2)/c^3/x-1/143*(-13*A*c+8*B*b)*x*(c*x^4+b*x^2)^(5/2)/c^2+1/13*B*x^3*(c*x
^4+b*x^2)^(5/2)/c
```

Rubi [A] time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2016, 2002, 2014}

$$\frac{16b^3 \left(bx^2+cx^4\right)^{5/2} (8bB-13Ac)}{15015c^5x^5}-\frac{8b^2 \left(bx^2+cx^4\right)^{5/2} (8bB-13Ac)}{3003c^4x^3}+\frac{2b \left(bx^2+cx^4\right)^{5/2} (8bB-13Ac)}{429c^3x}-\frac{x \left(bx^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (16*b^3*(8*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (8*b^2*(8
*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*c^4*x^3) + (2*b*(8*b*B - 13*A*c
)*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - ((8*b*B - 13*A*c)*x*(b*x^2 + c*x^4)^
(5/2))/(143*c^2) + (B*x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
```

```
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^4\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}dx=\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\int x^4\left(bx^2+cx^4\right)^{3/2}dx}{13c}$$
$$=-\frac{(8bB-13Ac)x\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}+\frac{(6b(8bB-13Ac))\int x}{143}$$
$$=\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-13Ac)x\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{Bx^3\left(bx^2+\right.}{13c}$$
$$=-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-}{}$$
$$=\frac{16b^3(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8b}{}$$

Mathematica [A] time = 0.09, size = 113, normalized size = 0.67

$$\frac{x\left(b+cx^2\right)^3\left(-16b^3c\left(13A+20Bx^2\right)+40b^2c^2x^2\left(13A+14Bx^2\right)-70bc^3x^4\left(13A+12Bx^2\right)+105c^4x^6\left(13A+11B\right.\right.}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(128*b^4*B + 105*c^4*x^6*(13*A + 11*B*x^2) - 70*b*c^3*x^4*
(13*A + 12*B*x^2) + 40*b^2*c^2*x^2*(13*A + 14*B*x^2) - 16*b^3*c*(13*A + 20*
B*x^2)))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.87, size = 154, normalized size = 0.92

$$\frac{\left(1155\,Bc^6x^{12}+105\left(14\,Bbc^5+13\,Ac^6\right)x^{10}+35\left(Bb^2c^4+52\,Abc^5\right)x^8+128\,Bb^6-208\,Ab^5c-5\left(8\,Bb^3c^3-13\,Ab^2\right.\right.}{15015\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^6*x^12 + 105*(14*B*b*c^5 + 13*A*c^6)*x^10 + 35*(B*b^2*c^4
 + 52*A*b*c^5)*x^8 + 128*B*b^6 - 208*A*b^5*c - 5*(8*B*b^3*c^3 - 13*A*b^2*c^
4)*x^6 + 6*(8*B*b^4*c^2 - 13*A*b^3*c^3)*x^4 - 8*(8*B*b^5*c - 13*A*b^4*c^2)*
x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

giac [A] time = 0.34, size = 175, normalized size = 1.04

$$-\frac{16\left(8\,Bb^{\frac{13}{2}}-13\,Ab^{\frac{11}{2}}c\right)\mathrm{sgn}(x)}{15015\,c^5}+\frac{1155\left(cx^2+b\right)^{\frac{13}{2}}B\mathrm{sgn}(x)-5460\left(cx^2+b\right)^{\frac{11}{2}}Bb\mathrm{sgn}(x)+10010\left(cx^2+b\right)^{\frac{9}{2}}Bb^2\mathrm{sg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15015*(8*B*b^(13/2) - 13*A*b^(11/2)*c)*sgn(x)/c^5 + 1/15015*(1155*(c*x^
2 + b)^(13/2)*B*sgn(x) - 5460*(c*x^2 + b)^(11/2)*B*b*sgn(x) + 10010*(c*x^2
```

```
+ b)^(9/2)*B*b^2*sgn(x) - 8580*(c*x^2 + b)^(7/2)*B*b^3*sgn(x) + 3003*(c*x^2
 + b)^(5/2)*B*b^4*sgn(x) + 1365*(c*x^2 + b)^(11/2)*A*c*sgn(x) - 5005*(c*x^2
 + b)^(9/2)*A*b*c*sgn(x) + 6435*(c*x^2 + b)^(7/2)*A*b^2*c*sgn(x) - 3003*(c*
x^2 + b)^(5/2)*A*b^3*c*sgn(x))/c^5
```

maple [A] time = 0.05, size = 115, normalized size = 0.68

$$-\frac{\left(c\,x^2+b\right)\left(-1155B\,x^8c^4-1365A\,c^4x^6+840Bb\,c^3x^6+910Ab\,c^3x^4-560B\,b^2c^2x^4-520A\,b^2c^2x^2+320B\,b^3c\,x\right.}{15015c^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/15015*(c*x^2+b)*(-1155*B*c^4*x^8-1365*A*c^4*x^6+840*B*b*c^3*x^6+910*A*b*
c^3*x^4-560*B*b^2*c^2*x^4-520*A*b^2*c^2*x^2+320*B*b^3*c*x^2+208*A*b^3*c-128
*B*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

maxima [A] time = 1.55, size = 150, normalized size = 0.89

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^2+b}\,A}{1155\,c^4}+\frac{\left(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*A/c^4 + 1/15015*(1155*c^6*x^12 + 1470*b*c
^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 +
 128*b^6)*sqrt(c*x^2 + b)*B/c^5
```

mupad [B] time = 0.35, size = 143, normalized size = 0.85

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{128\,B\,b^6-208\,A\,b^5\,c}{15015\,c^5}+\frac{x^{10}\,\left(1365\,A\,c^6+1470\,B\,b\,c^5\right)}{15015\,c^5}+\frac{B\,c\,x^{12}}{13}+\frac{b^2\,x^6\,\left(13\,A\,c-8\,B\,b\right)}{3003\,c^2}-\frac{2\,b^3\,x^4\,\left(13\,A\,c-8\,B\,b\right)}{5005\,c^3}+\frac{8\,b^4\,x^2\,\left(13\,A\right.}{1501}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((128*B*b^6 - 208*A*b^5*c)/(15015*c^5) + (x^10*(1365
*A*c^6 + 1470*B*b*c^5))/(15015*c^5) + (B*c*x^12)/13 + (b^2*x^6*(13*A*c - 8*
B*b))/(3003*c^2) - (2*b^3*x^4*(13*A*c - 8*B*b))/(5005*c^3) + (8*b^4*x^2*(13
*A*c - 8*B*b))/(15015*c^4) + (b*x^8*(52*A*c + B*b))/(429*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.120 $\int x^2 (A + Bx^2) (bx^2 + cx^4)^{3/2} dx$

Optimal. Leaf size=131

$$-\frac{8b^2 (bx^2+cx^4)^{5/2} (6bB-11Ac)}{3465c^4x^5}+\frac{4b (bx^2+cx^4)^{5/2} (6bB-11Ac)}{693c^3x^3}-\frac{(bx^2+cx^4)^{5/2} (6bB-11Ac)}{99c^2x}+\frac{Bx (bx^2+cx^4)^{5/2}}{11c}$$

```
[Out] -8/3465*b^2*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^5+4/693*b*(-11*A*c+6*
B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^3-1/99*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)/c
^2/x+1/11*B*x*(c*x^4+b*x^2)^(5/2)/c
```

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2016, 2002, 2014}

$$-\frac{8b^2 (bx^2+cx^4)^{5/2} (6bB-11Ac)}{3465c^4x^5}-\frac{(bx^2+cx^4)^{5/2} (6bB-11Ac)}{99c^2x}+\frac{4b (bx^2+cx^4)^{5/2} (6bB-11Ac)}{693c^3x^3}+\frac{Bx (bx^2+cx^4)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^2*(6*b*B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*c^4*x^5) + (4*b*(6*b*
B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*c^3*x^3) - ((6*b*B - 11*A*c)*(b*x^2
 + c*x^4)^(5/2))/(99*c^2*x) + (B*x*(b*x^2 + c*x^4)^(5/2))/(11*c)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\int x^2\left(bx^2+cx^4\right)^{3/2}\,dx}{11c}$$
$$=-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}+\frac{(4b(6bB-11Ac))\int}{99c^2}$$
$$=\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx\left(bx^2-\right.}{11}$$
$$=-\frac{8b^2(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{3465c^4x^5}+\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bI}{}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.70

$$\frac{x\left(b+cx^2\right)^3\left(8b^2c\left(11A+15Bx^2\right)-10bc^2x^2\left(22A+21Bx^2\right)+35c^3x^4\left(11A+9Bx^2\right)-48b^3B\right)}{3465c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(b + c*x^2)^3*(-48*b^3*B + 35*c^3*x^4*(11*A + 9*B*x^2) + 8*b^2*c*(11*A +
 15*B*x^2) - 10*b*c^2*x^2*(22*A + 21*B*x^2)))/(3465*c^4*Sqrt[x^2*(b + c*x^2
)])
```

fricas [A] time = 0.93, size = 131, normalized size = 1.00

$$\frac{\left(315\,Bc^5x^{10}+35\left(12\,Bbc^4+11\,Ac^5\right)x^8+5\left(3\,Bb^2c^3+110\,Abc^4\right)x^6-48\,Bb^5+88\,Ab^4c-3\left(6\,Bb^3c^2-11\,Ab^2c^3\right.\right.}{3465\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/3465*(315*B*c^5*x^10 + 35*(12*B*b*c^4 + 11*A*c^5)*x^8 + 5*(3*B*b^2*c^3 +
110*A*b*c^4)*x^6 - 48*B*b^5 + 88*A*b^4*c - 3*(6*B*b^3*c^2 - 11*A*b^2*c^3)*x
^4 + 4*(6*B*b^4*c - 11*A*b^3*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

giac [A] time = 0.21, size = 140, normalized size = 1.07

$$\frac{8\left(6\,Bb^{\frac{11}{2}}-11\,Ab^{\frac{9}{2}}c\right)\mathrm{sgn}(x)}{3465\,c^4}+\frac{315\left(cx^2+b\right)^{\frac{11}{2}}B\mathrm{sgn}(x)-1155\left(cx^2+b\right)^{\frac{9}{2}}Bb\mathrm{sgn}(x)+1485\left(cx^2+b\right)^{\frac{7}{2}}Bb^2\mathrm{sgn}(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] 8/3465*(6*B*b^(11/2) - 11*A*b^(9/2)*c)*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + b)
^(11/2)*B*sgn(x) - 1155*(c*x^2 + b)^(9/2)*B*b*sgn(x) + 1485*(c*x^2 + b)^(7/
2)*B*b^2*sgn(x) - 693*(c*x^2 + b)^(5/2)*B*b^3*sgn(x) + 385*(c*x^2 + b)^(9/2
)*A*c*sgn(x) - 990*(c*x^2 + b)^(7/2)*A*b*c*sgn(x) + 693*(c*x^2 + b)^(5/2)*A
*b^2*c*sgn(x))/c^4
```

maple [A] time = 0.05, size = 91, normalized size = 0.69

$$\frac{\left(c\,x^2+b\right)\left(315B\,c^3x^6+385A\,c^3x^4-210Bb\,c^2x^4-220Ab\,c^2x^2+120B\,b^2c\,x^2+88A\,b^2c-48B\,b^3\right)\left(c\,x^4+b\,x^2\right)}{3465c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/3465*(c*x^2+b)*(315*B*c^3*x^6+385*A*c^3*x^4-210*B*b*c^2*x^4-220*A*b*c^2*x
^2+120*B*b^2*c*x^2+88*A*b^2*c-48*B*b^3)*(c*x^4+b*x^2)^(3/2)/c^4/x^3
```

maxima [A] time = 1.53, size = 128, normalized size = 0.98

$$\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^2+b}\,A}{315\,c^3}+\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4c\right.}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*A/c^3 + 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 -
 6*b^3*c^2*x^4 + 8*b^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*B/c^4
```

mupad [B] time = 0.29, size = 124, normalized size = 0.95

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{x^8\,\left(385\,A\,c^5+420\,B\,b\,c^4\right)}{3465\,c^4}-\frac{48\,B\,b^5-88\,A\,b^4\,c}{3465\,c^4}+\frac{B\,c\,x^{10}}{11}+\frac{b^2\,x^4\,\left(11\,A\,c-6\,B\,b\right)}{1155\,c^2}-\frac{4\,b^3\,x^2\,\left(11\,A\,c-6\,B\,b\right)}{3465\,c^3}+\frac{b\,x^6\,\left(110\,A\,c+3\,B\,b\right)}{693\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((x^8*(385*A*c^5 + 420*B*b*c^4))/(3465*c^4) - (48*B*
b^5 - 88*A*b^4*c)/(3465*c^4) + (B*c*x^10)/11 + (b^2*x^4*(11*A*c - 6*B*b))/(
1155*c^2) - (4*b^3*x^2*(11*A*c - 6*B*b))/(3465*c^3) + (b*x^6*(110*A*c + 3*B
*b))/(693*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.121 $\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{2b\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{315c^3x^5}-\frac{\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{63c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

```
[Out] 2/315*b*(-9*A*c+4*B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^5-1/63*(-9*A*c+4*B*b)*(c*x
^4+b*x^2)^(5/2)/c^2/x^3+1/9*B*(c*x^4+b*x^2)^(5/2)/c/x
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1145, 2002, 2014}

$$-\frac{\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{63c^2x^3}+\frac{2b\left(bx^2+cx^4\right)^{5/2}(4bB-9Ac)}{315c^3x^5}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*b*(4*b*B - 9*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - ((4*b*B - 9*A*c
)*(b*x^2 + c*x^4)^(5/2))/(63*c^2*x^3) + (B*(b*x^2 + c*x^4)^(5/2))/(9*c*x)
```

Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{(4bB-9Ac)\int \left(bx^2+cx^4\right)^{3/2}\,dx}{9c}\\ &= -\frac{(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}+\frac{(2b(4bB-9Ac))\int \frac{\left(bx^2+cx^4\right)^{5/2}}{x^2}}{63c^2}\\ &= \frac{2b(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.74

$$\frac{x\left(b+cx^2\right)^3\left(-2bc\left(9A+10Bx^2\right)+5c^2x^2\left(9A+7Bx^2\right)+8b^2B\right)}{315c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(8*b^2*B + 5*c^2*x^2*(9*A + 7*B*x^2) - 2*b*c*(9*A + 10*B*x
^2)))/(315*c^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.06, size = 106, normalized size = 1.10

$$\frac{\left(35\,Bc^4x^8+5\left(10\,Bbc^3+9\,Ac^4\right)x^6+8\,Bb^4-18\,Ab^3c+3\left(Bb^2c^2+24\,Abc^3\right)x^4-\left(4\,Bb^3c-9\,Ab^2c^2\right)x^2\right)\sqrt{cx^4+bx}}{315\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*c^4*x^8 + 5*(10*B*b*c^3 + 9*A*c^4)*x^6 + 8*B*b^4 - 18*A*b^3*c +
 3*(B*b^2*c^2 + 24*A*b*c^3)*x^4 - (4*B*b^3*c - 9*A*b^2*c^2)*x^2)*sqrt(c*x^4
 + b*x^2)/(c^3*x)
```

giac [A] time = 0.17, size = 105, normalized size = 1.09

$$-\frac{2\left(4\,Bb^{\frac{9}{2}}-9\,Ab^{\frac{7}{2}}c\right)\mathrm{sgn}(x)}{315\,c^3}+\frac{35\left(cx^2+b\right)^{\frac{9}{2}}B\mathrm{sgn}(x)-90\left(cx^2+b\right)^{\frac{7}{2}}Bb\mathrm{sgn}(x)+63\left(cx^2+b\right)^{\frac{5}{2}}Bb^2\mathrm{sgn}(x)+45\left(cx\right.}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/315*(4*B*b^(9/2) - 9*A*b^(7/2)*c)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9/
2)*B*sgn(x) - 90*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*B*b^2*
sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*sgn(x) - 63*(c*x^2 + b)^(5/2)*A*b*c*sgn(x
))/c^3
```

maple [A] time = 0.05, size = 67, normalized size = 0.70

$$-\frac{\left(c\,x^2+b\right)\left(-35B\,c^2x^4-45A\,c^2x^2+20Bbc\,x^2+18Abc-8B\,b^2\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{315c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/315*(c*x^2+b)*(-35*B*c^2*x^4-45*A*c^2*x^2+20*B*b*c*x^2+18*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

maxima [A] time = 1.56, size = 105, normalized size = 1.09

$$\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^2+b}\,A}{35\,c^2}+\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^2+b}\,B}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*A/c^2 +
1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*B/c^3
```

mupad [B] time = 0.26, size = 103, normalized size = 1.07

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^4-18\,A\,b^3\,c}{315\,c^3}+\frac{x^6\,\left(45\,A\,c^4+50\,B\,b\,c^3\right)}{315\,c^3}+\frac{B\,c\,x^8}{9}+\frac{b^2\,x^2\,(9\,A\,c-4\,B\,b)}{315\,c^2}+\frac{b\,x^4\,(24\,A\,c+B\,b)}{105\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((8*B*b^4 - 18*A*b^3*c)/(315*c^3) + (x^6*(45*A*c^4 +
 50*B*b*c^3))/(315*c^3) + (B*c*x^8)/9 + (b^2*x^2*(9*A*c - 4*B*b))/(315*c^2)
 + (b*x^4*(24*A*c + B*b))/(105*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.122 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=61

$$\frac{B(bx^2+cx^4)^{5/2}}{7cx^3}-\frac{(bx^2+cx^4)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

```
[Out] -1/35*(-7*A*c+2*B*b)*(c*x^4+b*x^2)^(5/2)/c^2/x^5+1/7*B*(c*x^4+b*x^2)^(5/2)/
c/x^3
```

Rubi [A] time = 0.16, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2039, 2014}

$$\frac{B(bx^2+cx^4)^{5/2}}{7cx^3}-\frac{(bx^2+cx^4)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

```
[Out] -((2*b*B - 7*A*c)*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (B*(b*x^2 + c*x^4)^
(5/2))/(7*c*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} \, dx &= \frac{B(bx^2+cx^4)^{5/2}}{7cx^3}-\frac{(2bB-7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^2} \, dx}{7c}\\ &= -\frac{(2bB-7Ac)(bx^2+cx^4)^{5/2}}{35c^2x^5}+\frac{B(bx^2+cx^4)^{5/2}}{7cx^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.79

$$\frac{x(b+cx^2)^3(7Ac-2bB+5Bcx^2)}{35c^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

```
[Out] (x*(b + c*x^2)^3*(-2*b*B + 7*A*c + 5*B*c*x^2))/(35*c^2*Sqrt[x^2*(b + c*x^2)
])
```

fricas [A] time = 0.91, size = 80, normalized size = 1.31

$$\frac{\left(5\,Bc^3x^6 + \left(8\,Bbc^2 + 7\,Ac^3\right)x^4 - 2\,Bb^3 + 7\,Ab^2c + \left(Bb^2c + 14\,Abc^2\right)x^2\right)\sqrt{cx^4 + bx^2}}{35\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*c^3*x^6 + (8*B*b*c^2 + 7*A*c^3)*x^4 - 2*B*b^3 + 7*A*b^2*c + (B*b^
2*c + 14*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

giac [A] time = 0.21, size = 72, normalized size = 1.18

$$\frac{\left(2\,Bb^{\frac{7}{2}} - 7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{35\,c^2} + \frac{5\left(cx^2 + b\right)^{\frac{7}{2}}B\mathrm{sgn}(x) - 7\left(cx^2 + b\right)^{\frac{5}{2}}Bb\mathrm{sgn}(x) + 7\left(cx^2 + b\right)^{\frac{5}{2}}Ac\mathrm{sgn}(x)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/35*(2*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*B
*sgn(x) - 7*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 7*(c*x^2 + b)^(5/2)*A*c*sgn(x))/
c^2
```

maple [A] time = 0.05, size = 45, normalized size = 0.74

$$\frac{\left(c\,x^2 + b\right)\left(5Bc\,x^2 + 7Ac - 2bB\right)\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}}{35c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x)
```

```
[Out] 1/35*(c*x^2+b)*(5*B*c*x^2+7*A*c-2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

maxima [A] time = 1.55, size = 80, normalized size = 1.31

$$\frac{\left(c^2x^4 + 2\,bcx^2 + b^2\right)\sqrt{cx^2 + b}\,A}{5\,c} + \frac{\left(5\,c^3x^6 + 8\,bc^2x^4 + b^2cx^2 - 2\,b^3\right)\sqrt{cx^2 + b}\,B}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)*A/c + 1/35*(5*c^3*x^6 + 8*b
*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*B/c^2
```

mupad [B] time = 0.23, size = 83, normalized size = 1.36

$$\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(\frac{x^4\,\left(7\,A\,c^3+8\,B\,b\,c^2\right)}{35\,c^2} - \frac{2\,B\,b^3-7\,A\,b^2\,c}{35\,c^2} + \frac{B\,c\,x^6}{7} + \frac{b\,x^2\,\left(14\,A\,c+B\,b\right)}{35\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((x^4*(7*A*c^3 + 8*B*b*c^2))/(35*c^2) - (2*B*b^3 - 7
*A*b^2*c)/(35*c^2) + (B*c*x^6)/7 + (b*x^2*(14*A*c + B*b))/(35*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**2, x)
```

3.123 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=102

$$-Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

```
[Out] 1/3*A*(c*x^4+b*x^2)^(3/2)/x^3+1/5*B*(c*x^4+b*x^2)^(5/2)/c/x^5-A*b^(3/2)*arc
tanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))+A*b*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] time = 0.21, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2039, 2021, 2008, 206}

$$-Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3}+\frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

```
[Out] (A*b*Sqrt[b*x^2 + c*x^4])/x + (A*(b*x^2 + c*x^4)^(3/2))/(3*x^3) + (B*(b*x^2
 + c*x^4)^(5/2))/(5*c*x^5) - A*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^4}\,dx = \frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5} + A\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}\,dx$$
$$= \frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5} + (Ab)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$$
$$= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5} + \left(Ab^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}$$
$$= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5} - \left(Ab^2\right)\text{Subst}\left(\int \frac{1}{1-b\ldots}\right.$$
$$= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A\left(bx^2+cx^4\right)^{3/2}}{3x^3} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{5cx^5} - Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\ldots}{\sqrt{bx^2+\ldots}}\right.$$

Mathematica [A] time = 0.10, size = 109, normalized size = 1.07

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(-15Ab^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+5Ac\left(b+cx^2\right)^{3/2}+15Abc\sqrt{b+cx^2}+3B\left(b+cx^2\right)^{5/2}\right)}{15cx^3\left(b+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(15*A*b*c*Sqrt[b + c*x^2] + 5*A*c*(b + c*x^2)^(3/2
) + 3*B*(b + c*x^2)^(5/2) - 15*A*b^(3/2)*c*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]
))/(15*c*x^3*(b + c*x^2)^(3/2))
```

fricas [A] time = 0.87, size = 206, normalized size = 2.02

$$\left[\frac{15\,Ab^{\frac{3}{2}}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(3\,Bc^2x^4+3\,Bb^2+20\,Abc+\left(6\,Bbc+5\,Ac^2\right)x^2\right)\sqrt{cx^4+bx^2}}{30\,cx}, \frac{15\,A\sqrt{-\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*A*b^(3/2)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b)
)/x^3) + 2*(3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c + 5*A*c^2)*x^2)*sqr
t(c*x^4 + b*x^2))/(c*x), 1/15*(15*A*sqrt(-b)*b*c*x*arctan(sqrt(c*x^4 + b*x^
2)*sqrt(-b)/(c*x^3 + b*x)) + (3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c +
 5*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x)]
```

giac [A] time = 0.18, size = 140, normalized size = 1.37

$$\frac{Ab^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}} - \frac{\left(15\,Ab^2c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,B\sqrt{-b}\,b^{\frac{5}{2}}+20\,A\sqrt{-b}\,b^{\frac{3}{2}}c\right)\text{sgn}(x)}{15\,\sqrt{-b}\,c} + \frac{3\left(cx^2+b\right)^{\frac{5}{2}}Bc^4\text{sgn}(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] A*b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/15*(15*A*b^2*c*a
rctan(sqrt(b)/sqrt(-b)) + 3*B*sqrt(-b)*b^(5/2) + 20*A*sqrt(-b)*b^(3/2)*c)*s
```

```
gn(x)/(sqrt(-b)*c) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^4*sgn(x) + 5*(c*x^2 + b)
^(3/2)*A*c^5*sgn(x) + 15*sqrt(c*x^2 + b)*A*b*c^5*sgn(x))/c^5
```

maple [A] time = 0.05, size = 99, normalized size = 0.97

$$-\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(15A\,b^{\frac{3}{2}}c\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-15\sqrt{c\,x^{2}+b}\,Abc-5\left(c\,x^{2}+b\right)^{\frac{3}{2}}Ac-3\left(c\,x^{2}+b\right)^{\frac{5}{2}}B\right)}{15\left(c\,x^{2}+b\right)^{\frac{3}{2}}c\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/15*(c*x^4+b*x^2)^(3/2)*(-3*B*(c*x^2+b)^(5/2)+15*A*b^(3/2)*ln(2*(b+(c*x^2
+b)^(1/2)*b^(1/2))/x)*c-5*A*(c*x^2+b)^(3/2)*c-15*A*(c*x^2+b)^(1/2)*b*c)/x^3
/(c*x^2+b)^(3/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(Bx^{2}+A\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^{2}+A\right)\,\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**4, x)
```

3.124 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{bx^2+cx^4}\,(3Ac+2bB)}{2x}-\frac{1}{2}\sqrt{b}\,(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{\left(bx^2+cx^4\right)^{3/2}(3Ac+2bB)}{6bx^3}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{2bx^7}$$

```
[Out] 1/6*(3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^3-1/2*A*(c*x^4+b*x^2)^(5/2)/b/x^7
-1/2*(3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^(1/2)+1/2*(3*A*
c+2*B*b)*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] time = 0.22, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2021, 2008, 206}

$$\frac{\left(bx^2+cx^4\right)^{3/2}(3Ac+2bB)}{6bx^3}+\frac{\sqrt{bx^2+cx^4}\,(3Ac+2bB)}{2x}-\frac{1}{2}\sqrt{b}\,(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)-\frac{A\left(bx^2+cx^4\right)^{5/2}}{2bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

```
[Out] ((2*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*x) + ((2*b*B + 3*A*c)*(b*x^2 + c*x
^4)^(3/2))/(6*b*x^3) - (A*(b*x^2 + c*x^4)^(5/2))/(2*b*x^7) - (Sqrt[b]*(2*b*
B + 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{(-2bB-3Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx}{2b}$$
$$= \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}(-2bB-3Ac)\int \frac{\sqrt{bx^2+}}{x^2}$$
$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} +$$
$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} -$$
$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} -$$

Mathematica [A] time = 0.07, size = 109, normalized size = 0.82

$$\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\left(-3Ab+6Acx^2+8bBx^2+2Bcx^4\right)-3\sqrt{b}\,x^2(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{6x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(-3*A*b + 8*b*B*x^2 + 6*A*c*x^2 + 2
*B*c*x^4) - 3*Sqrt[b]*(2*b*B + 3*A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])
)/(6*x^3*Sqrt[b + c*x^2])
```

fricas [A] time = 1.13, size = 195, normalized size = 1.47

$$\left[\frac{3(2Bb+3Ac)\sqrt{b}\,x^3\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(2Bcx^4+2(4Bb+3Ac)x^2-3Ab\right)\sqrt{cx^4+bx^2}}{12x^3}, \frac{3(2Bl}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*b + 3*A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b
*x^2)*sqrt(b))/x^3) + 2*(2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b)*sqrt(c*
x^4 + b*x^2))/x^3, 1/6*(3*(2*B*b + 3*A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 +
b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b
)*sqrt(c*x^4 + b*x^2))/x^3]
```

giac [A] time = 0.28, size = 115, normalized size = 0.86

$$\frac{2(cx^2+b)^{\frac{3}{2}}Bc\operatorname{sgn}(x)+6\sqrt{cx^2+b}\,Bbc\operatorname{sgn}(x)+6\sqrt{cx^2+b}\,Ac^2\operatorname{sgn}(x)-\frac{3\sqrt{cx^2+b}\,Abc\operatorname{sgn}(x)}{x^2}+\frac{3(2Bb^2c\operatorname{sgn}(x)+3Abc}{}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + b)^(3/2)*B*c*sgn(x) + 6*sqrt(c*x^2 + b)*B*b*c*sgn(x) + 6*sq
rt(c*x^2 + b)*A*c^2*sgn(x) - 3*sqrt(c*x^2 + b)*A*b*c*sgn(x)/x^2 + 3*(2*B*b^
2*c*sgn(x) + 3*A*b*c^2*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b))/c
```

maple [A] time = 0.06, size = 172, normalized size = 1.29

$$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(9A\,b^{\frac{3}{2}}c\,x^2\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)+6B\,b^{\frac{5}{2}}x^2\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-9\sqrt{c\,x^2+b}\,A b c\,x^2-6\sqrt{c\,x^2+b}\,B}{6\left(c\,x^2+b\right)^{\frac{3}{2}}b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/6*(c*x^4+b*x^2)^(3/2)*(9*A*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x
^2*c-3*(c*x^2+b)^(3/2)*A*c*x^2+6*B*b^(5/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))
/x)*x^2-2*(c*x^2+b)^(3/2)*B*b*x^2+3*A*(c*x^2+b)^(5/2)-9*A*(c*x^2+b)^(1/2)*x
^2*b*c-6*B*(c*x^2+b)^(1/2)*x^2*b^2)/x^5/(c*x^2+b)^(3/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\left(c\,x^4+b\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**6, x)
```

3.125 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=135

$$\frac{3c\sqrt{bx^2+cx^4}\,(Ac+4bB)}{8bx}-\frac{3c(Ac+4bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{\left(bx^2+cx^4\right)^{3/2}(Ac+4bB)}{8bx^5}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9}$$

```
[Out] -1/8*(A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^5-1/4*A*(c*x^4+b*x^2)^(5/2)/b/x^9-
3/8*c*(A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+3/8*c*(A*c
+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

Rubi [A] time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2020, 2021, 2008, 206}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}(Ac+4bB)}{8bx^5}+\frac{3c\sqrt{bx^2+cx^4}\,(Ac+4bB)}{8bx}-\frac{3c(Ac+4bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

```
[Out] (3*c*(4*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x) - ((4*b*B + A*c)*(b*x^2 + c
*x^4)^(3/2))/(8*b*x^5) - (A*(b*x^2 + c*x^4)^(5/2))/(4*b*x^9) - (3*c*(4*b*B
+ A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
```

```
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{4bx^9} - \frac{(-4bB-Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx}{4b}$$
$$= -\frac{(4bB+Ac)(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} + \frac{(3c(4bB+Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx}{8b}$$
$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} + \frac{1}{8}(\ldots$$
$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} - \frac{1}{8}(\ldots$$
$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} - \frac{3c\ldots}{\ldots}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.47

$$\frac{(x^2(b+cx^2))^{5/2}\left(cx^4(Ac+4bB)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{b}+1\right)-5Ab^2\right)}{20b^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b^2 + c*(4*b*B + A*c)*x^4*Hypergeometric2F1[
2, 5/2, 7/2, 1 + (c*x^2)/b]))/(20*b^3*x^9)
```

fricas [A] time = 1.10, size = 217, normalized size = 1.61

$$\left[\frac{3(4Bbc+Ac^2)\sqrt{b}\,x^5\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(8Bbcx^4-2Ab^2-(4Bb^2+5Abc)x^2\right)\sqrt{cx^4+bx^2}}{16bx^5}, \frac{3(4B\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*B*b*c + A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 +
 b*x^2)*sqrt(b))/x^3) + 2*(8*B*b*c*x^4 - 2*A*b^2 - (4*B*b^2 + 5*A*b*c)*x^2)
*sqrt(c*x^4 + b*x^2))/(b*x^5), 1/8*(3*(4*B*b*c + A*c^2)*sqrt(-b)*x^5*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (8*B*b*c*x^4 - 2*A*b^2 - (4*
B*b^2 + 5*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^5)]
```

giac [A] time = 0.26, size = 145, normalized size = 1.07

$$\frac{8\sqrt{cx^2+b}\,Bc^2\mathrm{sgn}(x) + \frac{3(4Bbc^2\mathrm{sgn}(x)+Ac^3\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4(cx^2+b)^{\frac{3}{2}}Bbc^2\mathrm{sgn}(x)-4\sqrt{cx^2+b}\,Bb^2c^2\mathrm{sgn}(x)+5(cx^2+b)^{\frac{3}{2}}Ac^3\mathrm{sgn}\ldots}{c^2x^4}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + b)*B*c^2*sgn(x) + 3*(4*B*b*c^2*sgn(x) + A*c^3*sgn(x))*a
rctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn
(x) - 4*sqrt(c*x^2 + b)*B*b^2*c^2*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c^3*sgn(x)
 - 3*sqrt(c*x^2 + b)*A*b*c^3*sgn(x))/(c^2*x^4))/c
```

maple [A] time = 0.06, size = 213, normalized size = 1.58

$$-\frac{\left(c x^4+b x^2\right)^{\frac{3}{2}}\left(3A b^{\frac{3}{2}} c^2 x^4 \ln\left(\frac{2b+2\sqrt{c x^2+b}\,\sqrt{b}}{x}\right)+12B b^{\frac{5}{2}} c x^4 \ln\left(\frac{2b+2\sqrt{c x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c x^2+b}\,A b c^2 x^4-12\sqrt{c x}\cdots\right.}{8\left(c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(-A*(c*x^2+b)^(3/2)*x^4*c^2+3*A*b^(3/2)*ln(2*(b+(c
*x^2+b)^(1/2)*b^(1/2))/x)*x^4*c^2-4*B*(c*x^2+b)^(3/2)*x^4*b*c+12*B*b^(5/2)*
ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*c+A*(c*x^2+b)^(5/2)*x^2*c-3*A*(c*x^
2+b)^(1/2)*x^4*b*c^2+4*B*(c*x^2+b)^(5/2)*x^2*b-12*B*(c*x^2+b)^(1/2)*x^4*b^2
*c+2*A*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**8, x)
```

3.126 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=140

$$-\frac{c^2(6bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{\left(bx^2+cx^4\right)^{3/2}(6bB-Ac)}{24bx^7}-\frac{c\sqrt{bx^2+cx^4}\,(6bB-Ac)}{16bx^3}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}}$$

```
[Out] -1/24*(-A*c+6*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^7-1/6*A*(c*x^4+b*x^2)^(5/2)/b/x^
11-1/16*c^2*(-A*c+6*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/1
6*c*(-A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2020, 2008, 206}

$$-\frac{c^2(6bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{\left(bx^2+cx^4\right)^{3/2}(6bB-Ac)}{24bx^7}-\frac{c\sqrt{bx^2+cx^4}\,(6bB-Ac)}{16bx^3}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] -(c*(6*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - ((6*b*B - A*c)*(b*x^2 +
 c*x^4)^(3/2))/(24*b*x^7) - (A*(b*x^2 + c*x^4)^(5/2))/(6*b*x^11) - (c^2*(6*
b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx = -\frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} - \frac{(-6bB+Ac)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx}{6b}$$

$$= -\frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} + \frac{(c(6bB-Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,c}{8b}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} +$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} -$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{6bx^{11}} -$$

Mathematica [A] time = 0.14, size = 121, normalized size = 0.86

$$-\frac{\left(b+cx^2\right)\left(A\left(8b^2+14bcx^2+3c^2x^4\right)+6bBx^2\left(2b+5cx^2\right)\right)+3c^2x^6\sqrt{\frac{cx^2}{b}+1}\,(6bB-Ac)\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{48bx^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] -1/48*((b + c*x^2)*(6*b*B*x^2*(2*b + 5*c*x^2) + A*(8*b^2 + 14*b*c*x^2 + 3*c
^2*x^4)) + 3*c^2*(6*b*B - A*c)*x^6*Sqrt[1 + (c*x^2)/b]*ArcTanh[Sqrt[1 + (c*
x^2)/b]])/(b*x^5*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.94, size = 250, normalized size = 1.79

$$\left[-\frac{3\left(6\,Bbc^2-Ac^3\right)\sqrt{b}\,x^7\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(3\left(10\,Bb^2c+Abc^2\right)x^4+8\,Ab^3+2\left(6\,Bb^3+7\,Ab^2c\right)\right.}{96\,b^2x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*b*c^2 - A*c^3)*sqrt(b)*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^
4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6
*B*b^3 + 7*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^2*x^7), 1/48*(3*(6*B*b*c^2
 - A*c^3)*sqrt(-b)*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) -
 (3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6*B*b^3 + 7*A*b^2*c)*x^2)*sqr
t(c*x^4 + b*x^2))/(b^2*x^7)]
```

giac [A] time = 0.27, size = 175, normalized size = 1.25

$$\frac{\frac{3\left(6\,Bbc^3\mathrm{sgn}(x)-Ac^4\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b} - \frac{30\left(cx^2+b\right)^{\frac{5}{2}}Bbc^3\mathrm{sgn}(x)-48\left(cx^2+b\right)^{\frac{3}{2}}Bb^2c^3\mathrm{sgn}(x)+18\,\sqrt{cx^2+b}\,Bb^3c^3\mathrm{sgn}(x)+3\left(cx^2+b\right)^{\frac{5}{2}}Ac^4\mathrm{sgr}}{bc^3x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*b*c^3*sgn(x) - A*c^4*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/
(sqrt(-b)*b) - (30*(c*x^2 + b)^(5/2)*B*b*c^3*sgn(x) - 48*(c*x^2 + b)^(3/2)*
B*b^2*c^3*sgn(x) + 18*sqrt(c*x^2 + b)*B*b^3*c^3*sgn(x) + 3*(c*x^2 + b)^(5/2
)*A*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*A*b*c^4*sgn(x) - 3*sqrt(c*x^2 + b)*A*b
^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

maple [B] time = 0.08, size = 259, normalized size = 1.85

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3A\,b^{\frac{3}{2}}c^3x^6\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-18B\,b^{\frac{5}{2}}c^2x^6\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,A b\,c^3x^6+18\sqrt{c\,x^2+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(-A*(c*x^2+b)^(3/2)*x^6*c^3+3*A*b^(3/2)*ln(2*(b+(c
*x^2+b)^(1/2)*b^(1/2))/x)*x^6*c^3+6*B*(c*x^2+b)^(3/2)*x^6*b*c^2-18*B*b^(5/2
)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^6*c^2+A*(c*x^2+b)^(5/2)*x^4*c^2-3*A
*(c*x^2+b)^(1/2)*x^6*b*c^3-6*B*(c*x^2+b)^(5/2)*x^4*b*c+18*B*(c*x^2+b)^(1/2)
*x^6*b^2*c^2+2*A*(c*x^2+b)^(5/2)*x^2*b*c-12*B*(c*x^2+b)^(5/2)*x^2*b^2-8*A*(
c*x^2+b)^(5/2)*b^2)/x^9/(c*x^2+b)^(3/2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**10, x)
```

3.127 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=177

$$\frac{c^3(8bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c^2\sqrt{bx^2+cx^4}\,(8bB-3Ac)}{128b^2x^3}-\frac{(bx^2+cx^4)^{3/2}\,(8bB-3Ac)}{48bx^9}-\frac{c\sqrt{bx^2+cx^4}\,(8bB}{64bx^5}$$

```
[Out] -1/48*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^9-1/8*A*(c*x^4+b*x^2)^(5/2)/b/
x^13+1/128*c^3*(-3*A*c+8*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2
)-1/64*c*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^5-1/128*c^2*(-3*A*c+8*B*b)*
(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] time = 0.28, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2020, 2025, 2008, 206}

$$-\frac{c^2\sqrt{bx^2+cx^4}\,(8bB-3Ac)}{128b^2x^3}+\frac{c^3(8bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c\sqrt{bx^2+cx^4}\,(8bB-3Ac)}{64bx^5}-\frac{(bx^2+cx^4)^{3/2}\,(8b}{48bx^9}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] -(c*(8*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) - (c^2*(8*b*B - 3*A*c)*
Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - ((8*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2)
)/(48*b*x^9) - (A*(b*x^2 + c*x^4)^(5/2))/(8*b*x^13) + (c^3*(8*b*B - 3*A*c)*
ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{12}}\,dx = -\frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}} - \frac{(-8bB+3Ac)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx}{8b}$$

$$= -\frac{(8bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{48bx^9} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}} + \frac{(c(8bB-3Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,d}{16b}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{(8bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{48bx^9} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}} +$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)\left(bx^2+\right.}{48bx^9}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)\left(bx^2+\right.}{48bx^9}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)\left(bx^2+\right.}{48bx^9}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.37

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(c^3x^8(8bB-3Ac)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{b}+1\right)-5Ab^4\right)}{40b^5x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b^4 + c^3*(8*b*B - 3*A*c)*x^8*Hypergeometric
2F1[5/2, 4, 7/2, 1 + (c*x^2)/b]))/(40*b^5*x^13)
```

fricas [A] time = 1.00, size = 299, normalized size = 1.69

$$\left[-\frac{3\left(8\,Bbc^3-3\,Ac^4\right)\sqrt{b}\,x^9\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(3\left(8\,Bb^2c^2-3\,Abc^3\right)x^6+48\,Ab^4+2\left(56\,Bb^3c+3\,Ab\right.\right.}{768\,b^3x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(b)*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c
*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(8*B*b^2*c^2 - 3*A*b*c^3)*x^6 + 48*A*b^4
 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c)*x^2)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), -1/384*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(-b)*x^9*arctan
```

(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (3*(8*B*b^2*c^2 - 3*A*b*c^3)
*x^6 + 48*A*b^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c
)*x^2)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]

giac [A] time = 0.32, size = 214, normalized size = 1.21

$$-\frac{\frac{3\left(8\,Bbc^4\mathrm{sgn}(x)-3\,Ac^5\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}+\frac{24\left(cx^2+b\right)^{\frac{7}{2}}Bbc^4\mathrm{sgn}(x)+40\left(cx^2+b\right)^{\frac{5}{2}}Bb^2c^4\mathrm{sgn}(x)-88\left(cx^2+b\right)^{\frac{3}{2}}Bb^3c^4\mathrm{sgn}(x)+24\sqrt{cx^2+b}\,Bb}{}}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/384*(3*(8*B*b*c^4*sgn(x) - 3*A*c^5*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-
b))/(sqrt(-b)*b^2) + (24*(c*x^2 + b)^(7/2)*B*b*c^4*sgn(x) + 40*(c*x^2 + b)^
(5/2)*B*b^2*c^4*sgn(x) - 88*(c*x^2 + b)^(3/2)*B*b^3*c^4*sgn(x) + 24*sqrt(c*
x^2 + b)*B*b^4*c^4*sgn(x) - 9*(c*x^2 + b)^(7/2)*A*c^5*sgn(x) + 33*(c*x^2 +
b)^(5/2)*A*b*c^5*sgn(x) + 33*(c*x^2 + b)^(3/2)*A*b^2*c^5*sgn(x) - 9*sqrt(c*
x^2 + b)*A*b^3*c^5*sgn(x))/(b^2*c^4*x^8))/c
```

maple [A] time = 0.08, size = 302, normalized size = 1.71

$$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(9A\,b^{\frac{3}{2}}c^4x^8\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-24B\,b^{\frac{5}{2}}c^3x^8\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-9\sqrt{c\,x^2+b}\,Ab\,c^4x^8+24\sqrt{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x)
```

```
[Out] -1/384*(c*x^4+b*x^2)^(3/2)*(-3*A*(c*x^2+b)^(3/2)*x^8*c^4+9*A*b^(3/2)*ln(2*(
b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^8*c^4+8*B*(c*x^2+b)^(3/2)*x^8*b*c^3-24*B*b^
(5/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^8*c^3+3*A*(c*x^2+b)^(5/2)*x^6*c
^3-9*A*(c*x^2+b)^(1/2)*x^8*b*c^4-8*B*(c*x^2+b)^(5/2)*x^6*b*c^2+24*B*(c*x^2+
b)^(1/2)*x^8*b^2*c^3+6*A*(c*x^2+b)^(5/2)*x^4*b*c^2-16*B*(c*x^2+b)^(5/2)*x^4
*b^2*c-24*A*(c*x^2+b)^(5/2)*x^2*b^2*c+64*B*(c*x^2+b)^(5/2)*x^2*b^3+48*A*(c*
x^2+b)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**12, x)
```

3.128 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{14}} \, dx$

Optimal. Leaf size=214

$$-\frac{3c^4(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}+\frac{3c^3\sqrt{bx^2+cx^4}\,(2bB-Ac)}{256b^3x^3}-\frac{c^2\sqrt{bx^2+cx^4}\,(2bB-Ac)}{128b^2x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}(2b\ldots}{16bx^{11}}$$

```
[Out] -1/16*(-A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^11-1/10*A*(c*x^4+b*x^2)^(5/2)/b/
x^15-3/256*c^4*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)-
1/32*c*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^7-1/128*c^2*(-A*c+2*B*b)*(c*x^4
+b*x^2)^(1/2)/b^2/x^5+3/256*c^3*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^3
```

Rubi [A] time = 0.34, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2038, 2020, 2025, 2008, 206}

$$\frac{3c^3\sqrt{bx^2+cx^4}\,(2bB-Ac)}{256b^3x^3}-\frac{c^2\sqrt{bx^2+cx^4}\,(2bB-Ac)}{128b^2x^5}-\frac{3c^4(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{c\sqrt{bx^2+cx^4}\,(2bB-\ldots}{32bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

```
[Out] -(c*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(32*b*x^7) - (c^2*(2*b*B - A*c)*Sqrt
[b*x^2 + c*x^4])/(128*b^2*x^5) + (3*c^3*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/
(256*b^3*x^3) - ((2*b*B - A*c)*(b*x^2 + c*x^4)^(3/2))/(16*b*x^11) - (A*(b*x
^2 + c*x^4)^(5/2))/(10*b*x^15) - (3*c^4*(2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{(-10bB+5Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx}{10b}$$

$$= -\frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(3c(2bB-Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx}{16b}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(c}{}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{(2bB-Ac)(bx^2+cx^4}{16bx^{11}}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+}}{256b^3x^3}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+}}{256b^3x^3}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+}}{256b^3x^3}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.30

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(Ab^5+c^4x^{10}(2bB-Ac)\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{cx^2}{b}+1\right)\right)}{10b^6x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

```
[Out] -1/10*((x^2*(b + c*x^2))^(5/2)*(A*b^5 + c^4*(2*b*B - A*c)*x^10*Hypergeometr
ic2F1[5/2, 5, 7/2, 1 + (c*x^2)/b]))/(b^6*x^15)
```

fricas [A] time = 1.04, size = 345, normalized size = 1.61

$$\left[-\frac{15\left(2\,Bbc^4-Ac^5\right)\sqrt{b}\,x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\left(15\left(2\,Bb^2c^3-Abc^4\right)x^8-10\left(2\,Bb^3c^2-Ab^2c^3\right)x^6-12\right.}{2560\,b^4x^{11}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*b*c^4 - A*c^5)*sqrt(b)*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(
c*x^4 + b*x^2)*sqrt(b))/x^3) - 2*(15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*
b^3*c^2 - A*b^2*c^3)*x^6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*
(10*B*b^5 + 11*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), 1/1280*(15*(2
*B*b*c^4 - A*c^5)*sqrt(-b)*x^11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*b^3*c^2 - A*b^2*c^3)*x^
6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*(10*B*b^5 + 11*A*b^4*c)
*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11)]
```

giac [A] time = 0.31, size = 234, normalized size = 1.09

$$\frac{\frac{15\left(2\,Bbc^5\mathrm{sgn}(x)-Ac^6\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^3}+\frac{30\left(cx^2+b\right)^{\frac{9}{2}}Bbc^5\mathrm{sgn}(x)-140\left(cx^2+b\right)^{\frac{7}{2}}Bb^2c^5\mathrm{sgn}(x)+140\left(cx^2+b\right)^{\frac{3}{2}}Bb^4c^5\mathrm{sgn}(x)-30\sqrt{cx^2+b}\,Bb}{}}{1280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/1280*(15*(2*B*b*c^5*sgn(x) - A*c^6*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b
))/(sqrt(-b)*b^3) + (30*(c*x^2 + b)^(9/2)*B*b*c^5*sgn(x) - 140*(c*x^2 + b)^
(7/2)*B*b^2*c^5*sgn(x) + 140*(c*x^2 + b)^(3/2)*B*b^4*c^5*sgn(x) - 30*sqrt(c
*x^2 + b)*B*b^5*c^5*sgn(x) - 15*(c*x^2 + b)^(9/2)*A*c^6*sgn(x) + 70*(c*x^2
+ b)^(7/2)*A*b*c^6*sgn(x) - 128*(c*x^2 + b)^(5/2)*A*b^2*c^6*sgn(x) - 70*(c*
x^2 + b)^(3/2)*A*b^3*c^6*sgn(x) + 15*sqrt(c*x^2 + b)*A*b^4*c^6*sgn(x))/(b^3
*c^5*x^10))/c
```

maple [A] time = 0.09, size = 344, normalized size = 1.61

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(15A\,b^{\frac{3}{2}}c^5x^{10}\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-30B\,b^{\frac{5}{2}}c^4x^{10}\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-15\sqrt{c\,x^2+b}\,Ab\,c^5x^{10}+30\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(-5*A*(c*x^2+b)^(3/2)*x^10*c^5+15*A*b^(3/2)*ln(2
*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^10*c^5+10*B*(c*x^2+b)^(3/2)*x^10*b*c^4-30
*B*b^(5/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^10*c^4+5*A*(c*x^2+b)^(5/2)
*x^8*c^4-15*A*(c*x^2+b)^(1/2)*x^10*b*c^5-10*B*(c*x^2+b)^(5/2)*x^8*b*c^3+30*
B*(c*x^2+b)^(1/2)*x^10*b^2*c^4+10*A*(c*x^2+b)^(5/2)*x^6*b*c^3-20*B*(c*x^2+b
)^(5/2)*x^6*b^2*c^2-40*A*(c*x^2+b)^(5/2)*x^4*b^2*c^2+80*B*(c*x^2+b)^(5/2)*x
^4*b^3*c+80*A*(c*x^2+b)^(5/2)*x^2*b^3*c-160*B*(c*x^2+b)^(5/2)*x^2*b^4-128*A
*(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/2)/b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^14, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \left(A + Bx^2\right)}{x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**14, x)
```

3.129 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=251

$$\frac{c^5(12bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}}-\frac{c^4\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1024b^4x^3}+\frac{c^3\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1536b^3x^5}-\frac{c^2\sqrt{bx^2+cx^4}\,(\cdots}{1920b\cdots}$$

```
[Out] -1/120*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^13-1/12*A*(c*x^4+b*x^2)^(5/2
)/b/x^17+1/1024*c^5*(-7*A*c+12*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/
b^(9/2)-1/320*c*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^9-1/1920*c^2*(-7*A*
c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^7+1/1536*c^3*(-7*A*c+12*B*b)*(c*x^4+b*x
^2)^(1/2)/b^3/x^5-1/1024*c^4*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^3
```

Rubi [A] time = 0.39, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2020, 2025, 2008, 206}

$$-\frac{c^4\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1024b^4x^3}+\frac{c^3\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1536b^3x^5}-\frac{c^2\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1920b^2x^7}+\frac{c^5(12bB-7Ac)\tanh\cdots}{1024b^{9/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

```
[Out] -(c*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(320*b*x^9) - (c^2*(12*b*B - 7*A*
c)*Sqrt[b*x^2 + c*x^4])/(1920*b^2*x^7) + (c^3*(12*b*B - 7*A*c)*Sqrt[b*x^2 +
 c*x^4])/(1536*b^3*x^5) - (c^4*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(1024*
b^4*x^3) - ((12*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(120*b*x^13) - (A*(b*x^
2 + c*x^4)^(5/2))/(12*b*x^17) + (c^5*(12*b*B - 7*A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(1024*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} - \frac{(-12bB+7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx}{12b}$$

$$= -\frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c(12bB-7Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{10}}}{40b}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} - \frac{(12bB-7Ac)(bx}{120bx^{13}}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{}}{1536b^3x}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{}}{1536b^3x}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{}}{1536b^3x}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{}}{1536b^3x}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.26

$$\frac{(x^2(b+cx^2))^{5/2}\left(c^5x^{12}(12bB-7Ac)\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{cx^2}{b}+1\right)-5Ab^6\right)}{60b^7x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b^6 + c^5*(12*b*B - 7*A*c)*x^12*Hypergeometr
ic2F1[5/2, 6, 7/2, 1 + (c*x^2)/b]))/(60*b^7*x^17)
```

fricas [A] time = 1.33, size = 393, normalized size = 1.57

$$\left[-\frac{15\left(12\,Bbc^5-7\,Ac^6\right)\sqrt{b}\,x^{13}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(15\left(12\,Bb^2c^4-7\,Abc^5\right)x^{10}-10\left(12\,Bb^3c^3-7\,Ab^2c\right.\right.}{3072}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(12*B*b*c^5 - 7*A*c^6)*sqrt(b)*x^13*log(-(c*x^3 + 2*b*x - 2*s
qrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(15*(12*B*b^2*c^4 - 7*A*b*c^5)*x^10 -
10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b^6 + 8*(12*B*b^4*c^2 - 7*A*b^
3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 + 128*(12*B*b^6 + 13*A*b^5*c)*
x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13), -1/15360*(15*(12*B*b*c^5 - 7*A*c^6)*s
qrt(-b)*x^13*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*(12*B
*b^2*c^4 - 7*A*b*c^5)*x^10 - 10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b
^6 + 8*(12*B*b^4*c^2 - 7*A*b^3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 +
 128*(12*B*b^6 + 13*A*b^5*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13)]
```

giac [A] time = 0.37, size = 294, normalized size = 1.17

$$-\frac{\frac{15\left(12\,Bbc^6\mathrm{sgn}(x)-7\,Ac^7\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4}+\frac{180\left(cx^2+b\right)^{\frac{11}{2}}Bbc^6\mathrm{sgn}(x)-1020\left(cx^2+b\right)^{\frac{9}{2}}Bb^2c^6\mathrm{sgn}(x)+2376\left(cx^2+b\right)^{\frac{7}{2}}Bb^3c^6\mathrm{sgn}(x)-696}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/15360*(15*(12*B*b*c^6*sgn(x) - 7*A*c^7*sgn(x))*arctan(sqrt(c*x^2 + b)/sq
rt(-b))/(sqrt(-b)*b^4) + (180*(c*x^2 + b)^(11/2)*B*b*c^6*sgn(x) - 1020*(c*x
^2 + b)^(9/2)*B*b^2*c^6*sgn(x) + 2376*(c*x^2 + b)^(7/2)*B*b^3*c^6*sgn(x) -
696*(c*x^2 + b)^(5/2)*B*b^4*c^6*sgn(x) - 1020*(c*x^2 + b)^(3/2)*B*b^5*c^6*s
gn(x) + 180*sqrt(c*x^2 + b)*B*b^6*c^6*sgn(x) - 105*(c*x^2 + b)^(11/2)*A*c^7
*sgn(x) + 595*(c*x^2 + b)^(9/2)*A*b*c^7*sgn(x) - 1386*(c*x^2 + b)^(7/2)*A*b
^2*c^7*sgn(x) + 1686*(c*x^2 + b)^(5/2)*A*b^3*c^7*sgn(x) + 595*(c*x^2 + b)^(
3/2)*A*b^4*c^7*sgn(x) - 105*sqrt(c*x^2 + b)*A*b^5*c^7*sgn(x))/(b^4*c^6*x^12
))/c
```

maple [A] time = 0.15, size = 386, normalized size = 1.54

$$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(105A\,b^{\frac{3}{2}}c^6x^{12}\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-180B\,b^{\frac{5}{2}}c^5x^{12}\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-105\sqrt{c\,x^2+b}\,Ab\,c^6x^{12}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x)
```

```
[Out] -1/15360*(c*x^4+b*x^2)^(3/2)*(-35*A*(c*x^2+b)^(3/2)*x^12*c^6+105*A*b^(3/2)*
ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^12*c^6+60*B*(c*x^2+b)^(3/2)*x^12*b*c^
5-180*B*b^(5/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^12*c^5+35*A*(c*x^2+b)
^(5/2)*x^10*c^5-105*A*(c*x^2+b)^(1/2)*x^12*b*c^6-60*B*(c*x^2+b)^(5/2)*x^10*
b*c^4+180*B*(c*x^2+b)^(1/2)*x^12*b^2*c^5+70*A*(c*x^2+b)^(5/2)*x^8*b*c^4-120
*B*(c*x^2+b)^(5/2)*x^8*b^2*c^3-280*A*(c*x^2+b)^(5/2)*x^6*b^2*c^3+480*B*(c*x
^2+b)^(5/2)*x^6*b^3*c^2+560*A*(c*x^2+b)^(5/2)*x^4*b^3*c^2-960*B*(c*x^2+b)^(
5/2)*x^4*b^4*c-896*A*(c*x^2+b)^(5/2)*x^2*b^4*c+1536*B*(c*x^2+b)^(5/2)*x^2*b
^5+1280*A*(c*x^2+b)^(5/2)*b^5)/x^15/(c*x^2+b)^(3/2)/b^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^16, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{16}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2} \left(b + c x^{2}\right)\right)^{\frac{3}{2}} \left(A + B x^{2}\right)}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**16,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**16, x)
```

3.130 $\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=176

$$\frac{5b^3(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}-\frac{5b^2\sqrt{bx^2+cx^4}\,(7bB-8Ac)}{128c^4}+\frac{5bx^2\sqrt{bx^2+cx^4}\,(7bB-8Ac)}{192c^3}-\frac{x^4\sqrt{bx^2+cx}}{48}$$

```
[Out] 5/128*b^3*(-8*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-5
/128*b^2*(-8*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/c^4+5/192*b*(-8*A*c+7*B*b)*x^2*
(c*x^4+b*x^2)^(1/2)/c^3-1/48*(-8*A*c+7*B*b)*x^4*(c*x^4+b*x^2)^(1/2)/c^2+1/8
*B*x^6*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.33, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 794, 670, 640, 620, 206}

$$-\frac{5b^2\sqrt{bx^2+cx^4}\,(7bB-8Ac)}{128c^4}+\frac{5b^3(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}-\frac{x^4\sqrt{bx^2+cx^4}\,(7bB-8Ac)}{48c^2}+\frac{5bx^2\sqrt{bx^2+c}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-5*b^2*(7*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(128*c^4) + (5*b*(7*b*B - 8*A*
c)*x^2*Sqrt[b*x^2 + c*x^4])/(192*c^3) - ((7*b*B - 8*A*c)*x^4*Sqrt[b*x^2 + c
*x^4])/(48*c^2) + (B*x^6*Sqrt[b*x^2 + c*x^4])/(8*c) + (5*b^3*(7*b*B - 8*A*c
)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
```

```
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{\left(3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{8c}$$

$$= -\frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{(5b(7bB-8Ac))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,\right.}{96c^2}$$

$$= \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} - \frac{\left(5b^2(7bB\right.}{}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2}$$

Mathematica [A] time = 0.26, size = 145, normalized size = 0.82

$$\frac{x\left(15b^3\sqrt{b+cx^2}\,(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)-\sqrt{c}\,x\left(b+cx^2\right)\left(-10b^2c\left(12A+7Bx^2\right)+8bc^2x^2\left(10A+7Bx^2\right)-\right.\right.}{384c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(-(Sqrt[c]*x*(b + c*x^2)*(105*b^3*B - 16*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c
^2*x^2*(10*A + 7*B*x^2) - 10*b^2*c*(12*A + 7*B*x^2))) + 15*b^3*(7*b*B - 8*A
*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(384*c^(9/2)*Sqr
t[x^2*(b + c*x^2)])
```

fricas [A] time = 1.00, size = 275, normalized size = 1.56

$$\left[-\frac{15\left(7\,Bb^4-8\,Ab^3c\right)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8\left(7\,Bbc^3\right.\right.}{768\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7*B*b
*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)
)/c^5, -1/384*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)
*sqrt(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7
*B*b*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*
x^2))/c^5]
```

giac [A] time = 0.22, size = 149, normalized size = 0.85

$$\frac{1}{384}\sqrt{cx^4+bx^2}\left(2\left(4\left(\frac{6\,Bx^2}{c}-\frac{7\,Bbc^2-8\,Ac^3}{c^4}\right)x^2+\frac{5\left(7\,Bb^2c-8\,Abc^2\right)}{c^4}\right)x^2-\frac{15\left(7\,Bb^3-8\,Ab^2c\right)}{c^4}\right)-\frac{5\left(7\,Bb^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(c*x^4 + b*x^2)*(2*(4*(6*B*x^2/c - (7*B*b*c^2 - 8*A*c^3)/c^4)*x^2
 + 5*(7*B*b^2*c - 8*A*b*c^2)/c^4)*x^2 - 15*(7*B*b^3 - 8*A*b^2*c)/c^4) - 5/2
56*(7*B*b^4 - 8*A*b^3*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqr
t(c) - b))/c^(9/2)
```

maple [A] time = 0.06, size = 211, normalized size = 1.20

$$\frac{\sqrt{c\,x^2+b}\,\left(48\sqrt{c\,x^2+b}\;B\,c^{\frac{9}{2}}x^7+64\sqrt{c\,x^2+b}\;A\,c^{\frac{9}{2}}x^5-56\sqrt{c\,x^2+b}\;Bb\,c^{\frac{7}{2}}x^5-80\sqrt{c\,x^2+b}\;Ab\,c^{\frac{7}{2}}x^3+70\sqrt{c\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/384*x*(c*x^2+b)^(1/2)*(48*B*(c*x^2+b)^(1/2)*c^(9/2)*x^7+64*A*(c*x^2+b)^(1
/2)*c^(9/2)*x^5-56*B*(c*x^2+b)^(1/2)*c^(7/2)*x^5*b-80*A*(c*x^2+b)^(1/2)*c^(
7/2)*x^3*b+70*B*(c*x^2+b)^(1/2)*c^(5/2)*x^3*b^2+120*A*(c*x^2+b)^(1/2)*c^(5/
2)*x*b^2-105*B*(c*x^2+b)^(1/2)*c^(3/2)*x*b^3-120*A*ln(c^(1/2)*x+(c*x^2+b)^(
1/2))*b^3*c^2+105*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^4*c)/(c*x^4+b*x^2)^(1/2
)/c^(11/2)
```

maxima [A] time = 1.38, size = 231, normalized size = 1.31

$$\frac{1}{96}\left(\frac{16\,\sqrt{cx^4+bx^2}\,x^4}{c}-\frac{20\,\sqrt{cx^4+bx^2}\,bx^2}{c^2}-\frac{15\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}\,b^2}{c^3}\right)A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(16*sqrt(c*x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15*
b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^
4 + b*x^2)*b^2/c^3)*A + 1/768*(96*sqrt(c*x^4 + b*x^2)*x^6/c - 112*sqrt(c*x^
4 + b*x^2)*b*x^4/c^2 + 140*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^3 + 105*b^4*log(2*
c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2
)*b^3/c^4)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**7*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.131 $\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=139

$$-\frac{b^2(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{b\sqrt{bx^2+cx^4}(5bB-6Ac)}{16c^3}-\frac{x^2\sqrt{bx^2+cx^4}(5bB-6Ac)}{24c^2}+\frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

```
[Out] -1/16*b^2*(-6*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+1
/16*b*(-6*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^3-1/24*(-6*A*c+5*B*b)*x^2*(c*x^4
+b*x^2)^(1/2)/c^2+1/6*B*x^4*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.27, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 794, 670, 640, 620, 206}

$$-\frac{b^2(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{x^2\sqrt{bx^2+cx^4}(5bB-6Ac)}{24c^2}+\frac{b\sqrt{bx^2+cx^4}(5bB-6Ac)}{16c^3}+\frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*(5*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(16*c^3) - ((5*b*B - 6*A*c)*x^2*Sqr
t[b*x^2 + c*x^4])/(24*c^2) + (B*x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (b^2*(5*b*
B - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
```

```
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{\left(2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{6c}$$

$$= -\frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{(b(5bB-6Ac))\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x\right.}{16c^2}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(b^2(5bB-6A\right.}{}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(b^2(5bB-6A\right.}{}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{b^2(5bB-6Ac}{}$$

Mathematica [A] time = 0.18, size = 123, normalized size = 0.88

$$\frac{x\left(\sqrt{c}\,x\left(b+cx^2\right)\left(-2bc\left(9A+5Bx^2\right)+4c^2x^2\left(3A+2Bx^2\right)+15b^2B\right)-3b^2\sqrt{b+cx^2}\,(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right.}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(15*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) - 2*b*c*(9*
A + 5*B*x^2)) - 3*b^2*(5*b*B - 6*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/S
qrt[b + c*x^2]]))/(48*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.98, size = 226, normalized size = 1.63

$$\left[-\frac{3\left(5\,Bb^3-6\,Ab^2c\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(8\,Bc^3x^4+15\,Bb^2c-18\,Abc^2-2\left(5\,Bbc^2-6\,Ac\right.\right.}{96\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b
*x^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 -
6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(
-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 15*B*
b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]
```

giac [A] time = 0.25, size = 119, normalized size = 0.86

$$\frac{1}{48}\sqrt{cx^4+bx^2}\left(2\left(\frac{4\,Bx^2}{c}-\frac{5\,Bbc-6\,Ac^2}{c^3}\right)x^2+\frac{3\left(5\,Bb^2-6\,Abc\right)}{c^3}\right)+\frac{\left(5\,Bb^3-6\,Ab^2c\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4\cdots}\right.\right.\right.}{32\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2)*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2 + 3*(
5*B*b^2 - 6*A*b*c)/c^3) + 1/32*(5*B*b^3 - 6*A*b^2*c)*log(abs(-2*(sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(7/2)
```

maple [A] time = 0.06, size = 169, normalized size = 1.22

$$\frac{\sqrt{c\,x^2+b}\,\left(8\sqrt{c\,x^2+b}\,B\,c^{\frac{7}{2}}x^5+12\sqrt{c\,x^2+b}\,A\,c^{\frac{7}{2}}x^3-10\sqrt{c\,x^2+b}\,Bb\,c^{\frac{5}{2}}x^3+18A\,b^2c^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\cdots\right.}{48\sqrt{c\,x^4+b\,x^2}\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*x*(c*x^2+b)^(1/2)*(8*B*(c*x^2+b)^(1/2)*c^(7/2)*x^5+12*A*(c*x^2+b)^(1/2
)*c^(7/2)*x^3-10*B*(c*x^2+b)^(1/2)*c^(5/2)*x^3*b-18*A*(c*x^2+b)^(1/2)*c^(5/
2)*x*b+15*B*(c*x^2+b)^(1/2)*c^(3/2)*x*b^2+18*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
)*b^2*c^2-15*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(
9/2)
```

maxima [A] time = 1.45, size = 183, normalized size = 1.32

$$\frac{1}{16}\left(\frac{4\sqrt{cx^4+bx^2}\,x^2}{c}+\frac{3\,b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\sqrt{cx^4+bx^2}\,b}{c^2}\right)A+\frac{1}{96}\left(\frac{16\sqrt{cx^4+bx^2}\,x^4}{c}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*A + 1/96*(16*sqrt(c*
x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15*b^3*log(2*c*x^2
+ b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b^2/c
^3)*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**5*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.132 $\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=83

$$\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{bx^2+cx^4}\,\left(-4Ac+3bB-2Bcx^2\right)}{8c^2}$$

```
[Out] 1/8*b*(-4*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-1/8*(
-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 779, 620, 206}

$$\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{bx^2+cx^4}\,\left(-4Ac+3bB-2Bcx^2\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (b*(3*b*B - 4*
A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^3 \left(A + Bx^2\right)}{\sqrt{bx^2 + cx^4}} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x(A + Bx)}{\sqrt{bx + cx^2}} \, dx, x, x^2\right)$$

$$= -\frac{\left(3bB - 4Ac - 2Bcx^2\right)\sqrt{bx^2 + cx^4}}{8c^2} + \frac{(b(3bB - 4Ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}} \, dx, x, x^2\right)}{16c^2}$$

$$= -\frac{\left(3bB - 4Ac - 2Bcx^2\right)\sqrt{bx^2 + cx^4}}{8c^2} + \frac{(b(3bB - 4Ac))\,\text{Subst}\left(\int \frac{1}{1-cx^2} \, dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^2}$$

$$= -\frac{\left(3bB - 4Ac - 2Bcx^2\right)\sqrt{bx^2 + cx^4}}{8c^2} + \frac{b(3bB - 4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}$$

Mathematica [A] time = 0.13, size = 97, normalized size = 1.17

$$\frac{x\left(\sqrt{c}\,x\left(b + cx^2\right)\left(4Ac - 3bB + 2Bcx^2\right) + b\sqrt{b + cx^2}\,(3bB - 4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b + cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(-3*b*B + 4*A*c + 2*B*c*x^2) + b*(3*b*B - 4*A*c)*
Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*
(b + c*x^2)])
```

fricas [A] time = 0.77, size = 177, normalized size = 2.13

$$\left[-\frac{\left(3\,Bb^2 - 4\,Abc\right)\sqrt{c}\,\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right) - 2\left(2\,Bc^2x^2 - 3\,Bbc + 4\,Ac^2\right)\sqrt{cx^4 + bx^2}}{16\,c^3}, -\frac{\left(3\,Bb^2 - 4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3,
-1/8*((3*B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x
^2 + b)) - (2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

giac [A] time = 0.22, size = 91, normalized size = 1.10

$$\frac{1}{8}\sqrt{cx^4 + bx^2}\left(\frac{2\,Bx^2}{c} - \frac{3\,Bb - 4\,Ac}{c^2}\right) - \frac{\left(3\,Bb^2 - 4\,Abc\right)\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b^2 -
 4*A*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(5
/2)
```

maple [A] time = 0.05, size = 127, normalized size = 1.53

$$\frac{\sqrt{c\,x^2+b}\,\left(2\sqrt{c\,x^2+b}\,B\,c^{\frac{5}{2}}x^3-4Abc^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+3B\,b^2c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+4\sqrt{c\,x^2+b}\,A\right.}{8\sqrt{c\,x^4+b\,x^2}\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*B*c^(5/2)*(c*x^2+b)^(1/2)*x^3+4*A*c^(5/2)*(c*x^2+b
)^(1/2)*x-3*B*c^(3/2)*(c*x^2+b)^(1/2)*x*b-4*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))
*b*c^2+3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2)
```

maxima [A] time = 1.46, size = 134, normalized size = 1.61

$$\frac{1}{16}\left(\frac{4\sqrt{cx^4+bx^2}\,x^2}{c}+\frac{3\,b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\sqrt{cx^4+bx^2}\,b}{c^2}\right)B-\frac{1}{4}\,A\left(\frac{b\log\left(2\,cx^2+b+2\right.}{c^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*B - 1/4*A*(b*log(2*c
*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqrt(c*x^4 + b*x^2)/c
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.133 $\int \frac{x\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=66

$$\frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*B*(c
*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2034, 640, 620, 206}

$$\frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b
*x^2 + c*x^4]])/(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{B\sqrt{bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c}\\ &= \frac{B\sqrt{bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\ &= \frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.23

$$\frac{x\left(B\sqrt{c}\,x\left(b+cx^2\right)-\sqrt{b+cx^2}\,(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(B*Sqrt[c]*x*(b + c*x^2) - (b*B - 2*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c
]*x)/Sqrt[b + c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.66, size = 131, normalized size = 1.98

$$\left[\frac{2\sqrt{cx^4+bx^2}\,Bc-(Bb-2\,Ac)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,c^2},\ \frac{\sqrt{cx^4+bx^2}\,Bc+(Bb-2\,Ac)\sqrt{-c}\,\text{arct}}{2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^4 + b*x^2)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-2*c*x^2 - b -
2*sqrt(c*x^4 + b*x^2)*sqrt(c)))/c^2, 1/2*(sqrt(c*x^4 + b*x^2)*B*c + (B*b -
2*A*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)))/c^2]
```

giac [A] time = 0.21, size = 67, normalized size = 1.02

$$\frac{\sqrt{cx^4+bx^2}\,B}{2\,c}+\frac{(Bb-2\,Ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2)*B/c + 1/4*(B*b - 2*A*c)*log(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
```

maple [A] time = 0.06, size = 88, normalized size = 1.33

$$\frac{\sqrt{c\,x^2+b}\,\left(2A\,c^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-Bbc\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+\sqrt{c\,x^2+b}\,B\,c^{\frac{3}{2}}x\right)x}{2\sqrt{c\,x^4+b\,x^2}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(B*c^(3/2)*(c*x^2+b)^(1/2)*x+2*A*ln(c^(1/2)*x+(c*x^2+
b)^(1/2))*c^2-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(1/2)/c^(5
/2)
```

maxima [A] time = 1.44, size = 88, normalized size = 1.33

$$-\frac{1}{4}B\left(\frac{b\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}-\frac{2\sqrt{cx^4+bx^2}}{c}\right)+\frac{A\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*B*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqrt
(c*x^4 + b*x^2)/c) + 1/2*A*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
/sqrt(c)
```

mupad [B] time = 0.81, size = 89, normalized size = 1.35

$$\frac{B\sqrt{c\,x^4+b\,x^2}}{2\,c}+\frac{A\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}-\frac{B\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (B*(b*x^2 + c*x^4)^(1/2))/(2*c) + (A*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c
*x^4)^(1/2)))/(2*c^(1/2)) - (B*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.134 $\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)-A*(c*x^4+b*x^2)^(1/2)/b/
x^2
```

Rubi [A] time = 0.15, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 620, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[b*x^2 + c*x^4])/(b*x^2)) + (B*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{bx^2} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{bx^2} + B\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{bx^2} + \frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.30

$$\frac{bBx\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right) - A\sqrt{c}\,\left(b+cx^2\right)}{b\sqrt{c}\,\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(A*Sqrt[c]*(b + c*x^2)) + b*B*x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[
b + c*x^2]])/(b*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.88, size = 136, normalized size = 2.39

$$\left[\frac{Bb\sqrt{c}\,x^2\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right) - 2\,\sqrt{cx^4+bx^2}\,Ac}{2\,bcx^2}, -\frac{Bb\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right) + \sqrt{cx^4+bx^2}}{bcx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2
*sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2), -(B*b*sqrt(-c)*x^2*arctan(sqrt(c*x^4 +
 b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2)]
```

giac [A] time = 0.22, size = 66, normalized size = 1.16

$$-\frac{B\log\left(\left|2\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2}\right)\sqrt{c} + b\right|\right)}{2\,\sqrt{c}} + \frac{A}{\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*B*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b))/sqrt(c)
+ A/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))
```

maple [A] time = 0.06, size = 67, normalized size = 1.18

$$-\frac{\sqrt{c\,x^2+b}\,\left(-Bbx\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2+b}\right) + \sqrt{c\,x^2+b}\,A\sqrt{c}\right)}{\sqrt{c\,x^4+b\,x^2}\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -(c*x^2+b)^(1/2)*(-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x*b+A*(c*x^2+b)^(1/2)*c^
(1/2))/(c*x^4+b*x^2)^(1/2)/c^(1/2)/b
```

maxima [A] time = 1.41, size = 56, normalized size = 0.98

$$\frac{B\log\left(2\,cx^2 + b + 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}} - \frac{\sqrt{cx^4 + bx^2}\,A}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) - sqrt(c*x^4
 + b*x^2)*A/(b*x^2)
```

mupad [B] time = 0.51, size = 57, normalized size = 1.00

$$\frac{B\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}} + \sqrt{c\,x^4 + b\,x^2}\right)}{2\,\sqrt{c}} - \frac{A\,\sqrt{c\,x^4 + b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(1/2)) - (A*(b*
x^2 + c*x^4)^(1/2))/(b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(x**2*(b + c*x**2))), x)
```

3.135 $\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{bx^2+cx^4}\,(3bB-2Ac)}{3b^2x^2}-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -1/3*A*(c*x^4+b*x^2)^(1/2)/b/x^4-1/3*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^2
/x^2
```

Rubi [A] time = 0.17, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 650}

$$-\frac{\sqrt{bx^2+cx^4}\,(3bB-2Ac)}{3b^2x^2}-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(3*b*x^4) - ((3*b*B - 2*A*c)*Sqrt[b*x^2 + c*x^4])/
(3*b^2*x^2)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} + \frac{\left(-2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{3b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.70

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A\left(b-2cx^2\right)+3bBx^2\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
[Out] -1/3*(Sqrt[x^2*(b + c*x^2)]*(3*b*B*x^2 + A*(b - 2*c*x^2)))/(b^2*x^4)
```

fricas [A] time = 0.91, size = 38, normalized size = 0.62

$$-\frac{\sqrt{cx^4+bx^2}\left((3\,Bb-2\,Ac)x^2+Ab\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(c*x^4 + b*x^2)*((3*B*b - 2*A*c)*x^2 + A*b)/(b^2*x^4)
```

giac [A] time = 0.20, size = 88, normalized size = 1.44

$$\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^2 B + 3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)A\sqrt{c} + Ab}{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^2*B + 3*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2))*A*sqrt(c) + A*b)/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^3
```

maple [A] time = 0.05, size = 47, normalized size = 0.77

$$-\frac{\left(c\,x^2+b\right)\left(-2Ac\,x^2+3Bb\,x^2+Ab\right)}{3\sqrt{c\,x^4+b\,x^2}\;b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/3*(c*x^2+b)*(-2*A*c*x^2+3*B*b*x^2+A*b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.47, size = 70, normalized size = 1.15

$$\frac{1}{3}A\left(\frac{2\sqrt{cx^4+bx^2}\,c}{b^2x^2} - \frac{\sqrt{cx^4+bx^2}}{bx^4}\right) - \frac{\sqrt{cx^4+bx^2}\,B}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) - s
qrt(c*x^4 + b*x^2)*B/(b*x^2)
```

mupad [B] time = 0.20, size = 39, normalized size = 0.64

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(A\,b-2\,A\,c\,x^2+3\,B\,b\,x^2\right)}{3\,b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(A*b - 2*A*c*x^2 + 3*B*b*x^2))/(3*b^2*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

3.136 $\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=96

$$\frac{2c\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{15b^3x^2}-\frac{\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{15b^2x^4}-\frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

```
[Out] -1/5*A*(c*x^4+b*x^2)^(1/2)/b/x^6-1/15*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^
2/x^4+2/15*c*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^2
```

Rubi [A] time = 0.21, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{2c\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{15b^3x^2}-\frac{\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{15b^2x^4}-\frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(5*b*x^6) - ((5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/
(15*b^2*x^4) + (2*c*(5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6} + \frac{\left(-3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{5b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6} - \frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4} - \frac{(c(5bB-4Ac))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{15b^2}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6} - \frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4} + \frac{2c(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^3x^2}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A\left(-3b^2+4bcx^2-8c^2x^4\right)-5bBx^2\left(b-2cx^2\right)\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b*B*x^2*(b - 2*c*x^2) + A*(-3*b^2 + 4*b*c*x^2 -
8*c^2*x^4)))/(15*b^3*x^6)
```

fricas [A] time = 0.68, size = 62, normalized size = 0.65

$$\frac{\left(2\left(5\,Bbc-4\,Ac^2\right)x^4-3\,Ab^2-\left(5\,Bb^2-4\,Abc\right)x^2\right)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*b*c - 4*A*c^2)*x^4 - 3*A*b^2 - (5*B*b^2 - 4*A*b*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^3*x^6)
```

giac [A] time = 0.21, size = 153, normalized size = 1.59

$$\frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3B\sqrt{c}+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^2Bb+20\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^2Ac+15\left(\sqrt{c}\,x^2-\sqrt{cx^4}\right.}{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^3*B*sqrt(c) + 5*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2))^2*B*b + 20*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^2*A*c
+ 15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*A*b*sqrt(c) + 3*A*b^2)/(sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2))^5
```

maple [A] time = 0.05, size = 70, normalized size = 0.73

$$-\frac{\left(c\,x^2+b\right)\left(8A\,c^2x^4-10Bbc\,x^4-4Abc\,x^2+5B\,b^2x^2+3b^2A\right)}{15\sqrt{c\,x^4+b\,x^2}\;b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(8*A*c^2*x^4-10*B*b*c*x^4-4*A*b*c*x^2+5*B*b^2*x^2+3*A*b^2)/
x^4/b^3/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.49, size = 119, normalized size = 1.24

$$\frac{1}{3}B\left(\frac{2\sqrt{cx^4+bx^2}\,c}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}}{bx^4}\right)-\frac{1}{15}A\left(\frac{8\sqrt{cx^4+bx^2}\,c^2}{b^3x^2}-\frac{4\sqrt{cx^4+bx^2}\,c}{b^2x^4}+\frac{3\sqrt{cx^4+bx^2}}{bx^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) - 1
/15*A*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^2*x
^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6))
```

mupad [B] time = 0.25, size = 62, normalized size = 0.65

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^2\,x^2+3\,A\,b^2-10\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2+8\,A\,c^2\,x^4\right)}{15\,b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*A*b^2 + 5*B*b^2*x^2 + 8*A*c^2*x^4 - 4*A*b*c*x^2
- 10*B*b*c*x^4))/(15*b^3*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{x^5\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

3.137 $\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=133

$$-\frac{8c^2\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{105b^4x^2}+\frac{4c\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{105b^3x^4}-\frac{\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{35b^2x^6}-\frac{A\sqrt{bx^2+cx^4}}{7bx^8}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(1/2)/b/x^8-1/35*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^
2/x^6+4/105*c*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^4-8/105*c^2*(-6*A*c+
7*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Rubi [A] time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$-\frac{8c^2\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{105b^4x^2}+\frac{4c\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{105b^3x^4}-\frac{\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{35b^2x^6}-\frac{A\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(7*b*x^8) - ((7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/
(35*b^2*x^6) + (4*c*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^4) - (8
*c^2*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^4*x^2)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
```

```
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} + \frac{\left(-4(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{7b}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{(2c(7bB-6Ac))\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx,x\right.}{35b^2}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} + \frac{\left(4c^2(7bB-\right.}{}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} - \frac{8c^2(7bB-}{1}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.67

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3A\left(5b^3-6b^2cx^2+8bc^2x^4-16c^3x^6\right)+7bBx^2\left(3b^2-4bcx^2+8c^2x^4\right)\right)}{105b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/105*(Sqrt[x^2*(b + c*x^2)]*(7*b*B*x^2*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4) +
3*A*(5*b^3 - 6*b^2*c*x^2 + 8*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^8)
```

fricas [A] time = 0.95, size = 86, normalized size = 0.65

$$-\frac{\left(8\left(7\,Bbc^2-6\,Ac^3\right)x^6-4\left(7\,Bb^2c-6\,Abc^2\right)x^4+15\,Ab^3+3\left(7\,Bb^3-6\,Ab^2c\right)x^2\right)\sqrt{cx^4+bx^2}}{105\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*b*c^2 - 6*A*c^3)*x^6 - 4*(7*B*b^2*c - 6*A*b*c^2)*x^4 + 15*A*
b^3 + 3*(7*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^8)
```

giac [A] time = 0.31, size = 219, normalized size = 1.65

$$\frac{140\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^4Bc+105\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3Bb\sqrt{c}+210\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3Ac^{\frac{3}{2}}+21\left(\sqrt{c}\right.}{105\left(\sqrt{c}\,x^2-\sqrt{cx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(140*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^4*B*c + 105*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2))^3*B*b*sqrt(c) + 210*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))
^3*A*c^(3/2) + 21*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^2*B*b^2 + 252*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2))^2*A*b*c + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2))*A*b^2*sqrt(c) + 15*A*b^3)/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^7
```

maple [A] time = 0.05, size = 94, normalized size = 0.71

$$-\frac{\left(c\,x^{2}+b\right)\left(-48A\,c^{3}x^{6}+56Bb\,c^{2}x^{6}+24Ab\,c^{2}x^{4}-28B\,b^{2}c\,x^{4}-18A\,b^{2}c\,x^{2}+21B\,b^{3}x^{2}+15A\,b^{3}\right)}{105\sqrt{c\,x^{4}+b\,x^{2}}\;b^{4}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/105*(c*x^2+b)*(-48*A*c^3*x^6+56*B*b*c^2*x^6+24*A*b*c^2*x^4-28*B*b^2*c*x^
4-18*A*b^2*c*x^2+21*B*b^3*x^2+15*A*b^3)/x^6/b^4/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.49, size = 167, normalized size = 1.26

$$-\frac{1}{15}\,B\left(\frac{8\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{b^{3}x^{2}}-\frac{4\,\sqrt{cx^{4}+bx^{2}}\,c}{b^{2}x^{4}}+\frac{3\,\sqrt{cx^{4}+bx^{2}}}{bx^{6}}\right)+\frac{1}{35}\,A\left(\frac{16\,\sqrt{cx^{4}+bx^{2}}\,c^{3}}{b^{4}x^{2}}-\frac{8\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{b^{3}x^{4}}+\frac{6\,\sqrt{cx^{4}}}{b^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*B*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^2
*x^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6)) + 1/35*A*(16*sqrt(c*x^4 + b*x^2)*c^3
/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c/
(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^8))
```

mupad [B] time = 0.30, size = 121, normalized size = 0.91

$$\frac{\left(6\,A\,c-7\,B\,b\right)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^6}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,b\,x^8}-\frac{\left(24\,A\,c^2-28\,B\,b\,c\right)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^4}+\frac{\left(48\,A\,c^3-56\,B\,b\,c^2\right)\,\sqrt{c\,x^4+b}}{105\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] ((6*A*c - 7*B*b)*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (A*(b*x^2 + c*x^4)^(
1/2))/(7*b*x^8) - ((24*A*c^2 - 28*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x^
4) + ((48*A*c^3 - 56*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(105*b^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^{2}}{x^{7}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

3.138 $\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=170

$$\frac{16c^3\sqrt{bx^2+cx^4}\,(9bB-8Ac)}{315b^5x^2}-\frac{8c^2\sqrt{bx^2+cx^4}\,(9bB-8Ac)}{315b^4x^4}+\frac{2c\sqrt{bx^2+cx^4}\,(9bB-8Ac)}{105b^3x^6}-\frac{\sqrt{bx^2+cx^4}\,(9bB-8A}{63b^2x^8}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(1/2)/b/x^10-1/63*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b
^2/x^8+2/105*c*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^6-8/315*c^2*(-8*A*c
+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^4+16/315*c^3*(-8*A*c+9*B*b)*(c*x^4+b*x^2)
^(1/2)/b^5/x^2
```

Rubi [A] time = 0.30, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 650}

$$\frac{16c^3\sqrt{bx^2+cx^4}\,(9bB-8Ac)}{315b^5x^2}-\frac{8c^2\sqrt{bx^2+cx^4}\,(9bB-8Ac)}{315b^4x^4}+\frac{2c\sqrt{bx^2+cx^4}\,(9bB-8Ac)}{105b^3x^6}-\frac{\sqrt{bx^2+cx^4}\,(9bB-8A}{63b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(9*b*x^10) - ((9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])
/(63*b^2*x^8) + (2*c*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^6) - (
8*c^2*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^4*x^4) + (16*c^3*(9*b*B -
 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^5*x^2)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
```

```
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} + \frac{\left(-5(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{9b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} - \frac{(c(9bB-8Ac))\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{21b^2}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} + \frac{\left(4c^2(9bB-8A\right.}{}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(9bB-8A}{315b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(9bB-8A}{315b}$$

Mathematica [A] time = 0.10, size = 94, normalized size = 0.55

$$\frac{x^2\left(\frac{cx^2}{b}+1\right)\left(5b^3-6b^2cx^2+8bc^2x^4-16c^3x^6\right)(8Ac-9bB)-35Ab^3\left(b+cx^2\right)}{315b^4x^8\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-35*A*b^3*(b + c*x^2) + (-9*b*B + 8*A*c)*x^2*(1 + (c*x^2)/b)*(5*b^3 - 6*b^
2*c*x^2 + 8*b*c^2*x^4 - 16*c^3*x^6))/(315*b^4*x^8*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.24, size = 110, normalized size = 0.65

$$\frac{\left(16\left(9\,Bbc^3-8\,Ac^4\right)x^8-8\left(9\,Bb^2c^2-8\,Abc^3\right)x^6-35\,Ab^4+6\left(9\,Bb^3c-8\,Ab^2c^2\right)x^4-5\left(9\,Bb^4-8\,Ab^3c\right)x^2\right)\sqrt{cx^4}}{315\,b^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(16*(9*B*b*c^3 - 8*A*c^4)*x^8 - 8*(9*B*b^2*c^2 - 8*A*b*c^3)*x^6 - 35*
A*b^4 + 6*(9*B*b^3*c - 8*A*b^2*c^2)*x^4 - 5*(9*B*b^4 - 8*A*b^3*c)*x^2)*sqrt
(c*x^4 + b*x^2)/(b^5*x^10)
```

giac [A] time = 0.27, size = 287, normalized size = 1.69

$$\frac{630\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^5Bc^{\frac{3}{2}}+756\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^4Bbc+1008\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^4Ac^2+315\left(\sqrt{c}\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(630*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^5*B*c^(3/2) + 756*(sqrt(c)*x
^2 - sqrt(c*x^4 + b*x^2))^4*B*b*c + 1008*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2)
)^4*A*c^2 + 315*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^3*B*b^2*sqrt(c) + 1680*
(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^3*A*b*c^(3/2) + 45*(sqrt(c)*x^2 - sqrt(
c*x^4 + b*x^2))^2*B*b^3 + 1080*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^2*A*b^2*
c + 315*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*A*b^3*sqrt(c) + 35*A*b^4)/(sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2))^9
```

maple [A] time = 0.05, size = 118, normalized size = 0.69

$$-\frac{\left(c\,x^{2}+b\right)\left(128A\,c^{4}x^{8}-144Bb\,c^{3}x^{8}-64Ab\,c^{3}x^{6}+72B\,b^{2}c^{2}x^{6}+48A\,b^{2}c^{2}x^{4}-54B\,b^{3}c\,x^{4}-40A\,b^{3}c\,x^{2}+45B}{315\sqrt{c\,x^{4}+b\,x^{2}}\;b^{5}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/315*(c*x^2+b)*(128*A*c^4*x^8-144*B*b*c^3*x^8-64*A*b*c^3*x^6+72*B*b^2*c^2
*x^6+48*A*b^2*c^2*x^4-54*B*b^3*c*x^4-40*A*b^3*c*x^2+45*B*b^4*x^2+35*A*b^4)/
x^8/b^5/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.54, size = 215, normalized size = 1.26

$$\frac{1}{35}B\left(\frac{16\sqrt{cx^4+bx^2}\,c^3}{b^4x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^2}{b^3x^4}+\frac{6\sqrt{cx^4+bx^2}\,c}{b^2x^6}-\frac{5\sqrt{cx^4+bx^2}}{bx^8}\right)-\frac{1}{315}A\left(\frac{128\sqrt{cx^4+bx^2}\,c^4}{b^5x^2}-\frac{64}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/35*B*(16*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(b
^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^8)
) - 1/315*A*(128*sqrt(c*x^4 + b*x^2)*c^4/(b^5*x^2) - 64*sqrt(c*x^4 + b*x^2)
*c^3/(b^4*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^6) - 40*sqrt(c*x^4 + b*x
^2)*c/(b^2*x^8) + 35*sqrt(c*x^4 + b*x^2)/(b*x^10))
```

mupad [B] time = 0.32, size = 156, normalized size = 0.92

$$\frac{(8\,A\,c-9\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{63\,b^2\,x^8}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{9\,b\,x^{10}}-\frac{\left(16\,A\,c^2-18\,B\,b\,c\right)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^6}+\frac{\left(64\,A\,c^3-72\,B\,b\,c^2\right)\,\sqrt{c\,x^4+}}{315\,b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^9*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] ((8*A*c - 9*B*b)*(b*x^2 + c*x^4)^(1/2))/(63*b^2*x^8) - (A*(b*x^2 + c*x^4)^(
1/2))/(9*b*x^10) - ((16*A*c^2 - 18*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x
^6) + ((64*A*c^3 - 72*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^4) - ((128
*A*c^4 - 144*B*b*c^3)*(b*x^2 + c*x^4)^(1/2))/(315*b^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{x^9\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**9/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**9*sqrt(x**2*(b + c*x**2))), x)
```

3.139 $\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=131

$$-\frac{8b^2\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{105c^4x}+\frac{4bx\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{105c^3}-\frac{x^3\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] -8/105*b^2*(-7*A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x+4/105*b*(-7*A*c+6*B*b)*
x*(c*x^4+b*x^2)^(1/2)/c^3-1/35*(-7*A*c+6*B*b)*x^3*(c*x^4+b*x^2)^(1/2)/c^2+1
/7*B*x^5*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2039, 2016, 1588}

$$-\frac{8b^2\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{105c^4x}-\frac{x^3\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{35c^2}+\frac{4bx\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{105c^3}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-8*b^2*(6*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(105*c^4*x) + (4*b*(6*b*B - 7*
A*c)*x*Sqrt[b*x^2 + c*x^4])/(105*c^3) - ((6*b*B - 7*A*c)*x^3*Sqrt[b*x^2 + c
*x^4])/(35*c^2) + (B*x^5*Sqrt[b*x^2 + c*x^4])/(7*c)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} - \frac{(6bB-7Ac)\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} + \frac{(4b(6bB-7Ac))\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{35c^2}$$

$$= \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} - \frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} - \frac{(8b^2(6b\ldots}{}$$

$$= -\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x} + \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} - \frac{(6bB-7Ac)x^3\sqrt{bx^2+c\ldots}}{35c^2}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.65

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(8b^2c\left(7A+3Bx^2\right)-2bc^2x^2\left(14A+9Bx^2\right)+3c^3x^4\left(7A+5Bx^2\right)-48b^3B\right)}{105c^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-48*b^3*B + 8*b^2*c*(7*A + 3*B*x^2) + 3*c^3*x^4*(7*
A + 5*B*x^2) - 2*b*c^2*x^2*(14*A + 9*B*x^2)))/(105*c^4*x)
```

fricas [A] time = 0.82, size = 83, normalized size = 0.63

$$\frac{\left(15\,Bc^3x^6-3\left(6\,Bbc^2-7\,Ac^3\right)x^4-48\,Bb^3+56\,Ab^2c+4\left(6\,Bb^2c-7\,Abc^2\right)x^2\right)\sqrt{cx^4+bx^2}}{105\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*c^3*x^6 - 3*(6*B*b*c^2 - 7*A*c^3)*x^4 - 48*B*b^3 + 56*A*b^2*c +
 4*(6*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^6}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^6/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.05, size = 89, normalized size = 0.68

$$\frac{\left(c\,x^2+b\right)\left(15B\,c^3x^6+21A\,c^3x^4-18Bb\,c^2x^4-28Ab\,c^2x^2+24B\,b^2c\,x^2+56A\,b^2c-48B\,b^3\right)x}{105\sqrt{c\,x^4+b\,x^2}\;c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/105*(c*x^2+b)*(15*B*c^3*x^6+21*A*c^3*x^4-18*B*b*c^2*x^4-28*A*b*c^2*x^2+24
*B*b^2*c*x^2+56*A*b^2*c-48*B*b^3)*x/c^4/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.53, size = 106, normalized size = 0.81

$$\frac{\left(3\,c^3x^6 - bc^2x^4 + 4\,b^2cx^2 + 8\,b^3\right)A}{15\,\sqrt{cx^2+b}\,c^3} + \frac{\left(5\,c^4x^8 - bc^3x^6 + 2\,b^2c^2x^4 - 8\,b^3cx^2 - 16\,b^4\right)B}{35\,\sqrt{cx^2+b}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^3*x^6 - b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*A/(sqrt(c*x^2 + b)*c^3)
+ 1/35*(5*c^4*x^8 - b*c^3*x^6 + 2*b^2*c^2*x^4 - 8*b^3*c*x^2 - 16*b^4)*B/(sq
rt(c*x^2 + b)*c^4)
```

mupad [B] time = 0.26, size = 87, normalized size = 0.66

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{48\,B\,b^3-56\,A\,b^2\,c}{105\,c^4}-\frac{B\,x^6}{7\,c}-\frac{x^4\,\left(21\,A\,c^3-18\,B\,b\,c^2\right)}{105\,c^4}+\frac{4\,b\,x^2\,\left(7\,A\,c-6\,B\,b\right)}{105\,c^3}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*((48*B*b^3 - 56*A*b^2*c)/(105*c^4) - (B*x^6)/(7*c)
- (x^4*(21*A*c^3 - 18*B*b*c^2))/(105*c^4) + (4*b*x^2*(7*A*c - 6*B*b))/(105*
c^3)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**6*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.140 $\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=94

$$\frac{2b\sqrt{bx^2+cx^4}\,(4bB-5Ac)}{15c^3x}-\frac{x\sqrt{bx^2+cx^4}\,(4bB-5Ac)}{15c^2}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

```
[Out] 2/15*b*(-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x-1/15*(-5*A*c+4*B*b)*x*(c*x^
4+b*x^2)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.19, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2039, 2016, 1588}

$$-\frac{x\sqrt{bx^2+cx^4}\,(4bB-5Ac)}{15c^2}+\frac{2b\sqrt{bx^2+cx^4}\,(4bB-5Ac)}{15c^3x}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(15*c^3*x) - ((4*b*B - 5*A*c)*x*S
qrt[b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[b*x^2 + c*x^4])/(5*c)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^4 \left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} - \frac{(4bB-5Ac)\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$
$$= -\frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} + \frac{(2b(4bB-5Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}$$
$$= \frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x} - \frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-2bc\left(5A+2Bx^2\right)+c^2x^2\left(5A+3Bx^2\right)+8b^2B\right)}{15c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(8*b^2*B - 2*b*c*(5*A + 2*B*x^2) + c^2*x^2*(5*A + 3*
B*x^2)))/(15*c^3*x)
```

fricas [A] time = 1.12, size = 59, normalized size = 0.63

$$\frac{\left(3\,Bc^2x^4+8\,Bb^2-10\,Abc-\left(4\,Bbc-5\,Ac^2\right)x^2\right)\sqrt{cx^4+bx^2}}{15\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^4 + 8*B*b^2 - 10*A*b*c - (4*B*b*c - 5*A*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^4}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.05, size = 65, normalized size = 0.69

$$-\frac{\left(c\,x^2+b\right)\left(-3B\,c^2x^4-5A\,c^2x^2+4Bbc\,x^2+10Abc-8B\,b^2\right)x}{15\sqrt{c\,x^4+b\,x^2}\;c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(-3*B*c^2*x^4-5*A*c^2*x^2+4*B*b*c*x^2+10*A*b*c-8*B*b^2)*x/c
^3/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.54, size = 83, normalized size = 0.88

$$\frac{\left(c^2x^4-bcx^2-2\,b^2\right)A}{3\,\sqrt{cx^2+b}\,c^2} + \frac{\left(3\,c^3x^6-bc^2x^4+4\,b^2cx^2+8\,b^3\right)B}{15\,\sqrt{cx^2+b}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*A/(sqrt(c*x^2 + b)*c^2) + 1/15*(3*c^3*x^6 -
 b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*B/(sqrt(c*x^2 + b)*c^3)
```

mupad [B] time = 0.23, size = 64, normalized size = 0.68

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^2-10\,A\,b\,c}{15\,c^3}+\frac{x^2\,\left(5\,A\,c^2-4\,B\,b\,c\right)}{15\,c^3}+\frac{B\,x^4}{5\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((8*B*b^2 - 10*A*b*c)/(15*c^3) + (x^2*(5*A*c^2 - 4*B
*b*c))/(15*c^3) + (B*x^4)/(5*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.141 $\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c}-\frac{\sqrt{bx^2+cx^4}\,(2bB-3Ac)}{3c^2x}$$

```
[Out] -1/3*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*B*x*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.14, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2039, 1588}

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c}-\frac{\sqrt{bx^2+cx^4}\,(2bB-3Ac)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (B*x*Sqrt[b*x^2 + c*x^4]
)/(3*c)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{Bx\sqrt{bx^2+cx^4}}{3c}-\frac{(2bB-3Ac)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\ &= -\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x}+\frac{Bx\sqrt{bx^2+cx^4}}{3c}\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.68

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3Ac-2bB+Bcx^2\right)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*b*B + 3*A*c + B*c*x^2))/(3*c^2*x)
```

fricas [A] time = 0.68, size = 36, normalized size = 0.61

$$\frac{\sqrt{cx^4+bx^2}\left(Bcx^2-2\,Bb+3\,Ac\right)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*B*b + 3*A*c)/(c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^2+A\right)x^2}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.05, size = 42, normalized size = 0.71

$$\frac{\left(c\,x^2+b\right)\left(Bc\,x^2+3Ac-2bB\right)x}{3\sqrt{c\,x^4+b\,x^2}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
[Out] 1/3*(c*x^2+b)*(B*c*x^2+3*A*c-2*B*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.46, size = 50, normalized size = 0.85

$$\frac{\sqrt{cx^2+b}\,A}{c}+\frac{\left(c^2x^4-bcx^2-2\,b^2\right)B}{3\,\sqrt{cx^2+b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(c*x^2 + b)*A/c + 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*B/(sqrt(c*x^2 + b)*c^
2)
```

mupad [B] time = 0.19, size = 41, normalized size = 0.69

$$\frac{\left(\frac{3\,A\,c-2\,B\,b}{3\,c^2}+\frac{B\,x^2}{3\,c}\right)\sqrt{c\,x^4+b\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] (((3*A*c - 2*B*b)/(3*c^2) + (B*x^2)/(3*c))*(b*x^2 + c*x^4)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.142 $\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=55

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+B*(c*x^4+b*x^2)^(1/2)/c/x
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1145, 2008, 206}

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(c*x) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} \, dx &= \frac{B\sqrt{bx^2+cx^4}}{cx}+A\int \frac{1}{\sqrt{bx^2+cx^4}} \, dx\\ &= \frac{B\sqrt{bx^2+cx^4}}{cx}-A\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.33

$$\frac{x\left(\sqrt{b}\,B\left(b+cx^2\right)-Ac\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{\sqrt{b}\,c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[b]*B*(b + c*x^2) - A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(Sqrt[b]*c*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.70, size = 138, normalized size = 2.51

$$\left[\frac{A\sqrt{b}\,cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,Bb}{2\,bcx},\ \frac{A\sqrt{-b}\,cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,Bb}{bcx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) + 2*sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x), (A*sqrt(-b)*c*x*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x)]
```

giac [A] time = 0.33, size = 60, normalized size = 1.09

$$\frac{A\log\left(\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2\right)}{2\,\sqrt{b}}-\frac{2\,B\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*A*log((sqrt(c + b/x^2) - sqrt(b)/x)^2)/sqrt(b) - 2*B*sqrt(b)/((sqrt(c +
 b/x^2) - sqrt(b)/x)^2 - c)
```

maple [A] time = 0.06, size = 72, normalized size = 1.31

$$-\frac{\sqrt{c\,x^2+b}\,\left(Ac\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^2+b}\,B\sqrt{b}\right)x}{\sqrt{c\,x^4+b\,x^2}\,\sqrt{b}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -x*(c*x^2+b)^(1/2)*(A*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*c-B*(c*x^2+b)^(1/
2)*b^(1/2))/(c*x^4+b*x^2)^(1/2)/c/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{Bx^2+A}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.143 $\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=68

$$-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] -1/2*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*A*(c*x
^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2038, 2008, 206}

$$-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(2*b*x^3) - ((2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqr
t[b*x^2 + c*x^4]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} - \frac{(-2bB+Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} + \frac{(-2bB+Ac)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} - \frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.09, size = 87, normalized size = 1.28

$$\frac{x\sqrt{b+cx^2}\left(-\frac{2\left(bB-\frac{Ac}{2}\right)\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{A\sqrt{b+cx^2}}{bx^2}\right)}{2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(-((A*Sqrt[b + c*x^2])/(b*x^2)) - (2*(b*B - (A*c)/2)*Arc
Tanh[Sqrt[b + c*x^2]/Sqrt[b]])/b^(3/2)))/(2*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.13, size = 152, normalized size = 2.24

$$\left[-\frac{(2\,Bb-Ac)\sqrt{b}\,x^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,Ab}{4\,b^2x^3}, \frac{(2\,Bb-Ac)\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)}{2\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*b - A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2
)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*A*b)/(b^2*x^3), 1/2*((2*B*b - A*c)*
sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^
4 + b*x^2)*A*b)/(b^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]-4*A/8/b/x*sqrt(b*(1/x)^2+c)+2*(-A*c+2*B
*b)/4/b/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.06, size = 105, normalized size = 1.54

$$-\frac{\sqrt{c\,x^2+b}\,\left(-Abc\,x^2\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)+2B\,b^2x^2\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)+\sqrt{c\,x^2+b}\,A\,b^{\frac{3}{2}}\right)}{2\sqrt{c\,x^4+b\,x^2}\,b^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(2*B*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*b^2-A*l
n(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*b*c+A*b^(3/2)*(c*x^2+b)^(1/2))/(c*x^
4+b*x^2)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\, x^2 + A}{x^2\, \sqrt{c\, x^4 + b\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2 \sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

3.144 $\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=103

$$\frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}-\frac{\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{8b^2x^3}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] 1/8*c*(-3*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*A*(
c*x^4+b*x^2)^(1/2)/b/x^5-1/8*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2038, 2025, 2008, 206}

$$-\frac{\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{8b^2x^3}+\frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[b*x^2 + c*x^4])/(4*b*x^5) - ((4*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/
(8*b^2*x^3) + (c*(4*b*B - 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/
(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(-4bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{(c(4bB-3Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{(c(4bB-3Ac))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.22, size = 104, normalized size = 1.01

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(b\sqrt{\frac{cx^2}{b}+1}\left(2Ab-3Acx^2+4bBx^2\right)+cx^4(3Ac-4bB)\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)\right)}{8b^3x^5\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/8*(Sqrt[x^2*(b + c*x^2)]*(b*(2*A*b + 4*b*B*x^2 - 3*A*c*x^2)*Sqrt[1 + (c*
x^2)/b] + c*(-4*b*B + 3*A*c)*x^4*ArcTanh[Sqrt[1 + (c*x^2)/b]]))/(b^3*x^5*Sq
rt[1 + (c*x^2)/b])
```

fricas [A] time = 0.98, size = 199, normalized size = 1.93

$$\left[-\frac{\left(4\,Bbc-3\,Ac^2\right)\sqrt{b}\,x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,Ab^2+\left(4\,Bb^2-3\,Abc\right)x^2\right)}{16\,b^3x^5}, -\frac{\left(4\,Bbc-3\,A\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - 3*A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 - 3*A*b*
c)*x^2))/(b^3*x^5), -1/8*((4*B*b*c - 3*A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^
2 - 3*A*b*c)*x^2))/(b^3*x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
```

```
meters values [66.1769613782,93,91]2*(-4*b^2*A/32/b^3/x/x-(8*b^2*B-6*b*A*c)
/32/b^3)/x*sqrt(b*(1/x)^2+c)+2*(3*A*c^2-4*B*b*c)/16/b^2/sqrt(b)*ln(abs(sqrt
(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.06, size = 146, normalized size = 1.42

$$-\frac{\sqrt{c\,x^2+b}\,\left(3Ab\,c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-4B\,b^2c\,x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,A\,b^{\frac{3}{2}}c\,x^2+4\sqrt{c\,x^2+b}\right.}{8\sqrt{c\,x^4+b\,x^2}\,b^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*A*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*b*c^2+4*B
*(c*x^2+b)^(1/2)*b^(5/2)*x^2-4*B*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*b^
2*c-3*A*(c*x^2+b)^(1/2)*b^(3/2)*x^2*c+2*A*(c*x^2+b)^(1/2)*b^(5/2))/x^3/(c*x
^4+b*x^2)^(1/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^4\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

3.145 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}+\frac{5b\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{16c^4}-\frac{5x^2\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{24c^3}+\frac{x^4\sqrt{bx^2+cx^4}\,(7}{6bc^2}$$

```
[Out] -5/16*b^2*(-6*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-(
-A*c+B*b)*x^8/b/c/(c*x^4+b*x^2)^(1/2)+5/16*b*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(
1/2)/c^4-5/24*(-6*A*c+7*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/c^3+1/6*(-6*A*c+7*B*b)
*x^4*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] time = 0.33, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 788, 670, 640, 620, 206}

$$-\frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}+\frac{x^4\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{6bc^2}-\frac{5x^2\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{24c^3}+\frac{5b\sqrt{bx^2+cx^4}\,(7}{16c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^8)/(b*c*Sqrt[b*x^2 + c*x^4])) + (5*b*(7*b*B - 6*A*c)*Sqrt[
b*x^2 + c*x^4])/(16*c^4) - (5*(7*b*B - 6*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(24*
c^3) + ((7*b*B - 6*A*c)*x^4*Sqrt[b*x^2 + c*x^4])/(6*b*c^2) - (5*b^2*(7*b*B
- 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{6A}{b}+\frac{7B}{c}\right)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{(5(7bB-6Ac))\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x\right.}{12c^2}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} + \frac{(5b(7bI}{}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-}{}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-}{}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-}{}$$

Mathematica [A] time = 0.26, size = 136, normalized size = 0.74

$$\frac{x\left(\sqrt{c}\,x\left(b^2\left(35Bcx^2-90Ac\right)-2bc^2x^2\left(15A+7Bx^2\right)+4c^3x^4\left(3A+2Bx^2\right)+105b^3B\right)-15b^{5/2}\sqrt{\frac{cx^2}{b}+1}\,(7bB-\right.}{48c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(105*b^3*B + 4*c^3*x^4*(3*A + 2*B*x^2) - 2*b*c^2*x^2*(15*A +
7*B*x^2) + b^2*(-90*A*c + 35*B*c*x^2)) - 15*b^(5/2)*(7*b*B - 6*A*c)*Sqrt[1
+ (c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(48*c^(9/2)*Sqrt[x^2*(b + c*x^2
)])
```

fricas [A] time = 0.68, size = 340, normalized size = 1.85

$$\left[-\frac{15\left(7\,Bb^4-6\,Ab^3c+\left(7\,Bb^3c-6\,Ab^2c^2\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(8\,Bc^4x^6+105\,Bb^3c-9\right.}{96\left(c^6x^2+bc^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(7*B*b^4 - 6*A*b^3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(c)*lo
g(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(8*B*c^4*x^6 + 105*B*b^
3*c - 90*A*b^2*c^2 - 2*(7*B*b*c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c
^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^6*x^2 + b*c^5), 1/48*(15*(7*B*b^4 - 6*A*b^
3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sq
rt(-c)/(c*x^2 + b)) + (8*B*c^4*x^6 + 105*B*b^3*c - 90*A*b^2*c^2 - 2*(7*B*b*
c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/
(c^6*x^2 + b*c^5)]
```

giac [A] time = 0.31, size = 176, normalized size = 0.96

$$\frac{1}{48}\,\sqrt{cx^4+bx^2}\left(2\,x^2\left(\frac{4\,Bx^2}{c^2}-\frac{11\,Bbc^{10}-6\,Ac^{11}}{c^{13}}\right)+\frac{3\left(19\,Bb^2c^9-14\,Abc^{10}\right)}{c^{13}}\right)+\frac{5\left(7\,Bb^3-6\,Ab^2c\right)\log\left(\left|-2\left(\sqrt{c}\,x^2\right.\right.\right.}{32\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2)*(2*x^2*(4*B*x^2/c^2 - (11*B*b*c^10 - 6*A*c^11)/c^1
3) + 3*(19*B*b^2*c^9 - 14*A*b*c^10)/c^13) + 5/32*(7*B*b^3 - 6*A*b^2*c)*log(
abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(9/2) + (B*b^4 -
 A*b^3*c)/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*c + b*sqrt(c))*c^4)
```

maple [A] time = 0.06, size = 166, normalized size = 0.90

$$\frac{\left(c\,x^2+b\right)\left(8B\,c^{\frac{9}{2}}x^7+12A\,c^{\frac{9}{2}}x^5-14Bb\,c^{\frac{7}{2}}x^5-30Ab\,c^{\frac{7}{2}}x^3+35B\,b^2c^{\frac{5}{2}}x^3-90A\,b^2c^{\frac{5}{2}}x+105B\,b^3c^{\frac{3}{2}}x+90\sqrt{c\,x^2+b}\right.}{48\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/48*x^3*(c*x^2+b)*(8*B*c^(9/2)*x^7+12*A*c^(9/2)*x^5-14*B*c^(7/2)*x^5*b-30*
A*c^(7/2)*x^3*b+35*B*c^(5/2)*x^3*b^2-90*A*c^(5/2)*x*b^2+105*B*c^(3/2)*x*b^3
+90*A*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c^2-105*B*(c*x^2+b)
^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3*c)/(c*x^4+b*x^2)^(3/2)/c^(11/2)
```

maxima [A] time = 1.49, size = 237, normalized size = 1.29

$$\frac{1}{16}\left(\frac{4\,x^6}{\sqrt{cx^4+bx^2}\,c}-\frac{10\,bx^4}{\sqrt{cx^4+bx^2}\,c^2}-\frac{30\,b^2x^2}{\sqrt{cx^4+bx^2}\,c^3}+\frac{15\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}\right)A+\frac{1}{96}\left(\frac{16\,x^8}{\sqrt{cx^4+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) -
30*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(7/2))*A + 1/96*(16*x^8/(sqrt(c*x^4 + b*x^2)*c) - 28*
b*x^6/(sqrt(c*x^4 + b*x^2)*c^2) + 70*b^2*x^4/(sqrt(c*x^4 + b*x^2)*c^3) + 21
0*b^3*x^2/(sqrt(c*x^4 + b*x^2)*c^4) - 105*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(9/2))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{9} \left(A + B x^{2}\right)}{\left(x^{2} \left(b + c x^{2}\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.146 $\int \frac{x^7\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{3b(5bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{3\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{8c^3}+\frac{x^2\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{4bc^2}-\frac{x^6(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] 3/8*b*(-4*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)-(-A*c
+B*b)*x^6/b/c/(c*x^4+b*x^2)^(1/2)-3/8*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^
3+1/4*(-4*A*c+5*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] time = 0.28, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2034, 788, 670, 640, 620, 206}

$$\frac{x^2\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{4bc^2}-\frac{3\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{8c^3}+\frac{3b(5bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{x^6(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^6)/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(5*b*B - 4*A*c)*Sqrt[b*
x^2 + c*x^4])/(8*c^3) + ((5*b*B - 4*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(4*b*c^2)
 + (3*b*(5*b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
```

```
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{4A}{b}+\frac{5B}{c}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} - \frac{(3(5bB-4Ac))\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x\right.}{8c^2}$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{(3b(5bB-}{}$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{(3b(5bB-}{}$$
$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{3b(5bB-}{}$$

Mathematica [A] time = 0.21, size = 113, normalized size = 0.77

$$\frac{x\left(3b^{3/2}\sqrt{\frac{cx^2}{b}+1}\,(5bB-4Ac)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\left(bc\left(12A-5Bx^2\right)+2c^2x^2\left(2A+Bx^2\right)-15b^2B\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(-15*b^2*B + b*c*(12*A - 5*B*x^2) + 2*c^2*x^2*(2*A + B*x^2))
+ 3*b^(3/2)*(5*b*B - 4*A*c)*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]
]))/(8*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.02, size = 289, normalized size = 1.97

$$\left[-\frac{3\left(5\,Bb^3-4\,Ab^2c+\left(5\,Bb^2c-4\,Abc^2\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(2\,Bc^3x^4-15\,Bb^2c+1\right.}{16\left(c^5x^2+bc^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(2*B*c^3*x^4 - 15*B*b^2*c
+ 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2 + b
*c^4), -1/8*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(-c)
*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^3*x^4 - 15*B*b^2
*c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2
+ b*c^4)]
```

giac [A] time = 0.28, size = 147, normalized size = 1.00

$$\frac{1}{8}\sqrt{cx^4+bx^2}\left(\frac{2\,Bx^2}{c^2}-\frac{7\,Bbc^5-4\,Ac^6}{c^8}\right)-\frac{3\left(5\,Bb^2-4\,Abc\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{7}{2}}}-\frac{\cdots}{\left(\left(\sqrt{c}\,x^2-\sqrt{\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*(2*B*x^2/c^2 - (7*B*b*c^5 - 4*A*c^6)/c^8) - 3/16*(5
*B*b^2 - 4*A*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) -
b))/c^(7/2) - (B*b^3 - A*b^2*c)/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*c + b
*sqrt(c))*c^3)
```

maple [A] time = 0.06, size = 140, normalized size = 0.95

$$-\frac{\left(c\,x^2+b\right)\left(-2B\,c^{\frac{7}{2}}x^5-4A\,c^{\frac{7}{2}}x^3+5Bb\,c^{\frac{5}{2}}x^3-12Ab\,c^{\frac{5}{2}}x+15B\,b^2c^{\frac{3}{2}}x+12\sqrt{c\,x^2+b}\;Ab\,c^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)\cdots\right)}{8\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/8*x^3*(c*x^2+b)*(-2*B*c^(7/2)*x^5-4*A*c^(7/2)*x^3+5*B*c^(5/2)*x^3*b-12*A
*c^(5/2)*x*b+15*B*c^(3/2)*x*b^2+12*A*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)
^(1/2))*b*c^2-15*B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*
x^4+b*x^2)^(3/2)/c^(9/2)
```

maxima [A] time = 1.54, size = 187, normalized size = 1.27

$$\frac{1}{4}\left(\frac{2\,x^4}{\sqrt{cx^4+bx^2}\,c}+\frac{6\,bx^2}{\sqrt{cx^4+bx^2}\,c^2}-\frac{3\,b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)A+\frac{1}{16}\left(\frac{4\,x^6}{\sqrt{cx^4+bx^2}\,c}-\frac{10\,bx^4}{\sqrt{cx^4+bx^2}\,c^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3*
b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*A + 1/16*(4*x^6
/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 30*b^2*x^2/
(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*
sqrt(c))/c^(7/2))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \left(A + Bx^2\right)}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**7*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.147 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}+\frac{\sqrt{bx^2+cx^4}\,(3bB-2Ac)}{2bc^2}-\frac{x^4(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -1/2*(-2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-(-A*c+
B*b)*x^4/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b/c
^2
```

Rubi [A] time = 0.24, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2034, 788, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}\,(3bB-2Ac)}{2bc^2}-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}-\frac{x^4(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^4)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - 2*A*c)*Sqrt[b*x^
2 + c*x^4])/(2*b*c^2) - ((3*b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{1}{2}\left(-\frac{2A}{b}+\frac{3B}{c}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2}-\frac{(3bB-2Ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c^2}\\ &= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2}-\frac{(3bB-2Ac)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+c}}\right.}{2c^2}\\ &= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2}-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 91, normalized size = 0.81

$$\frac{x\left(\sqrt{c}x\left(-2Ac+3bB+Bcx^2\right)-\sqrt{b}\sqrt{\frac{cx^2}{b}+1}\,(3bB-2Ac)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b*B - 2*A*c + B*c*x^2) - Sqrt[b]*(3*b*B - 2*A*c)*Sqrt[1 +
(c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.01, size = 230, normalized size = 2.05

$$\left[-\frac{\left(3\,Bb^2-2\,Abc+\left(3\,Bbc-2\,Ac^2\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(Bc^2x^2+3\,Bbc-2\,Ac^2\right)\sqrt{c}}{4\left(c^4x^2+bc^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2*A*c^2)*x^2)*sqrt(c)*log(-2*c*x^2 -
 b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(B*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqr
t(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3), 1/2*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2
*A*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (B
*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqrt(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3)]
```

giac [A] time = 0.26, size = 116, normalized size = 1.04

$$\frac{\sqrt{cx^4+bx^2}\,B}{2\,c^2}+\frac{(3\,Bb-2\,Ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{5}{2}}}+\frac{Bb^2-Abc}{\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)c+b\sqrt{c}\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2)*B/c^2 + 1/4*(3*B*b - 2*A*c)*log(abs(-2*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(5/2) + (B*b^2 - A*b*c)/(((sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2))*c + b*sqrt(c))*c^2)
```

maple [A] time = 0.06, size = 115, normalized size = 1.03

$$-\frac{\left(c\,x^{2}+b\right)\left(-B\,c^{\frac{5}{2}}x^{3}+2A\,c^{\frac{5}{2}}x-3Bb\,c^{\frac{3}{2}}x-2\sqrt{c\,x^{2}+b}\;A\,c^{2}\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+b}\right)+3\sqrt{c\,x^{2}+b}\;Bbc\ln\left(\sqrt{c}\;x+\right.\right.}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x^3*(c*x^2+b)*(-B*c^(5/2)*x^3+2*A*c^(5/2)*x-3*B*c^(3/2)*x*b-2*A*ln(c^(
1/2)*x+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*c^2+3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2
))*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

maxima [A] time = 1.51, size = 138, normalized size = 1.23

$$\frac{1}{4}\left(\frac{2\,x^{4}}{\sqrt{cx^{4}+bx^{2}}\,c}+\frac{6\,bx^{2}}{\sqrt{cx^{4}+bx^{2}}\,c^{2}}-\frac{3\,b\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)B-\frac{1}{2}\,A\left(\frac{2\,x^{2}}{\sqrt{cx^{4}+bx^{2}}\,c}-\frac{\log\left(2\,cx^{2}+b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3*
b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*B - 1/2*A*(2*x^
2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
)/c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5}\,\left(B\,x^{2}+A\right)}{{\left(c\,x^{4}+b\,x^{2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}\left(A+Bx^{2}\right)}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.148 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)-(-A*c+B*b)*x^2/b/c/(c*x^
4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 777, 620, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^2)/(b*c*Sqrt[b*x^2 + c*x^4])) + (B*ArcTanh[(Sqrt[c]*x^2)/S
qrt[b*x^2 + c*x^4]])/c^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^3 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{2c}$$
$$= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}$$
$$= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 1.12

$$\frac{x\left(\sqrt{c}\,x(Ac-bB)+b^{3/2}B\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{bc^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*(-(b*B) + A*c)*x + b^(3/2)*B*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[
c]*x)/Sqrt[b]]))/(b*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.92, size = 188, normalized size = 2.81

$$\left[\frac{\left(Bbcx^2+Bb^2\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\,\left(Bbc-Ac^2\right)}{2\left(bc^3x^2+b^2c^2\right)}, -\frac{\left(Bbcx^2+Bb^2\right)\sqrt{-c}\,\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*c*x^2 + B*b^2)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2), -(
(B*b*c*x^2 + B*b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{-2,[1]%%%},[2,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%
%%}+%%%{-2,[0,4]%%%} / %%%{%%%{1,[2]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1]%%%},[0,2]%%%} Error: Bad
Argument Value
```

maple [A] time = 0.05, size = 75, normalized size = 1.12

$$\frac{\left(c\,x^{2}+b\right)\left(A\,c^{\frac{5}{2}}x-Bb\,c^{\frac{3}{2}}x+\sqrt{c\,x^{2}+b}\;Bbc\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+b}\right)\right)x^{3}}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(A*c^(5/2)*x-B*b*c^(3/2)*x+(c*x^2+b)^(1/2)*B*b*c*ln(c^(1/2)*x
+(c*x^2+b)^(1/2)))/(c*x^4+b*x^2)^(3/2)/c^(5/2)/b
```

maxima [A] time = 1.48, size = 79, normalized size = 1.18

$$-\frac{1}{2}B\left(\frac{2\,x^{2}}{\sqrt{cx^{4}+bx^{2}}\,c}-\frac{\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}\right)+\frac{Ax^{2}}{\sqrt{cx^{4}+bx^{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*B*(2*x^2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*
x^2)*sqrt(c))/c^(3/2)) + A*x^2/(sqrt(c*x^4 + b*x^2)*b)
```

mupad [B] time = 0.63, size = 78, normalized size = 1.16

$$\frac{B\,\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}}\right)}{2\,c^{3/2}}+\frac{A\,x^{2}}{b\,\sqrt{c\,x^{4}+b\,x^{2}}}-\frac{B\,x^{2}}{c\,\sqrt{c\,x^{4}+b\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(3/2)) + (A*x^2
)/(b*(b*x^2 + c*x^4)^(1/2)) - (B*x^2)/(c*(b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}\left(A+Bx^{2}\right)}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.149 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{Ab - x^2(bB - 2Ac)}{b^2\sqrt{bx^2 + cx^4}}$$

```
[Out] (-A*b+(-2*A*c+B*b)*x^2)/b^2/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2034, 636}

$$-\frac{Ab - x^2(bB - 2Ac)}{b^2\sqrt{bx^2 + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]

[Out] -((A*b - (b*B - 2*A*c)*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{Ab-(bB-2Ac)x^2}{b^2\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{bBx^2 - A\left(b + 2cx^2\right)}{b^2\sqrt{x^2\left(b + cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]

[Out] (b*B*x^2 - A*(b + 2*c*x^2))/(b^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.17, size = 49, normalized size = 1.32

$$\frac{\sqrt{cx^4+bx^2}\left((Bb-2\,Ac)x^2-Ab\right)}{b^2cx^4+b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2)
```

giac [A] time = 0.21, size = 36, normalized size = 0.97

$$\frac{\frac{(Bb-2\,Ac)x^2}{b^2}-\frac{A}{b}}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((B*b - 2*A*c)*x^2/b^2 - A/b)/sqrt(c*x^4 + b*x^2)
```

maple [A] time = 0.05, size = 47, normalized size = 1.27

$$-\frac{\left(c\,x^2+b\right)\left(2Ac\,x^2-Bb\,x^2+Ab\right)x^2}{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -(c*x^2+b)*x^2*(2*A*c*x^2-B*b*x^2+A*b)/b^2/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.49, size = 65, normalized size = 1.76

$$-A\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}\,b^2}+\frac{1}{\sqrt{cx^4+bx^2}\,b}\right)+\frac{Bx^2}{\sqrt{cx^4+bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -A*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + B*x^2/
(sqrt(c*x^4 + b*x^2)*b)
```

mupad [B] time = 0.18, size = 53, normalized size = 1.43

$$-\frac{\left(\frac{A}{b}-x^2\left(\frac{B}{b}-\frac{2\,A\,c}{b^2}\right)\right)\sqrt{c\,x^4+b\,x^2}}{x\left(c\,x^3+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -((A/b - x^2*(B/b - (2*A*c)/b^2))*(b*x^2 + c*x^4)^(1/2))/(x*(b*x + c*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(A+Bx^2\right)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.150 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\left(b+2cx^2\right)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}}-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

```
[Out] -1/3*A/b/x^2/(c*x^4+b*x^2)^(1/2)-1/3*(-4*A*c+3*B*b)*(2*c*x^2+b)/b^3/(c*x^4+
b*x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2034, 792, 613}

$$-\frac{\left(b+2cx^2\right)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}}-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -A/(3*b*x^2*Sqrt[b*x^2 + c*x^4]) - ((3*b*B - 4*A*c)*(b + 2*c*x^2))/(3*b^3*S
qrt[b*x^2 + c*x^4])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}} + \frac{\left(bB-Ac+\frac{1}{2}(bB-2Ac)\right)\text{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{3b}$$

$$= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}} - \frac{(3bB-4Ac)\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.97

$$\frac{A\left(-b^2+4bcx^2+8c^2x^4\right)-3bBx^2\left(b+2cx^2\right)}{3b^3x^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-3*b*B*x^2*(b + 2*c*x^2) + A*(-b^2 + 4*b*c*x^2 + 8*c^2*x^4))/(3*b^3*x^2*Sq
rt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.91, size = 72, normalized size = 1.09

$$-\frac{\left(2\left(3\,Bbc-4\,Ac^2\right)x^4+Ab^2+\left(3\,Bb^2-4\,Abc\right)x^2\right)\sqrt{cx^4+bx^2}}{3\left(b^3cx^6+b^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*b*c - 4*A*c^2)*x^4 + A*b^2 + (3*B*b^2 - 4*A*b*c)*x^2)*sqrt(c*x
^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x), x)
```

maple [A] time = 0.05, size = 66, normalized size = 1.00

$$-\frac{\left(c\,x^2+b\right)\left(-8A\,c^2x^4+6Bbc\,x^4-4Abc\,x^2+3B\,b^2x^2+b^2A\right)}{3\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-8*A*c^2*x^4+6*B*b*c*x^4-4*A*b*c*x^2+3*B*b^2*x^2+A*b^2)/b^3
/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.39, size = 112, normalized size = 1.70

$$-B\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}\,b^2}+\frac{1}{\sqrt{cx^4+bx^2}\,b}\right)+\frac{1}{3}\,A\left(\frac{8\,c^2x^2}{\sqrt{cx^4+bx^2}\,b^3}+\frac{4\,c}{\sqrt{cx^4+bx^2}\,b^2}-\frac{1}{\sqrt{cx^4+bx^2}\,bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + 1/3*A*
(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(s
qrt(c*x^4 + b*x^2)*b*x^2))
```

mupad [B] time = 0.27, size = 70, normalized size = 1.06

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,B\,b^2\,x^2+A\,b^2+6\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2-8\,A\,c^2\,x^4\right)}{3\,b^3\,x^4\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(A*b^2 + 3*B*b^2*x^2 - 8*A*c^2*x^4 - 4*A*b*c*x^2 +
6*B*b*c*x^4))/(3*b^3*x^4*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

3.151 $\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{4c\left(b+2cx^2\right)(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

```
[Out] -1/5*A/b/x^4/(c*x^4+b*x^2)^(1/2)+1/15*(6*A*c-5*B*b)/b^2/x^2/(c*x^4+b*x^2)^(
1/2)+4/15*c*(-6*A*c+5*B*b)*(2*c*x^2+b)/b^4/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 613}

$$\frac{4c\left(b+2cx^2\right)(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -A/(5*b*x^4*Sqrt[b*x^2 + c*x^4]) - (5*b*B - 6*A*c)/(15*b^2*x^2*Sqrt[b*x^2 +
 c*x^4]) + (4*c*(5*b*B - 6*A*c)*(b + 2*c*x^2))/(15*b^4*Sqrt[b*x^2 + c*x^4])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}} + \frac{\left(\frac{1}{2}(bB-2Ac)-2(-bB+Ac)\right)\operatorname{Subst}\left(\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{5b}$$

$$= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}} - \frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}} - \frac{(2c(5bB-6Ac))\operatorname{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx, x\right.}{15b^2}$$

$$= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}} - \frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}} + \frac{4c(5bB-6Ac)\left(b+2cx^2\right)}{15b^4\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 0.84

$$\frac{-3A\left(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6\right)-5bBx^2\left(b^2-4bcx^2-8c^2x^4\right)}{15b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b*B*x^2*(b^2 - 4*b*c*x^2 - 8*c^2*x^4) - 3*A*(b^3 - 2*b^2*c*x^2 + 8*b*c^
2*x^4 + 16*c^3*x^6))/(15*b^4*x^4*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.89, size = 98, normalized size = 0.97

$$\frac{\left(8\left(5\,Bbc^2-6\,Ac^3\right)x^6+4\left(5\,Bb^2c-6\,Abc^2\right)x^4-3\,Ab^3-\left(5\,Bb^3-6\,Ab^2c\right)x^2\right)\sqrt{cx^4+bx^2}}{15\left(b^4cx^8+b^5x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*b*c^2 - 6*A*c^3)*x^6 + 4*(5*B*b^2*c - 6*A*b*c^2)*x^4 - 3*A*b^3
 - (5*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*c*x^8 + b^5*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^3), x)
```

maple [A] time = 0.05, size = 94, normalized size = 0.93

$$-\frac{\left(c\,x^2+b\right)\left(48A\,c^3x^6-40Bb\,c^2x^6+24Ab\,c^2x^4-20B\,b^2c\,x^4-6A\,b^2c\,x^2+5B\,b^3x^2+3A\,b^3\right)}{15\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\,b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(48*A*c^3*x^6-40*B*b*c^2*x^6+24*A*b*c^2*x^4-20*B*b^2*c*x^4-
6*A*b^2*c*x^2+5*B*b^3*x^2+3*A*b^3)/x^2/b^4/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.50, size = 160, normalized size = 1.58

$$\frac{1}{3}\,B\left(\frac{8\,c^2x^2}{\sqrt{cx^4+bx^2}\,b^3}+\frac{4\,c}{\sqrt{cx^4+bx^2}\,b^2}-\frac{1}{\sqrt{cx^4+bx^2}\,bx^2}\right)-\frac{1}{5}\,A\left(\frac{16\,c^3x^2}{\sqrt{cx^4+bx^2}\,b^4}+\frac{8\,c^2}{\sqrt{cx^4+bx^2}\,b^3}-\frac{2\,c}{\sqrt{cx^4+bx^2}\,b^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2)
- 1/(sqrt(c*x^4 + b*x^2)*b*x^2)) - 1/5*A*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b
^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*b^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) +
 1/(sqrt(c*x^4 + b*x^2)*b*x^4))
```

mupad [B] time = 0.42, size = 95, normalized size = 0.94

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^3\,x^2+3\,A\,b^3-20\,B\,b^2\,c\,x^4-6\,A\,b^2\,c\,x^2-40\,B\,b\,c^2\,x^6+24\,A\,b\,c^2\,x^4+48\,A\,c^3\,x^6\right)}{15\,b^4\,x^6\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*A*b^3 + 5*B*b^3*x^2 + 48*A*c^3*x^6 - 6*A*b^2*c*x
^2 + 24*A*b*c^2*x^4 - 20*B*b^2*c*x^4 - 40*B*b*c^2*x^6))/(15*b^4*x^6*(b + c*
x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

3.152 $\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{8c^2\left(b+2cx^2\right)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

```
[Out] -1/7*A/b/x^6/(c*x^4+b*x^2)^(1/2)+1/35*(8*A*c-7*B*b)/b^2/x^4/(c*x^4+b*x^2)^(
1/2)+2/35*c*(-8*A*c+7*B*b)/b^3/x^2/(c*x^4+b*x^2)^(1/2)-8/35*c^2*(-8*A*c+7*B
*b)*(2*c*x^2+b)/b^5/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2034, 792, 658, 613}

$$-\frac{8c^2\left(b+2cx^2\right)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -A/(7*b*x^6*Sqrt[b*x^2 + c*x^4]) - (7*b*B - 8*A*c)/(35*b^2*x^4*Sqrt[b*x^2 +
 c*x^4]) + (2*c*(7*b*B - 8*A*c))/(35*b^3*x^2*Sqrt[b*x^2 + c*x^4]) - (8*c^2*
(7*b*B - 8*A*c)*(b + 2*c*x^2))/(35*b^5*Sqrt[b*x^2 + c*x^4])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
```

```
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} + \frac{\left(\frac{1}{2}(bB-2Ac)-3(-bB+Ac)\right)\text{Subst}\left(\int \frac{1}{x^2\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{7b}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} - \frac{(3c(7bB-8Ac))\text{Subst}\left(\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx, x, x\right.}{35b^2}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} + \frac{\left(4c^2(7bB-8Ac)\right)\text{Sub}}{3}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} - \frac{8c^2(7bB-8Ac)\left(b+2\right.}{35b^5\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.54

$$\frac{x^2\left(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6\right)(8Ac-7bB)-5Ab^4}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*A*b^4 + (-7*b*B + 8*A*c)*x^2*(b^3 - 2*b^2*c*x^2 + 8*b*c^2*x^4 + 16*c^3*
x^6))/(35*b^5*x^6*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 1.11, size = 121, normalized size = 0.88

$$-\frac{\left(16\left(7\,Bbc^3-8\,Ac^4\right)x^8+8\left(7\,Bb^2c^2-8\,Abc^3\right)x^6+5\,Ab^4-2\left(7\,Bb^3c-8\,Ab^2c^2\right)x^4+\left(7\,Bb^4-8\,Ab^3c\right)x^2\right)\sqrt{cx^4+}}{35\left(b^5cx^{10}+b^6x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*(7*B*b*c^3 - 8*A*c^4)*x^8 + 8*(7*B*b^2*c^2 - 8*A*b*c^3)*x^6 + 5*A
*b^4 - 2*(7*B*b^3*c - 8*A*b^2*c^2)*x^4 + (7*B*b^4 - 8*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^5), x)
```

maple [A] time = 0.05, size = 118, normalized size = 0.86

$$-\frac{\left(c\,x^{2}+b\right)\left(-128A\,c^{4}x^{8}+112Bb\,c^{3}x^{8}-64Ab\,c^{3}x^{6}+56B\,b^{2}c^{2}x^{6}+16A\,b^{2}c^{2}x^{4}-14B\,b^{3}c\,x^{4}-8A\,b^{3}c\,x^{2}+7B\,b\right.}{35\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-128*A*c^4*x^8+112*B*b*c^3*x^8-64*A*b*c^3*x^6+56*B*b^2*c^2
*x^6+16*A*b^2*c^2*x^4-14*B*b^3*c*x^4-8*A*b^3*c*x^2+7*B*b^4*x^2+5*A*b^4)/x^4
/b^5/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.48, size = 208, normalized size = 1.51

$$-\frac{1}{5}B\left(\frac{16\,c^{3}x^{2}}{\sqrt{cx^{4}+bx^{2}}\,b^{4}}+\frac{8\,c^{2}}{\sqrt{cx^{4}+bx^{2}}\,b^{3}}-\frac{2\,c}{\sqrt{cx^{4}+bx^{2}}\,b^{2}x^{2}}+\frac{1}{\sqrt{cx^{4}+bx^{2}}\,bx^{4}}\right)+\frac{1}{35}A\left(\frac{128\,c^{4}x^{2}}{\sqrt{cx^{4}+bx^{2}}\,b^{5}}+\frac{64}{\sqrt{cx^{4}+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*B*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*b
^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) + 1/(sqrt(c*x^4 + b*x^2)*b*x^4)) +
1/35*A*(128*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64*c^3/(sqrt(c*x^4 + b*x^2)
*b^4) - 16*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8*c/(sqrt(c*x^4 + b*x^2)*b^2
*x^4) - 5/(sqrt(c*x^4 + b*x^2)*b*x^6))
```

mupad [B] time = 0.56, size = 173, normalized size = 1.25

$$-\frac{\left(7\,B\,b^{2}-13\,A\,b\,c\right)\,\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b^{4}\,x^{6}}-\frac{\left(x^{2}\,\left(\frac{58\,A\,c^{4}-42\,B\,b\,c^{3}}{35\,b^{5}}-\frac{2\,c^{3}\,(93\,A\,c-77\,B\,b)}{35\,b^{5}}\right)-\frac{c^{2}\,(93\,A\,c-77\,B\,b)}{35\,b^{4}}\right)\,\sqrt{c\,x^{4}+b\,x^{2}}}{x^{2}\,\left(c\,x^{2}+b\right)}-\frac{A\,\sqrt{c}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] - ((7*B*b^2 - 13*A*b*c)*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^6) - ((x^2*((58*A*
c^4 - 42*B*b*c^3)/(35*b^5) - (2*c^3*(93*A*c - 77*B*b))/(35*b^5)) - (c^2*(93
*A*c - 77*B*b))/(35*b^4))*(b*x^2 + c*x^4)^(1/2))/(x^2*(b + c*x^2)) - (A*(b*
x^2 + c*x^4)^(1/2))/(7*b^2*x^8) - (c*(29*A*c - 21*B*b)*(b*x^2 + c*x^4)^(1/2
))/(35*b^4*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^{2}}{x^{5}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

3.153 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{8b\sqrt{bx^2+cx^4}\,(6bB-5Ac)}{15c^4x}-\frac{4x\sqrt{bx^2+cx^4}\,(6bB-5Ac)}{15c^3}+\frac{x^3\sqrt{bx^2+cx^4}\,(6bB-5Ac)}{5bc^2}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^7/b/c/(c*x^4+b*x^2)^(1/2)+8/15*b*(-5*A*c+6*B*b)*(c*x^4+b*x^2)
^(1/2)/c^4/x-4/15*(-5*A*c+6*B*b)*x*(c*x^4+b*x^2)^(1/2)/c^3+1/5*(-5*A*c+6*B*
b)*x^3*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] time = 0.25, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2037, 2016, 1588}

$$\frac{x^3\sqrt{bx^2+cx^4}\,(6bB-5Ac)}{5bc^2}-\frac{4x\sqrt{bx^2+cx^4}\,(6bB-5Ac)}{15c^3}+\frac{8b\sqrt{bx^2+cx^4}\,(6bB-5Ac)}{15c^4x}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^7)/(b*c*Sqrt[b*x^2 + c*x^4])) + (8*b*(6*b*B - 5*A*c)*Sqrt[
b*x^2 + c*x^4])/(15*c^4*x) - (4*(6*b*B - 5*A*c)*x*Sqrt[b*x^2 + c*x^4])/(15*
c^3) + ((6*b*B - 5*A*c)*x^3*Sqrt[b*x^2 + c*x^4])/(5*b*c^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^8 \left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}} + \frac{(6bB-5Ac)\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}} + \frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2} - \frac{(4(6bB-5Ac))\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{5c^2}$$

$$= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}} - \frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3} + \frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2} + \frac{(8b(6bB}{}$$

$$= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}} + \frac{8b(6bB-5Ac)\sqrt{bx^2+cx^4}}{15c^4x} - \frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3} + \frac{(6bB-5}{}$$

Mathematica [A] time = 0.08, size = 82, normalized size = 0.59

$$\frac{x\left(-8b^2c\left(5A-3Bx^2\right)-2bc^2x^2\left(10A+3Bx^2\right)+c^3x^4\left(5A+3Bx^2\right)+48b^3B\right)}{15c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(48*b^3*B - 8*b^2*c*(5*A - 3*B*x^2) + c^3*x^4*(5*A + 3*B*x^2) - 2*b*c^2*
x^2*(10*A + 3*B*x^2)))/(15*c^4*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.91, size = 93, normalized size = 0.67

$$\frac{\left(3\,Bc^3x^6-\left(6\,Bbc^2-5\,Ac^3\right)x^4+48\,Bb^3-40\,Ab^2c+4\left(6\,Bb^2c-5\,Abc^2\right)x^2\right)\sqrt{cx^4+bx^2}}{15\left(c^5x^3+bc^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/15*(3*B*c^3*x^6 - (6*B*b*c^2 - 5*A*c^3)*x^4 + 48*B*b^3 - 40*A*b^2*c + 4*(
6*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x^3 + b*c^4*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^8}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^8/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.05, size = 91, normalized size = 0.65

$$-\frac{\left(c\,x^2+b\right)\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40A\,b^2c-48B\,b^3\right)x^3}{15\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(-3*B*c^3*x^6-5*A*c^3*x^4+6*B*b*c^2*x^4+20*A*b*c^2*x^2-24*B
*b^2*c*x^2+40*A*b^2*c-48*B*b^3)*x^3/c^4/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.58, size = 82, normalized size = 0.59

$$\frac{\left(c^2x^4 - 4\,bcx^2 - 8\,b^2\right)A}{3\,\sqrt{cx^2+b}\,c^3} + \frac{\left(c^3x^6 - 2\,bc^2x^4 + 8\,b^2cx^2 + 16\,b^3\right)B}{5\,\sqrt{cx^2+b}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*A/(sqrt(c*x^2 + b)*c^3) + 1/5*(c^3*x^6 -
2*b*c^2*x^4 + 8*b^2*c*x^2 + 16*b^3)*B/(sqrt(c*x^2 + b)*c^4)
```

mupad [B] time = 0.40, size = 92, normalized size = 0.66

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(48\,B\,b^3+24\,B\,b^2\,c\,x^2-40\,A\,b^2\,c-6\,B\,b\,c^2\,x^4-20\,A\,b\,c^2\,x^2+3\,B\,c^3\,x^6+5\,A\,c^3\,x^4\right)}{15\,c^4\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(48*B*b^3 + 5*A*c^3*x^4 + 3*B*c^3*x^6 - 40*A*b^2*c -
 20*A*b*c^2*x^2 + 24*B*b^2*c*x^2 - 6*B*b*c^2*x^4))/(15*c^4*x*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**8*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.154 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{2\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{3c^3x}+\frac{x\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{3bc^2}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^5/b/c/(c*x^4+b*x^2)^(1/2)-2/3*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(1
/2)/c^3/x+1/3*(-3*A*c+4*B*b)*x*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] time = 0.20, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2037, 2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{3bc^2}-\frac{2\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{3c^3x}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^5)/(b*c*Sqrt[b*x^2 + c*x^4])) - (2*(4*b*B - 3*A*c)*Sqrt[b*
x^2 + c*x^4])/(3*c^3*x) + ((4*b*B - 3*A*c)*x*Sqrt[b*x^2 + c*x^4])/(3*b*c^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^6 \left(A + Bx^2\right)}{\left(bx^2 + cx^4\right)^{3/2}}\, dx = -\frac{(bB - Ac)x^5}{bc\sqrt{bx^2 + cx^4}} + \frac{(4bB - 3Ac)\int \frac{x^4}{\sqrt{bx^2+cx^4}}\, dx}{bc}$$

$$= -\frac{(bB - Ac)x^5}{bc\sqrt{bx^2 + cx^4}} + \frac{(4bB - 3Ac)x\sqrt{bx^2 + cx^4}}{3bc^2} - \frac{(2(4bB - 3Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\, dx}{3c^2}$$

$$= -\frac{(bB - Ac)x^5}{bc\sqrt{bx^2 + cx^4}} - \frac{2(4bB - 3Ac)\sqrt{bx^2 + cx^4}}{3c^3x} + \frac{(4bB - 3Ac)x\sqrt{bx^2 + cx^4}}{3bc^2}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.58

$$\frac{x\left(b\left(6Ac - 4Bcx^2\right) + c^2x^2\left(3A + Bx^2\right) - 8b^2B\right)}{3c^3\sqrt{x^2\left(b + cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(-8*b^2*B + c^2*x^2*(3*A + B*x^2) + b*(6*A*c - 4*B*c*x^2)))/(3*c^3*Sqrt[
x^2*(b + c*x^2)])
```

fricas [A] time = 1.16, size = 68, normalized size = 0.65

$$\frac{\left(Bc^2x^4 - 8\,Bb^2 + 6\,Abc - \left(4\,Bbc - 3\,Ac^2\right)x^2\right)\sqrt{cx^4 + bx^2}}{3\left(c^4x^3 + bc^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*c^2*x^4 - 8*B*b^2 + 6*A*b*c - (4*B*b*c - 3*A*c^2)*x^2)*sqrt(c*x^4 +
b*x^2)/(c^4*x^3 + b*c^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^6}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.05, size = 66, normalized size = 0.63

$$\frac{\left(c\,x^2 + b\right)\left(B\,c^2x^4 + 3A\,c^2x^2 - 4Bbc\,x^2 + 6Abc - 8B\,b^2\right)x^3}{3\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/3*(c*x^2+b)*(B*c^2*x^4+3*A*c^2*x^2-4*B*b*c*x^2+6*A*b*c-8*B*b^2)*x^3/c^3/(
c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.56, size = 59, normalized size = 0.57

$$\frac{\left(cx^2+2\,b\right)A}{\sqrt{cx^2+b}\,c^2}+\frac{\left(c^2x^4-4\,bcx^2-8\,b^2\right)B}{3\,\sqrt{cx^2+b}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)*A/(sqrt(c*x^2 + b)*c^2) + 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*B
/(sqrt(c*x^2 + b)*c^3)
```

mupad [B] time = 0.28, size = 67, normalized size = 0.64

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-8\,B\,b^2-4\,B\,b\,c\,x^2+6\,A\,b\,c+B\,c^2\,x^4+3\,A\,c^2\,x^2\right)}{3\,c^3\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(3*A*c^2*x^2 - 8*B*b^2 + B*c^2*x^4 + 6*A*b*c - 4*B*b
*c*x^2))/(3*c^3*x*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6\left(A+Bx^2\right)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.155 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$

Optimal. Leaf size=69

$$\frac{\sqrt{bx^2+cx^4}\,(2bB-Ac)}{bc^2x}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^3/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/
c^2/x
```

Rubi [A] time = 0.15, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2037, 1588}

$$\frac{\sqrt{bx^2+cx^4}\,(2bB-Ac)}{bc^2x}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^3)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((2*b*B - A*c)*Sqrt[b*x^2
+ c*x^4])/(b*c^2*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\ &= -\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\sqrt{bx^2+cx^4}}{bc^2x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.51

$$\frac{x\left(-Ac+2bB+Bcx^2\right)}{c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(2*b*B - A*c + B*c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.69, size = 45, normalized size = 0.65

$$\frac{\sqrt{cx^4 + bx^2}\left(Bcx^2 + 2\,Bb - Ac\right)}{c^3x^3 + bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)*(B*c*x^2 + 2*B*b - A*c)/(c^3*x^3 + b*c^2*x)
```

giac [A] time = 0.29, size = 60, normalized size = 0.87

$$-\frac{2\,B\sqrt{b}}{\left(\left(\sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x}\right)^2 - c\right)c} + \frac{Bb - Ac}{\sqrt{c + \frac{b}{x^2}}\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -2*B*sqrt(b)/(((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)*c) + (B*b - A*c)/(sqrt(
c + b/x^2)*c^2*x)
```

maple [A] time = 0.05, size = 44, normalized size = 0.64

$$-\frac{\left(c\,x^2 + b\right)\left(-Bc\,x^2 + Ac - 2bB\right)x^3}{\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
[Out] -(c*x^2+b)*(-B*c*x^2+A*c-2*B*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.57, size = 39, normalized size = 0.57

$$\frac{\left(cx^2 + 2\,b\right)B}{\sqrt{cx^2 + b}\,c^2} - \frac{A}{\sqrt{cx^2 + b}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] (c*x^2 + 2*b)*B/(sqrt(c*x^2 + b)*c^2) - A/(sqrt(c*x^2 + b)*c)
```

mupad [B] time = 0.21, size = 44, normalized size = 0.64

$$\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(B\,c\,x^2 - A\,c + 2\,B\,b\right)}{c^2\,x\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(2*B*b - A*c + B*c*x^2))/(c^2*x*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(A + Bx^2\right)}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.156 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-(-A*c+B*b)*x/b/c/(c*x^4+b
*x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2037, 2008, 206}

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x)/(b*c*Sqrt[b*x^2 + c*x^4])) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[
b*x^2 + c*x^4]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}+\frac{A\int\frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$=-\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}$$

$$=-\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.14

$$-\frac{x\left(\sqrt{b}\,(bB-Ac)+Ac\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{b^{3/2}c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(Sqrt[b]*(b*B - A*c) + A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(b^(3/2)*c*Sqrt[x^2*(b + c*x^2)]))
```

fricas [A] time = 0.93, size = 199, normalized size = 3.11

$$\left[\frac{\left(Ac^2x^3+Abcx\right)\sqrt{b}\,\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(Bb^2-Abc\right)}{2\left(b^2c^2x^3+b^3cx\right)},\ \frac{\left(Ac^2x^3+Abcx\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4}}{\ldots}\right.}{b^2c^2x\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*c^2*x^3 + A*b*c*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 +
b^3*c*x), ((A*c^2*x^3 + A*b*c*x)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-
b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 + b^3
*c*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]-2*(2*b*B-2*A*c)/4/b/c/x*sqrt(b*(1/x)^2+
c)/(b*(1/x)^2+c)+2*A/2/b/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.06, size = 79, normalized size = 1.23

$$\frac{\left(c\,x^{2}+b\right)\left(-\sqrt{c\,x^{2}+b}\;A b c \ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\;\sqrt{b}}{x}\right)+A\,b^{\frac{3}{2}} c-B\,b^{\frac{5}{2}}\right) x^{3}}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} b^{\frac{5}{2}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(A*b^(3/2)*c-A*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*(c*x^2+b)^
(1/2)*b*c-B*b^(5/2))/(c*x^4+b*x^2)^(3/2)/c/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{2}+A\right)x^{2}}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2}\left(B\,x^{2}+A\right)}{\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(A+Bx^{2}\right)}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.157 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}} \, dx$

Optimal. Leaf size=142

$$-\frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{2b^2cx^3}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

```
[Out] -1/2*(-3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/3*B/c/
x/(c*x^4+b*x^2)^(1/2)+1/3*(3*A*c-2*B*b)/b/c/x/(c*x^4+b*x^2)^(1/2)+1/2*(-3*A
*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/c/x^3
```

Rubi [A] time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1145, 2006, 2025, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{2b^2cx^3}-\frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -B/(3*c*x*Sqrt[b*x^2 + c*x^4]) - (2*b*B - 3*A*c)/(3*b*c*x*Sqrt[b*x^2 + c*x^
4]) + ((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b^2*c*x^3) - ((2*b*B - 3*A*c
)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1145

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[(e*(b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 3)*x), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{B}{3cx\sqrt{bx^2+cx^4}} + \frac{(-2bB+3Ac)\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(-2bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3} + \frac{(2bB-3Ac)\int \frac{1}{\sqrt{bx^2}}}{2b^2}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3} - \frac{(2bB-3Ac)\,\mathrm{Subst}}{}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3} - \frac{(2bB-3Ac)\tanh^{-1}}{2b^{5/2}}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.43

$$\frac{x^2(2bB-3Ac)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{b}+1\right)-Ab}{2b^2x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(A*b) + (2*b*B - 3*A*c)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/b
])/(2*b^2*x*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.84, size = 260, normalized size = 1.83

$$\left[-\frac{\left(\left(2\,Bbc-3\,Ac^2\right)x^5+\left(2\,Bb^2-3\,Abc\right)x^3\right)\sqrt{b}\,\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(Ab^2-\left(2\,Bb^2-3\right.\right.}{4\left(b^3cx^5+b^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B*b*c - 3*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(b)*log(-(c*
x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(
A*b^2 - (2*B*b^2 - 3*A*b*c)*x^2))/(b^3*c*x^5 + b^4*x^3), 1/2*(((2*B*b*c - 3
*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(A*b^2 - (2*B*b^2 - 3*A*b*c)*
x^2))/(b^3*c*x^5 + b^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.06, size = 129, normalized size = 0.91

$$-\frac{\left(c x^2+b\right)\left(-3\sqrt{c x^2+b}\, Abc\, x^2 \ln\left(\frac{2b+2\sqrt{c x^2+b}\,\sqrt{b}}{x}\right)+2\sqrt{c x^2+b}\, B b^2 x^2 \ln\left(\frac{2b+2\sqrt{c x^2+b}\,\sqrt{b}}{x}\right)+3A b^{\frac{3}{2}} c x^2-2B b^{\frac{5}{2}}\right.}{2\left(c x^4+b x^2\right)^{\frac{3}{2}} b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)*(3*A*b^(3/2)*x^2*c-3*A*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)
*(c*x^2+b)^(1/2)*x^2*b*c-2*B*b^(5/2)*x^2+2*B*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2
))/x)*(c*x^2+b)^(1/2)*x^2*b^2+A*b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.158 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}-\frac{3\sqrt{bx^2+cx^4}\,(4bB-5Ac)}{8b^3x^3}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

```
[Out] 3/8*c*(-5*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)-1/4*A/b
/x^3/(c*x^4+b*x^2)^(1/2)+1/4*(-5*A*c+4*B*b)/b^2/x/(c*x^4+b*x^2)^(1/2)-3/8*(
-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^3
```

Rubi **[A]** time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2038, 2006, 2025, 2008, 206}

$$-\frac{3\sqrt{bx^2+cx^4}\,(4bB-5Ac)}{8b^3x^3}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}+\frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -A/(4*b*x^3*Sqrt[b*x^2 + c*x^4]) + (4*b*B - 5*A*c)/(4*b^2*x*Sqrt[b*x^2 + c*
x^4]) - (3*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) + (3*c*(4*b*B -
 5*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
```

```
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} - \frac{(-4bB+5Ac)\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{4b}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} + \frac{(3(4bB-5Ac))\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} - \frac{(3c(4bB-5Ac))}{8b^3}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} + \frac{(3c(4bB-5Ac))}{}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} + \frac{3c(4bB-5Ac)\,\mathrm{ta}}{8b^{7}}$$

Mathematica [C] time = 0.05, size = 64, normalized size = 0.47

$$\frac{cx^4(5Ac-4bB)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{b}+1\right)-Ab^2}{4b^3x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(A*b^2) + c*(-4*b*B + 5*A*c)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x
^2)/b])/(4*b^3*x^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.92, size = 315, normalized size = 2.30

$$\left[-\frac{3\left(\left(4\,Bbc^2-5\,Ac^3\right)x^7+\left(4\,Bb^2c-5\,Abc^2\right)x^5\right)\sqrt{b}\,\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(3\left(4\,Bb^2c-5\,Abc^2\right)x^4+2\,A\right.}{16\left(b^4cx^7+b^5x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c - 5*A*b*c^2)*x^5)*sqrt(b)
*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(4*B*b^2*
c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^
2))/(b^4*c*x^7 + b^5*x^5), -1/8*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c
- 5*A*b*c^2)*x^5)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x
```

```
)) + (3*(4*B*b^2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*
sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

maple [A] time = 0.06, size = 157, normalized size = 1.15

$$-\frac{\left(c\,x^2+b\right)\left(15\sqrt{c\,x^2+b}\;A b\,c^2 x^4 \ln\left(\frac{2b+2\sqrt{c\,x^2+b}\;\sqrt{b}}{x}\right)-12\sqrt{c\,x^2+b}\;B\,b^2 c\,x^4 \ln\left(\frac{2b+2\sqrt{c\,x^2+b}\;\sqrt{b}}{x}\right)-15A\,b^{\frac{3}{2}} c^2 x^4 \cdots\right.}{8\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\,b^{\frac{9}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/8/x*(c*x^2+b)*(15*A*(c*x^2+b)^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*
x^4*b*c^2-15*A*b^(3/2)*x^4*c^2-12*B*(c*x^2+b)^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)
*b^(1/2))/x)*x^4*b^2*c+12*B*b^(5/2)*x^4*c-5*A*b^(5/2)*x^2*c+4*B*b^(7/2)*x^2
+2*A*b^(7/2))/(c*x^4+b*x^2)^(3/2)/b^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

mupad [B] time = 1.26, size = 89, normalized size = 0.65

$$-\frac{A\left(\frac{b}{c\,x^2}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\;\frac{9}{2};\;-\frac{b}{c\,x^2}\right)}{7\,x\left(c\,x^4+b\,x^2\right)^{3/2}}-\frac{B\,x\left(\frac{b}{c\,x^2}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\;\frac{7}{2};\;-\frac{b}{c\,x^2}\right)}{5\left(c\,x^4+b\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] - (A*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*
x^2 + c*x^4)^(3/2)) - (B*x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2,
 -b/(c*x^2)))/(5*(b*x^2 + c*x^4)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

3.159 $\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{17}x^{17/2}(Ac+bB)+\frac{2}{13}Abx^{13/2}+\frac{2}{21}Bcx^{21/2}$$

```
[Out] 2/13*A*b*x^(13/2)+2/17*(A*c+B*b)*x^(17/2)+2/21*B*c*x^(21/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{17}x^{17/2}(Ac+bB)+\frac{2}{13}Abx^{13/2}+\frac{2}{21}Bcx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(13/2))/13 + (2*(b*B + A*c)*x^(17/2))/17 + (2*B*c*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{11/2}+(bB+Ac)x^{15/2}+Bcx^{19/2}\right)dx\\ &= \frac{2}{13}Abx^{13/2}+\frac{2}{17}(bB+Ac)x^{17/2}+\frac{2}{21}Bcx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{13/2}\left(273x^2(Ac+bB)+357Ab+221Bcx^4\right)}{4641}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(13/2)*(357*A*b + 273*(b*B + A*c)*x^2 + 221*B*c*x^4))/4641
```

fricas [A] time = 0.98, size = 32, normalized size = 0.82

$$\frac{2}{4641}\left(221\,Bcx^{10}+273\,(Bb+Ac)x^8+357\,Abx^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/4641*(221*B*c*x^10 + 273*(B*b + A*c)*x^8 + 357*A*b*x^6)*sqrt(x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{21}\,Bcx^{\frac{21}{2}} + \frac{2}{17}\,Bbx^{\frac{17}{2}} + \frac{2}{17}\,Acx^{\frac{17}{2}} + \frac{2}{13}\,Abx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*B*b*x^(17/2) + 2/17*A*c*x^(17/2) + 2/13*A*b*x^(13/
2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(221Bc\,x^4 + 273Ac\,x^2 + 273Bb\,x^2 + 357Ab\right)x^{\frac{13}{2}}}{4641}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 2/4641*x^(13/2)*(221*B*c*x^4+273*A*c*x^2+273*B*b*x^2+357*A*b)
```

maxima [A] time = 1.34, size = 27, normalized size = 0.69

$$\frac{2}{21}\,Bcx^{\frac{21}{2}} + \frac{2}{17}\,(Bb + Ac)x^{\frac{17}{2}} + \frac{2}{13}\,Abx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*(B*b + A*c)*x^(17/2) + 2/13*A*b*x^(13/2)
```

mupad [B] time = 0.06, size = 31, normalized size = 0.79

$$\frac{2\,x^{13/2}\,\left(357\,A\,b + 273\,A\,c\,x^2 + 273\,B\,b\,x^2 + 221\,B\,c\,x^4\right)}{4641}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(13/2)*(357*A*b + 273*A*c*x^2 + 273*B*b*x^2 + 221*B*c*x^4))/4641
```

sympy [A] time = 20.14, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{13}{2}}}{13} + \frac{2Acx^{\frac{17}{2}}}{17} + \frac{2Bbx^{\frac{17}{2}}}{17} + \frac{2Bcx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(13/2)/13 + 2*A*c*x**(17/2)/17 + 2*B*b*x**(17/2)/17 + 2*B*c*x**(21
/2)/21
```

3.160 $\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{15}x^{15/2}(Ac+bB)+\frac{2}{11}Abx^{11/2}+\frac{2}{19}Bcx^{19/2}$$

```
[Out] 2/11*A*b*x^(11/2)+2/15*(A*c+B*b)*x^(15/2)+2/19*B*c*x^(19/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{15}x^{15/2}(Ac+bB)+\frac{2}{11}Abx^{11/2}+\frac{2}{19}Bcx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(11/2))/11 + (2*(b*B + A*c)*x^(15/2))/15 + (2*B*c*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{9/2}+(bB+Ac)x^{13/2}+Bcx^{17/2}\right)\,dx\\ &= \frac{2}{11}Abx^{11/2}+\frac{2}{15}(bB+Ac)x^{15/2}+\frac{2}{19}Bcx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{11/2}\left(209x^2(Ac+bB)+285Ab+165Bcx^4\right)}{3135}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(11/2)*(285*A*b + 209*(b*B + A*c)*x^2 + 165*B*c*x^4))/3135
```

fricas [A] time = 0.97, size = 32, normalized size = 0.82

$$\frac{2}{3135}\left(165\,Bcx^9+209\,(Bb+Ac)x^7+285\,Abx^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/3135*(165*B*c*x^9 + 209*(B*b + A*c)*x^7 + 285*A*b*x^5)*sqrt(x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{19}\,Bcx^{\frac{19}{2}} + \frac{2}{15}\,Bbx^{\frac{15}{2}} + \frac{2}{15}\,Acx^{\frac{15}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/19*B*c*x^(19/2) + 2/15*B*b*x^(15/2) + 2/15*A*c*x^(15/2) + 2/11*A*b*x^(11/
2)
```

maple [A] time = 0.06, size = 32, normalized size = 0.82

$$\frac{2\left(165Bc\,x^{4}+209Ac\,x^{2}+209Bb\,x^{2}+285Ab\right)x^{\frac{11}{2}}}{3135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 2/3135*x^(11/2)*(165*B*c*x^4+209*A*c*x^2+209*B*b*x^2+285*A*b)
```

maxima [A] time = 1.35, size = 27, normalized size = 0.69

$$\frac{2}{19}\,Bcx^{\frac{19}{2}} + \frac{2}{15}\,(Bb + Ac)x^{\frac{15}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/19*B*c*x^(19/2) + 2/15*(B*b + A*c)*x^(15/2) + 2/11*A*b*x^(11/2)
```

mupad [B] time = 0.11, size = 31, normalized size = 0.79

$$\frac{2\,x^{11/2}\,\left(285\,A\,b+209\,A\,c\,x^2+209\,B\,b\,x^2+165\,B\,c\,x^4\right)}{3135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(11/2)*(285*A*b + 209*A*c*x^2 + 209*B*b*x^2 + 165*B*c*x^4))/3135
```

sympy [A] time = 11.25, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Acx^{\frac{15}{2}}}{15} + \frac{2Bbx^{\frac{15}{2}}}{15} + \frac{2Bcx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(11/2)/11 + 2*A*c*x**(15/2)/15 + 2*B*b*x**(15/2)/15 + 2*B*c*x**(19
/2)/19
```

3.161 $\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{9}Abx^{9/2}+\frac{2}{17}Bcx^{17/2}$$

```
[Out] 2/9*A*b*x^(9/2)+2/13*(A*c+B*b)*x^(13/2)+2/17*B*c*x^(17/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{9}Abx^{9/2}+\frac{2}{17}Bcx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(9/2))/9 + (2*(b*B + A*c)*x^(13/2))/13 + (2*B*c*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{7/2}+(bB+Ac)x^{11/2}+Bcx^{15/2}\right)\,dx\\ &= \frac{2}{9}Abx^{9/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{17}Bcx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{9/2}\left(153x^2(Ac+bB)+221Ab+117Bcx^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(9/2)*(221*A*b + 153*(b*B + A*c)*x^2 + 117*B*c*x^4))/1989
```

fricas [A] time = 0.68, size = 32, normalized size = 0.82

$$\frac{2}{1989}\left(117\,Bcx^8+153\,(Bb+Ac)x^6+221\,Abx^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*B*c*x^8 + 153*(B*b + A*c)*x^6 + 221*A*b*x^4)*sqrt(x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{17}\,Bcx^{\frac{17}{2}} + \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{13}\,Acx^{\frac{13}{2}} + \frac{2}{9}\,Abx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/17*B*c*x^(17/2) + 2/13*B*b*x^(13/2) + 2/13*A*c*x^(13/2) + 2/9*A*b*x^(9/2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(117Bc\,x^{4}+153Ac\,x^{2}+153Bb\,x^{2}+221Ab\right)x^{\frac{9}{2}}}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] 2/1989*x^(9/2)*(117*B*c*x^4+153*A*c*x^2+153*B*b*x^2+221*A*b)
```

maxima [A] time = 1.33, size = 27, normalized size = 0.69

$$\frac{2}{17}\,Bcx^{\frac{17}{2}} + \frac{2}{13}\,(Bb + Ac)x^{\frac{13}{2}} + \frac{2}{9}\,Abx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/17*B*c*x^(17/2) + 2/13*(B*b + A*c)*x^(13/2) + 2/9*A*b*x^(9/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{9/2}\,\left(221\,A\,b+153\,A\,c\,x^2+153\,B\,b\,x^2+117\,B\,c\,x^4\right)}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(9/2)*(221*A*b + 153*A*c*x^2 + 153*B*b*x^2 + 117*B*c*x^4))/1989
```

sympy [A] time = 5.59, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{9}{2}}}{9} + \frac{2Acx^{\frac{13}{2}}}{13} + \frac{2Bbx^{\frac{13}{2}}}{13} + \frac{2Bcx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(9/2)/9 + 2*A*c*x**(13/2)/13 + 2*B*b*x**(13/2)/13 + 2*B*c*x**(17/2
)/17
```

3.162 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)dx$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{15}Bcx^{15/2}$$

```
[Out] 2/7*A*b*x^(7/2)+2/11*(A*c+B*b)*x^(11/2)+2/15*B*c*x^(15/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{15}Bcx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(7/2))/7 + (2*(b*B + A*c)*x^(11/2))/11 + (2*B*c*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)dx &= \int x^{5/2}\left(A+Bx^2\right)\left(b+cx^2\right)dx\\ &= \int \left(Abx^{5/2}+(bB+Ac)x^{9/2}+Bcx^{13/2}\right)dx\\ &= \frac{2}{7}Abx^{7/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{15}Bcx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{7/2}\left(105x^2(Ac+bB)+165Ab+77Bcx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(7/2)*(165*A*b + 105*(b*B + A*c)*x^2 + 77*B*c*x^4))/1155
```

fricas [A] time = 0.97, size = 32, normalized size = 0.82

$$\frac{2}{1155}\left(77\,Bcx^7+105\,(Bb+Ac)x^5+165\,Abx^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="fricas")
[Out] 2/1155*(77*B*c*x^7 + 105*(B*b + A*c)*x^5 + 165*A*b*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 29, normalized size = 0.74

$$\frac{2}{15}Bcx^{\frac{15}{2}} + \frac{2}{11}Bbx^{\frac{11}{2}} + \frac{2}{11}Acx^{\frac{11}{2}} + \frac{2}{7}Abx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="giac")
[Out] 2/15*B*c*x^(15/2) + 2/11*B*b*x^(11/2) + 2/11*A*c*x^(11/2) + 2/7*A*b*x^(7/2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(77Bc\,x^4 + 105Ac\,x^2 + 105Bb\,x^2 + 165Ab\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x)
[Out] 2/1155*x^(7/2)*(77*B*c*x^4+105*A*c*x^2+105*B*b*x^2+165*A*b)
```

maxima [A] time = 1.31, size = 27, normalized size = 0.69

$$\frac{2}{15}Bcx^{\frac{15}{2}} + \frac{2}{11}(Bb + Ac)x^{\frac{11}{2}} + \frac{2}{7}Abx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="maxima")
[Out] 2/15*B*c*x^(15/2) + 2/11*(B*b + A*c)*x^(11/2) + 2/7*A*b*x^(7/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{7/2}\,\left(165\,A\,b + 105\,A\,c\,x^2 + 105\,B\,b\,x^2 + 77\,B\,c\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] (2*x^(7/2)*(165*A*b + 105*A*c*x^2 + 105*B*b*x^2 + 77*B*c*x^4))/1155
```

sympy [A] time = 2.34, size = 37, normalized size = 0.95

$$\frac{2Abx^{\frac{7}{2}}}{7} + \frac{2Bcx^{\frac{15}{2}}}{15} + \frac{2x^{\frac{11}{2}}\left(Ac + Bb\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)*x**(1/2),x)
[Out] 2*A*b*x**(7/2)/7 + 2*B*c*x**(15/2)/15 + 2*x**(11/2)*(A*c + B*b)/11
```

3.163 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} \, dx$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{5}Abx^{5/2}+\frac{2}{13}Bcx^{13/2}$$

```
[Out] 2/5*A*b*x^(5/2)+2/9*(A*c+B*b)*x^(9/2)+2/13*B*c*x^(13/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{5}Abx^{5/2}+\frac{2}{13}Bcx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

```
[Out] (2*A*b*x^(5/2))/5 + (2*(b*B + A*c)*x^(9/2))/9 + (2*B*c*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} \, dx &= \int x^{3/2}\left(A+Bx^2\right)\left(b+cx^2\right) \, dx \\
&= \int \left(Abx^{3/2}+(bB+Ac)x^{7/2}+Bcx^{11/2}\right) \, dx \\
&= \frac{2}{5}Abx^{5/2}+\frac{2}{9}(bB+Ac)x^{9/2}+\frac{2}{13}Bcx^{13/2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{585}x^{5/2}\left(65x^2(Ac+bB)+117Ab+45Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

```
[Out] (2*x^(5/2)*(117*A*b + 65*(b*B + A*c)*x^2 + 45*B*c*x^4))/585
```

fricas [A] time = 0.70, size = 32, normalized size = 0.82

$$\frac{2}{585}\left(45\,Bcx^6+65\,(Bb+Ac)x^4+117\,Abx^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*c*x^6 + 65*(B*b + A*c)*x^4 + 117*A*b*x^2)*sqrt(x)
```

giac [A] time = 0.21, size = 29, normalized size = 0.74

$$\frac{2}{13} Bcx^{\frac{13}{2}} + \frac{2}{9} Bbx^{\frac{9}{2}} + \frac{2}{9} Acx^{\frac{9}{2}} + \frac{2}{5} Abx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*c*x^(13/2) + 2/9*B*b*x^(9/2) + 2/9*A*c*x^(9/2) + 2/5*A*b*x^(5/2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2 \left(45Bc\,x^{4}+65Ac\,x^{2}+65Bb\,x^{2}+117Ab\right) x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] 2/585*x^(5/2)*(45*B*c*x^4+65*A*c*x^2+65*B*b*x^2+117*A*b)
```

maxima [A] time = 1.37, size = 27, normalized size = 0.69

$$\frac{2}{13} Bcx^{\frac{13}{2}} + \frac{2}{9} (Bb + Ac)x^{\frac{9}{2}} + \frac{2}{5} Abx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*c*x^(13/2) + 2/9*(B*b + A*c)*x^(9/2) + 2/5*A*b*x^(5/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{5/2}\,\left(117\,A\,b+65\,A\,c\,x^2+65\,B\,b\,x^2+45\,B\,c\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(1/2),x)
```

```
[Out] (2*x^(5/2)*(117*A*b + 65*A*c*x^2 + 65*B*b*x^2 + 45*B*c*x^4))/585
```

sympy [A] time = 2.06, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Acx^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{9}{2}}}{9} + \frac{2Bcx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] 2*A*b*x**(5/2)/5 + 2*A*c*x**(9/2)/9 + 2*B*b*x**(9/2)/9 + 2*B*c*x**(13/2)/13
```

3.164 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{3}Abx^{3/2}+\frac{2}{11}Bcx^{11/2}$$

```
[Out] 2/3*A*b*x^(3/2)+2/7*(A*c+B*b)*x^(7/2)+2/11*B*c*x^(11/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{3}Abx^{3/2}+\frac{2}{11}Bcx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

```
[Out] (2*A*b*x^(3/2))/3 + (2*(b*B + A*c)*x^(7/2))/7 + (2*B*c*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx &= \int \sqrt{x}\,(A+Bx^2)(b+cx^2)\,dx\\ &= \int \left(Ab\sqrt{x}+(bB+Ac)x^{5/2}+Bcx^{9/2}\right)dx\\ &= \frac{2}{3}Abx^{3/2}+\frac{2}{7}(bB+Ac)x^{7/2}+\frac{2}{11}Bcx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{231}x^{3/2}\left(33x^2(Ac+bB)+77Ab+21Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

```
[Out] (2*x^(3/2)*(77*A*b + 33*(b*B + A*c)*x^2 + 21*B*c*x^4))/231
```

fricas [A] time = 0.93, size = 30, normalized size = 0.77

$$\frac{2}{231}\left(21\,Bcx^5+33\,(Bb+Ac)x^3+77\,Abx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
[Out] 2/231*(21*B*c*x^5 + 33*(B*b + A*c)*x^3 + 77*A*b*x)*sqrt(x)
```

giac [A] time = 0.16, size = 29, normalized size = 0.74

$$\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,Bbx^{\frac{7}{2}}+\frac{2}{7}\,Acx^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
[Out] 2/11*B*c*x^(11/2) + 2/7*B*b*x^(7/2) + 2/7*A*c*x^(7/2) + 2/3*A*b*x^(3/2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(21Bc\,x^{4}+33Ac\,x^{2}+33Bb\,x^{2}+77Ab\right)x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x)
[Out] 2/231*x^(3/2)*(21*B*c*x^4+33*A*c*x^2+33*B*b*x^2+77*A*b)
```

maxima [A] time = 1.38, size = 27, normalized size = 0.69

$$\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,(Bb+Ac)x^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
[Out] 2/11*B*c*x^(11/2) + 2/7*(B*b + A*c)*x^(7/2) + 2/3*A*b*x^(3/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{3/2}\,\left(77\,A\,b+33\,A\,c\,x^2+33\,B\,b\,x^2+21\,B\,c\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x)
[Out] (2*x^(3/2)*(77*A*b + 33*A*c*x^2 + 33*B*b*x^2 + 21*B*c*x^4))/231
```

sympy [A] time = 2.26, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Acx^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{7}{2}}}{7}+\frac{2Bcx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(3/2),x)
[Out] 2*A*b*x**(3/2)/3 + 2*A*c*x**(7/2)/7 + 2*B*b*x**(7/2)/7 + 2*B*c*x**(11/2)/11
```

3.165 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{9}Bcx^{9/2}$$

```
[Out] 2/5*(A*c+B*b)*x^(5/2)+2/9*B*c*x^(9/2)+2*A*b*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{5}x^{5/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{9}Bcx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

```
[Out] 2*A*b*Sqrt[x] + (2*(b*B + A*c)*x^(5/2))/5 + (2*B*c*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{\sqrt{x}}\,dx\\
&= \int \left(\frac{Ab}{\sqrt{x}}+(bB+Ac)x^{3/2}+Bcx^{7/2}\right)dx\\
&= 2Ab\sqrt{x}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{9}Bcx^{9/2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$\frac{2}{45}\sqrt{x}\left(9x^2(Ac+bB)+45Ab+5Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*(45*A*b + 9*(b*B + A*c)*x^2 + 5*B*c*x^4))/45
```

fricas [A] time = 0.85, size = 29, normalized size = 0.78

$$\frac{2}{45}\left(5\,Bcx^4+9\,(Bb+Ac)x^2+45\,Ab\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*c*x^4 + 9*(B*b + A*c)*x^2 + 45*A*b)*sqrt(x)
```

giac [A] time = 0.18, size = 29, normalized size = 0.78

$$\frac{2}{9}\,Bcx^{\frac{9}{2}} + \frac{2}{5}\,Bbx^{\frac{5}{2}} + \frac{2}{5}\,Acx^{\frac{5}{2}} + 2\,Ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*c*x^(9/2) + 2/5*B*b*x^(5/2) + 2/5*A*c*x^(5/2) + 2*A*b*sqrt(x)
```

maple [A] time = 0.05, size = 32, normalized size = 0.86

$$\frac{2\left(5Bc\,x^{4}+9Ac\,x^{2}+9Bb\,x^{2}+45Ab\right)\sqrt{x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x)
```

```
[Out] 2/45*x^(1/2)*(5*B*c*x^4+9*A*c*x^2+9*B*b*x^2+45*A*b)
```

maxima [A] time = 1.34, size = 27, normalized size = 0.73

$$\frac{2}{9}\,Bcx^{\frac{9}{2}} + \frac{2}{5}\,(Bb + Ac)x^{\frac{5}{2}} + 2\,Ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*c*x^(9/2) + 2/5*(B*b + A*c)*x^(5/2) + 2*A*b*sqrt(x)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.84

$$\frac{2\,\sqrt{x}\,\left(45\,A\,b+9\,A\,c\,x^2+9\,B\,b\,x^2+5\,B\,c\,x^4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x)
```

```
[Out] (2*x^(1/2)*(45*A*b + 9*A*c*x^2 + 9*B*b*x^2 + 5*B*c*x^4))/45
```

sympy [A] time = 2.62, size = 44, normalized size = 1.19

$$2Ab\sqrt{x} + \frac{2Acx^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{5}{2}}}{5} + \frac{2Bcx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*A*b*sqrt(x) + 2*A*c*x**(5/2)/5 + 2*B*b*x**(5/2)/5 + 2*B*c*x**(9/2)/9
```

3.166 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx$

Optimal. Leaf size=37

$$\frac{2}{3}x^{3/2}(Ac+bB)-\frac{2Ab}{\sqrt{x}}+\frac{2}{7}Bcx^{7/2}$$

```
[Out] 2/3*(A*c+B*b)*x^(3/2)+2/7*B*c*x^(7/2)-2*A*b/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{2}{3}x^{3/2}(Ac+bB)-\frac{2Ab}{\sqrt{x}}+\frac{2}{7}Bcx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
```

```
[Out] (-2*A*b)/Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*B*c*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^{3/2}}\,dx\\
&= \int \left(\frac{Ab}{x^{3/2}}+(bB+Ac)\sqrt{x}+Bcx^{5/2}\right)dx\\
&= -\frac{2Ab}{\sqrt{x}}+\frac{2}{3}(bB+Ac)x^{3/2}+\frac{2}{7}Bcx^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.95

$$\frac{2\left(-21Ab+7Acx^2+7bBx^2+3Bcx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
```

```
[Out] (2*(-21*A*b + 7*b*B*x^2 + 7*A*c*x^2 + 3*B*c*x^4))/(21*Sqrt[x])
```

fricas [A] time = 0.84, size = 29, normalized size = 0.78

$$\frac{2\left(3\,Bcx^4+7\,(Bb+Ac)x^2-21\,Ab\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
[Out] 2/21*(3*B*c*x^4 + 7*(B*b + A*c)*x^2 - 21*A*b)/sqrt(x)
```

giac [A] time = 0.19, size = 29, normalized size = 0.78

$$\frac{2}{7}\,Bcx^{\frac{7}{2}} + \frac{2}{3}\,Bbx^{\frac{3}{2}} + \frac{2}{3}\,Acx^{\frac{3}{2}} - \frac{2\,Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
[Out] 2/7*B*c*x^(7/2) + 2/3*B*b*x^(3/2) + 2/3*A*c*x^(3/2) - 2*A*b/sqrt(x)
```

maple [A] time = 0.04, size = 32, normalized size = 0.86

$$-\frac{2\left(-3Bc\,x^{4}-7Ac\,x^{2}-7Bb\,x^{2}+21Ab\right)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x)
[Out] -2/21/x^(1/2)*(-3*B*c*x^4-7*A*c*x^2-7*B*b*x^2+21*A*b)
```

maxima [A] time = 1.33, size = 27, normalized size = 0.73

$$\frac{2}{7}\,Bcx^{\frac{7}{2}} + \frac{2}{3}\,(Bb + Ac)x^{\frac{3}{2}} - \frac{2\,Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
[Out] 2/7*B*c*x^(7/2) + 2/3*(B*b + A*c)*x^(3/2) - 2*A*b/sqrt(x)
```

mupad [B] time = 0.10, size = 31, normalized size = 0.84

$$\frac{14\,A\,c\,x^2-42\,A\,b+14\,B\,b\,x^2+6\,B\,c\,x^4}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x)
[Out] (14*A*c*x^2 - 42*A*b + 14*B*b*x^2 + 6*B*c*x^4)/(21*x^(1/2))
```

sympy [A] time = 3.92, size = 44, normalized size = 1.19

$$-\frac{2Ab}{\sqrt{x}} + \frac{2Acx^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{3}{2}}}{3} + \frac{2Bcx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(7/2),x)
[Out] -2*A*b/sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*b*x**(3/2)/3 + 2*B*c*x**(7/2)/7
```

3.167 $\int x^{7/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=63

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

```
[Out] 2/17*A*b^2*x^(17/2)+2/21*b*(2*A*c+B*b)*x^(21/2)+2/25*c*(A*c+2*B*b)*x^(25/2)
+2/29*B*c^2*x^(29/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*c
)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx &= \int x^{15/2} \left(A+Bx^2\right) \left(b+cx^2\right)^2 \, dx\\
&= \int \left(Ab^2x^{15/2}+b(bB+2Ac)x^{19/2}+c(2bB+Ac)x^{23/2}+Bc^2x^{27/2}\right) dx\\
&= \frac{2}{17}Ab^2x^{17/2}+\frac{2}{21}b(bB+2Ac)x^{21/2}+\frac{2}{25}c(2bB+Ac)x^{25/2}+\frac{2}{29}Bc^2x^{29/2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.00

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*c
)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

fricas [A] time = 0.87, size = 56, normalized size = 0.89

$$\frac{2}{258825}\left(8925\,Bc^2x^{14}+10353\left(2\,Bbc+Ac^2\right)x^{12}+15225\,Ab^2x^8+12325\left(Bb^2+2\,Abc\right)x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/258825*(8925*B*c^2*x^14 + 10353*(2*B*b*c + A*c^2)*x^12 + 15225*A*b^2*x^8
+ 12325*(B*b^2 + 2*A*b*c)*x^10)*sqrt(x)
```

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{4}{25}\,Bbcx^{\frac{25}{2}}+\frac{2}{25}\,Ac^2x^{\frac{25}{2}}+\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{21}\,Abcx^{\frac{21}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 4/25*B*b*c*x^(25/2) + 2/25*A*c^2*x^(25/2) + 2/21*B*b^
2*x^(21/2) + 4/21*A*b*c*x^(21/2) + 2/17*A*b^2*x^(17/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(8925B\,c^2x^6+10353A\,c^2x^4+20706Bbc\,x^4+24650Abc\,x^2+12325B\,b^2x^2+15225b^2A\right)x^{\frac{17}{2}}}{258825}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/258825*x^(17/2)*(8925*B*c^2*x^6+10353*A*c^2*x^4+20706*B*b*c*x^4+24650*A*b
*c*x^2+12325*B*b^2*x^2+15225*A*b^2)
```

maxima [A] time = 1.30, size = 51, normalized size = 0.81

$$\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{2}{25}\left(2\,Bbc+Ac^2\right)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{21}\left(Bb^2+2\,Abc\right)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 2/25*(2*B*b*c + A*c^2)*x^(25/2) + 2/17*A*b^2*x^(17/2)
 + 2/21*(B*b^2 + 2*A*b*c)*x^(21/2)
```

mupad [B] time = 0.13, size = 51, normalized size = 0.81

$$x^{21/2}\left(\frac{2\,B\,b^2}{21}+\frac{4\,A\,c\,b}{21}\right)+x^{25/2}\left(\frac{2\,A\,c^2}{25}+\frac{4\,B\,b\,c}{25}\right)+\frac{2\,A\,b^2\,x^{17/2}}{17}+\frac{2\,B\,c^2\,x^{29/2}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(21/2)*((2*B*b^2)/21 + (4*A*b*c)/21) + x^(25/2)*((2*A*c^2)/25 + (4*B*b*c)
/25) + (2*A*b^2*x^(17/2))/17 + (2*B*c^2*x^(29/2))/29
```

sympy [A] time = 57.87, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{4Abcx^{\frac{21}{2}}}{21}+\frac{2Ac^2x^{\frac{25}{2}}}{25}+\frac{2Bb^2x^{\frac{21}{2}}}{21}+\frac{4Bbcx^{\frac{25}{2}}}{25}+\frac{2Bc^2x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(17/2)/17 + 4*A*b*c*x**(21/2)/21 + 2*A*c**2*x**(25/2)/25 + 2*B*
b**2*x**(21/2)/21 + 4*B*b*c*x**(25/2)/25 + 2*B*c**2*x**(29/2)/29
```

3.168 $\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{2}{15}Ab^2x^{15/2}+\frac{2}{23}cx^{23/2}(Ac+2bB)+\frac{2}{19}bx^{19/2}(2Ac+bB)+\frac{2}{27}Bc^2x^{27/2}$$

```
[Out] 2/15*A*b^2*x^(15/2)+2/19*b*(2*A*c+B*b)*x^(19/2)+2/23*c*(A*c+2*B*b)*x^(23/2)
+2/27*B*c^2*x^(27/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{15}Ab^2x^{15/2}+\frac{2}{23}cx^{23/2}(Ac+2bB)+\frac{2}{19}bx^{19/2}(2Ac+bB)+\frac{2}{27}Bc^2x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(15/2))/15 + (2*b*(b*B + 2*A*c)*x^(19/2))/19 + (2*c*(2*b*B + A*c
)*x^(23/2))/23 + (2*B*c^2*x^(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx &= \int x^{13/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{13/2}+b(bB+2Ac)x^{17/2}+c(2bB+Ac)x^{21/2}+Bc^2x^{25/2}\right)dx\\ &= \frac{2}{15}Ab^2x^{15/2}+\frac{2}{19}b(bB+2Ac)x^{19/2}+\frac{2}{23}c(2bB+Ac)x^{23/2}+\frac{2}{27}Bc^2x^{27/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{15/2}\left(3933Ab^2+2565cx^4(Ac+2bB)+3105bx^2(2Ac+bB)+2185Bc^2x^6\right)}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(15/2)*(3933*A*b^2 + 3105*b*(b*B + 2*A*c)*x^2 + 2565*c*(2*b*B + A*c)*x
^4 + 2185*B*c^2*x^6))/58995
```

fricas [A] time = 0.82, size = 56, normalized size = 0.89

$$\frac{2}{58995}\left(2185\,Bc^2x^{13}+2565\left(2\,Bbc+Ac^2\right)x^{11}+3933\,Ab^2x^7+3105\left(Bb^2+2\,Abc\right)x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*B*c^2*x^13 + 2565*(2*B*b*c + A*c^2)*x^11 + 3933*A*b^2*x^7 + 3
105*(B*b^2 + 2*A*b*c)*x^9)*sqrt(x)
```

giac [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{2}{27}\,Bc^2x^{\frac{27}{2}}+\frac{4}{23}\,Bbcx^{\frac{23}{2}}+\frac{2}{23}\,Ac^2x^{\frac{23}{2}}+\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{19}\,Abcx^{\frac{19}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 4/23*B*b*c*x^(23/2) + 2/23*A*c^2*x^(23/2) + 2/19*B*b^
2*x^(19/2) + 4/19*A*b*c*x^(19/2) + 2/15*A*b^2*x^(15/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(2185B\,c^2x^6+2565A\,c^2x^4+5130Bbc\,x^4+6210Abc\,x^2+3105B\,b^2x^2+3933b^2A\right)x^{\frac{15}{2}}}{58995}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/58995*x^(15/2)*(2185*B*c^2*x^6+2565*A*c^2*x^4+5130*B*b*c*x^4+6210*A*b*c*x
^2+3105*B*b^2*x^2+3933*A*b^2)
```

maxima [A] time = 1.34, size = 51, normalized size = 0.81

$$\frac{2}{27}\,Bc^2x^{\frac{27}{2}}+\frac{2}{23}\left(2\,Bbc+Ac^2\right)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{19}\left(Bb^2+2\,Abc\right)x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 2/23*(2*B*b*c + A*c^2)*x^(23/2) + 2/15*A*b^2*x^(15/2)
 + 2/19*(B*b^2 + 2*A*b*c)*x^(19/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{19/2}\left(\frac{2\,B\,b^2}{19}+\frac{4\,A\,c\,b}{19}\right)+x^{23/2}\left(\frac{2\,A\,c^2}{23}+\frac{4\,B\,b\,c}{23}\right)+\frac{2\,A\,b^2\,x^{15/2}}{15}+\frac{2\,B\,c^2\,x^{27/2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(19/2)*((2*B*b^2)/19 + (4*A*b*c)/19) + x^(23/2)*((2*A*c^2)/23 + (4*B*b*c)
/23) + (2*A*b^2*x^(15/2))/15 + (2*B*c^2*x^(27/2))/27
```

sympy [A] time = 39.82, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{4Abcx^{\frac{19}{2}}}{19}+\frac{2Ac^2x^{\frac{23}{2}}}{23}+\frac{2Bb^2x^{\frac{19}{2}}}{19}+\frac{4Bbcx^{\frac{23}{2}}}{23}+\frac{2Bc^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(15/2)/15 + 4*A*b*c*x**(19/2)/19 + 2*A*c**2*x**(23/2)/23 + 2*B*
b**2*x**(19/2)/19 + 4*B*b*c*x**(23/2)/23 + 2*B*c**2*x**(27/2)/27
```

3.169 $\int x^{3/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=63

$$\frac{2}{13}Ab^2x^{13/2}+\frac{2}{21}cx^{21/2}(Ac+2bB)+\frac{2}{17}bx^{17/2}(2Ac+bB)+\frac{2}{25}Bc^2x^{25/2}$$

```
[Out] 2/13*A*b^2*x^(13/2)+2/17*b*(2*A*c+B*b)*x^(17/2)+2/21*c*(A*c+2*B*b)*x^(21/2)
+2/25*B*c^2*x^(25/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{13}Ab^2x^{13/2}+\frac{2}{21}cx^{21/2}(Ac+2bB)+\frac{2}{17}bx^{17/2}(2Ac+bB)+\frac{2}{25}Bc^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*c
)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx &= \int x^{11/2} \left(A+Bx^2\right) \left(b+cx^2\right)^2 \, dx\\
&= \int \left(Ab^2x^{11/2}+b(bB+2Ac)x^{15/2}+c(2bB+Ac)x^{19/2}+Bc^2x^{23/2}\right) dx\\
&= \frac{2}{13}Ab^2x^{13/2}+\frac{2}{17}b(bB+2Ac)x^{17/2}+\frac{2}{21}c(2bB+Ac)x^{21/2}+\frac{2}{25}Bc^2x^{25/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{2}{13}Ab^2x^{13/2}+\frac{2}{21}cx^{21/2}(Ac+2bB)+\frac{2}{17}bx^{17/2}(2Ac+bB)+\frac{2}{25}Bc^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*c
)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

fricas [A] time = 0.80, size = 56, normalized size = 0.89

$$\frac{2}{116025}\left(4641\,Bc^2x^{12}+5525\left(2\,Bbc+Ac^2\right)x^{10}+8925\,Ab^2x^6+6825\left(Bb^2+2\,Abc\right)x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*B*c^2*x^12 + 5525*(2*B*b*c + A*c^2)*x^10 + 8925*A*b^2*x^6 +
6825*(B*b^2 + 2*A*b*c)*x^8)*sqrt(x)
```

giac [A] time = 0.17, size = 53, normalized size = 0.84

$$\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{4}{21}\,Bbcx^{\frac{21}{2}}+\frac{2}{21}\,Ac^2x^{\frac{21}{2}}+\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{17}\,Abcx^{\frac{17}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/25*B*c^2*x^(25/2) + 4/21*B*b*c*x^(21/2) + 2/21*A*c^2*x^(21/2) + 2/17*B*b^
2*x^(17/2) + 4/17*A*b*c*x^(17/2) + 2/13*A*b^2*x^(13/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(4641B\,c^2x^6+5525A\,c^2x^4+11050Bbc\,x^4+13650Abc\,x^2+6825B\,b^2x^2+8925b^2A\right)x^{\frac{13}{2}}}{116025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/116025*x^(13/2)*(4641*B*c^2*x^6+5525*A*c^2*x^4+11050*B*b*c*x^4+13650*A*b*
c*x^2+6825*B*b^2*x^2+8925*A*b^2)
```

maxima [A] time = 1.30, size = 51, normalized size = 0.81

$$\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{2}{21}\left(2\,Bbc+Ac^2\right)x^{\frac{21}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{17}\left(Bb^2+2\,Abc\right)x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/25*B*c^2*x^(25/2) + 2/21*(2*B*b*c + A*c^2)*x^(21/2) + 2/13*A*b^2*x^(13/2)
 + 2/17*(B*b^2 + 2*A*b*c)*x^(17/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{17/2}\left(\frac{2\,B\,b^2}{17}+\frac{4\,A\,c\,b}{17}\right)+x^{21/2}\left(\frac{2\,A\,c^2}{21}+\frac{4\,B\,b\,c}{21}\right)+\frac{2\,A\,b^2\,x^{13/2}}{13}+\frac{2\,B\,c^2\,x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(17/2)*((2*B*b^2)/17 + (4*A*b*c)/17) + x^(21/2)*((2*A*c^2)/21 + (4*B*b*c)
/21) + (2*A*b^2*x^(13/2))/13 + (2*B*c^2*x^(25/2))/25
```

sympy [A] time = 20.95, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{4Abcx^{\frac{17}{2}}}{17}+\frac{2Ac^2x^{\frac{21}{2}}}{21}+\frac{2Bb^2x^{\frac{17}{2}}}{17}+\frac{4Bbcx^{\frac{21}{2}}}{21}+\frac{2Bc^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(13/2)/13 + 4*A*b*c*x**(17/2)/17 + 2*A*c**2*x**(21/2)/21 + 2*B*
b**2*x**(17/2)/17 + 4*B*b*c*x**(21/2)/21 + 2*B*c**2*x**(25/2)/25
```

3.170 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{2}{11}Ab^2x^{11/2}+\frac{2}{19}cx^{19/2}(Ac+2bB)+\frac{2}{15}bx^{15/2}(2Ac+bB)+\frac{2}{23}Bc^2x^{23/2}$$

```
[Out] 2/11*A*b^2*x^(11/2)+2/15*b*(2*A*c+B*b)*x^(15/2)+2/19*c*(A*c+2*B*b)*x^(19/2)
+2/23*B*c^2*x^(23/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{11}Ab^2x^{11/2}+\frac{2}{19}cx^{19/2}(Ac+2bB)+\frac{2}{15}bx^{15/2}(2Ac+bB)+\frac{2}{23}Bc^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(11/2))/11 + (2*b*(b*B + 2*A*c)*x^(15/2))/15 + (2*c*(2*b*B + A*c
)*x^(19/2))/19 + (2*B*c^2*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx &= \int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{9/2}+b(bB+2Ac)x^{13/2}+c(2bB+Ac)x^{17/2}+Bc^2x^{21/2}\right)dx\\ &= \frac{2}{11}Ab^2x^{11/2}+\frac{2}{15}b(bB+2Ac)x^{15/2}+\frac{2}{19}c(2bB+Ac)x^{19/2}+\frac{2}{23}Bc^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{11/2}\left(6555Ab^2+3795cx^4(Ac+2bB)+4807bx^2(2Ac+bB)+3135Bc^2x^6\right)}{72105}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(11/2)*(6555*A*b^2 + 4807*b*(b*B + 2*A*c)*x^2 + 3795*c*(2*b*B + A*c)*x
^4 + 3135*B*c^2*x^6))/72105
```

fricas [A] time = 0.88, size = 56, normalized size = 0.89

$$\frac{2}{72105}\left(3135\,Bc^2x^{11}+3795\left(2\,Bbc+Ac^2\right)x^9+6555\,Ab^2x^5+4807\left(Bb^2+2\,Abc\right)x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/72105*(3135*B*c^2*x^11 + 3795*(2*B*b*c + A*c^2)*x^9 + 6555*A*b^2*x^5 + 48
07*(B*b^2 + 2*A*b*c)*x^7)*sqrt(x)
```

giac [A] time = 0.18, size = 53, normalized size = 0.84

$$\frac{2}{23}\,Bc^2x^{\frac{23}{2}}+\frac{4}{19}\,Bbcx^{\frac{19}{2}}+\frac{2}{19}\,Ac^2x^{\frac{19}{2}}+\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{15}\,Abcx^{\frac{15}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*B*c^2*x^(23/2) + 4/19*B*b*c*x^(19/2) + 2/19*A*c^2*x^(19/2) + 2/15*B*b^
2*x^(15/2) + 4/15*A*b*c*x^(15/2) + 2/11*A*b^2*x^(11/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(3135B\,c^2x^6+3795A\,c^2x^4+7590Bbc\,x^4+9614Abc\,x^2+4807B\,b^2x^2+6555b^2A\right)x^{\frac{11}{2}}}{72105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x)
```

```
[Out] 2/72105*x^(11/2)*(3135*B*c^2*x^6+3795*A*c^2*x^4+7590*B*b*c*x^4+9614*A*b*c*x
^2+4807*B*b^2*x^2+6555*A*b^2)
```

maxima [A] time = 1.38, size = 51, normalized size = 0.81

$$\frac{2}{23}\,Bc^2x^{\frac{23}{2}}+\frac{2}{19}\left(2\,Bbc+Ac^2\right)x^{\frac{19}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{15}\left(Bb^2+2\,Abc\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*c^2*x^(23/2) + 2/19*(2*B*b*c + A*c^2)*x^(19/2) + 2/11*A*b^2*x^(11/2)
 + 2/15*(B*b^2 + 2*A*b*c)*x^(15/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,b^2}{15}+\frac{4\,A\,c\,b}{15}\right)+x^{19/2}\left(\frac{2\,A\,c^2}{19}+\frac{4\,B\,b\,c}{19}\right)+\frac{2\,A\,b^2\,x^{11/2}}{11}+\frac{2\,B\,c^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(15/2)*((2*B*b^2)/15 + (4*A*b*c)/15) + x^(19/2)*((2*A*c^2)/19 + (4*B*b*c)
/19) + (2*A*b^2*x^(11/2))/11 + (2*B*c^2*x^(23/2))/23
```

sympy [A] time = 3.73, size = 66, normalized size = 1.05

$$\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{23}{2}}}{23}+\frac{2x^{\frac{19}{2}}\left(Ac^2+2Bbc\right)}{19}+\frac{2x^{\frac{15}{2}}\left(2Abc+Bb^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2*x**(1/2),x)
```

```
[Out] 2*A*b**2*x**(11/2)/11 + 2*B*c**2*x**(23/2)/23 + 2*x**(19/2)*(A*c**2 + 2*B*b
*c)/19 + 2*x**(15/2)*(2*A*b*c + B*b**2)/15
```

3.171 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{9}Ab^2x^{9/2} + \frac{2}{17}cx^{17/2}(Ac+2bB) + \frac{2}{13}bx^{13/2}(2Ac+bB) + \frac{2}{21}Bc^2x^{21/2}$$

```
[Out] 2/9*A*b^2*x^(9/2)+2/13*b*(2*A*c+B*b)*x^(13/2)+2/17*c*(A*c+2*B*b)*x^(17/2)+2
/21*B*c^2*x^(21/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1584, 448}

$$\frac{2}{9}Ab^2x^{9/2} + \frac{2}{17}cx^{17/2}(Ac+2bB) + \frac{2}{13}bx^{13/2}(2Ac+bB) + \frac{2}{21}Bc^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

```
[Out] (2*A*b^2*x^(9/2))/9 + (2*b*(b*B + 2*A*c)*x^(13/2))/13 + (2*c*(2*b*B + A*c)*
x^(17/2))/17 + (2*B*c^2*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx &= \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2 \, dx \\
&= \int \left(Ab^2x^{7/2} + b(bB+2Ac)x^{11/2} + c(2bB+Ac)x^{15/2} + Bc^2x^{19/2}\right) dx \\
&= \frac{2}{9}Ab^2x^{9/2} + \frac{2}{13}b(bB+2Ac)x^{13/2} + \frac{2}{17}c(2bB+Ac)x^{17/2} + \frac{2}{21}Bc^2x^{21/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547Ab^2 + 819cx^4(Ac+2bB) + 1071bx^2(2Ac+bB) + 663Bc^2x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

```
[Out] (2*x^(9/2)*(1547*A*b^2 + 1071*b*(b*B + 2*A*c)*x^2 + 819*c*(2*b*B + A*c)*x^4
 + 663*B*c^2*x^6))/13923
```

fricas [A] time = 0.88, size = 56, normalized size = 0.89

$$\frac{2}{13923}\left(663\,Bc^2x^{10}+819\left(2\,Bbc+Ac^2\right)x^8+1547\,Ab^2x^4+1071\left(Bb^2+2\,Abc\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*B*c^2*x^10 + 819*(2*B*b*c + A*c^2)*x^8 + 1547*A*b^2*x^4 + 1071
*(B*b^2 + 2*A*b*c)*x^6)*sqrt(x)
```

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{21}\,Bc^2x^{\frac{21}{2}}+\frac{4}{17}\,Bbcx^{\frac{17}{2}}+\frac{2}{17}\,Ac^2x^{\frac{17}{2}}+\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{13}\,Abcx^{\frac{13}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*c^2*x^(21/2) + 4/17*B*b*c*x^(17/2) + 2/17*A*c^2*x^(17/2) + 2/13*B*b^
2*x^(13/2) + 4/13*A*b*c*x^(13/2) + 2/9*A*b^2*x^(9/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(663B\,c^2x^6+819A\,c^2x^4+1638Bbc\,x^4+2142Abc\,x^2+1071B\,b^2x^2+1547b^2A\right)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] 2/13923*x^(9/2)*(663*B*c^2*x^6+819*A*c^2*x^4+1638*B*b*c*x^4+2142*A*b*c*x^2+
1071*B*b^2*x^2+1547*A*b^2)
```

maxima [A] time = 1.35, size = 51, normalized size = 0.81

$$\frac{2}{21}\,Bc^2x^{\frac{21}{2}}+\frac{2}{17}\left(2\,Bbc+Ac^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}+\frac{2}{13}\left(Bb^2+2\,Abc\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*c^2*x^(21/2) + 2/17*(2*B*b*c + A*c^2)*x^(17/2) + 2/9*A*b^2*x^(9/2) +
 2/13*(B*b^2 + 2*A*b*c)*x^(13/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,b^2}{13}+\frac{4\,A\,c\,b}{13}\right)+x^{17/2}\left(\frac{2\,A\,c^2}{17}+\frac{4\,B\,b\,c}{17}\right)+\frac{2\,A\,b^2\,x^{9/2}}{9}+\frac{2\,B\,c^2\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(1/2),x)
```

```
[Out] x^(13/2)*((2*B*b^2)/13 + (4*A*b*c)/13) + x^(17/2)*((2*A*c^2)/17 + (4*B*b*c)
/17) + (2*A*b^2*x^(9/2))/9 + (2*B*c^2*x^(21/2))/21
```

sympy [A] time = 9.94, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Abcx^{\frac{13}{2}}}{13}+\frac{2Ac^2x^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{13}{2}}}{13}+\frac{4Bbcx^{\frac{17}{2}}}{17}+\frac{2Bc^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 2*A*b**2*x**(9/2)/9 + 4*A*b*c*x**(13/2)/13 + 2*A*c**2*x**(17/2)/17 + 2*B*b*
*2*x**(13/2)/13 + 4*B*b*c*x**(17/2)/17 + 2*B*c**2*x**(21/2)/21
```

3.172 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{19}Bc^2x^{19/2}$$

```
[Out] 2/7*A*b^2*x^(7/2)+2/11*b*(2*A*c+B*b)*x^(11/2)+2/15*c*(A*c+2*B*b)*x^(15/2)+2
/19*B*c^2*x^(19/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{19}Bc^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

```
[Out] (2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)*
x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{3/2}}\,dx &= \int x^{5/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{5/2}+b(bB+2Ac)x^{9/2}+c(2bB+Ac)x^{13/2}+Bc^2x^{17/2}\right)dx\\ &= \frac{2}{7}Ab^2x^{7/2}+\frac{2}{11}b(bB+2Ac)x^{11/2}+\frac{2}{15}c(2bB+Ac)x^{15/2}+\frac{2}{19}Bc^2x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{19}Bc^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

```
[Out] (2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)*
x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

fricas [A] time = 0.83, size = 56, normalized size = 0.89

$$\frac{2}{21945}\left(1155\,Bc^2x^9+1463\left(2\,Bbc+Ac^2\right)x^7+3135\,Ab^2x^3+1995\left(Bb^2+2\,Abc\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*c^2*x^9 + 1463*(2*B*b*c + A*c^2)*x^7 + 3135*A*b^2*x^3 + 199
5*(B*b^2 + 2*A*b*c)*x^5)*sqrt(x)
```

giac [A] time = 0.21, size = 53, normalized size = 0.84

$$\frac{2}{19}\,Bc^2x^{\frac{19}{2}}+\frac{4}{15}\,Bbcx^{\frac{15}{2}}+\frac{2}{15}\,Ac^2x^{\frac{15}{2}}+\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{4}{11}\,Abcx^{\frac{11}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 4/15*B*b*c*x^(15/2) + 2/15*A*c^2*x^(15/2) + 2/11*B*b^
2*x^(11/2) + 4/11*A*b*c*x^(11/2) + 2/7*A*b^2*x^(7/2)
```

maple [A] time = 0.06, size = 56, normalized size = 0.89

$$\frac{2\left(1155B\,c^2x^6+1463A\,c^2x^4+2926Bbc\,x^4+3990Abc\,x^2+1995B\,b^2x^2+3135b^2A\right)x^{\frac{7}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*B*c^2*x^6+1463*A*c^2*x^4+2926*B*b*c*x^4+3990*A*b*c*x^
2+1995*B*b^2*x^2+3135*A*b^2)
```

maxima [A] time = 1.36, size = 51, normalized size = 0.81

$$\frac{2}{19}\,Bc^2x^{\frac{19}{2}}+\frac{2}{15}\left(2\,Bbc+Ac^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}+\frac{2}{11}\left(Bb^2+2\,Abc\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 2/15*(2*B*b*c + A*c^2)*x^(15/2) + 2/7*A*b^2*x^(7/2) +
 2/11*(B*b^2 + 2*A*b*c)*x^(11/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,b^2}{11}+\frac{4\,A\,c\,b}{11}\right)+x^{15/2}\left(\frac{2\,A\,c^2}{15}+\frac{4\,B\,b\,c}{15}\right)+\frac{2\,A\,b^2\,x^{7/2}}{7}+\frac{2\,B\,c^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x)
```

```
[Out] x^(11/2)*((2*B*b^2)/11 + (4*A*b*c)/11) + x^(15/2)*((2*A*c^2)/15 + (4*B*b*c)
/15) + (2*A*b^2*x^(7/2))/7 + (2*B*c^2*x^(19/2))/19
```

sympy [A] time = 10.48, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{7}{2}}}{7}+\frac{4Abcx^{\frac{11}{2}}}{11}+\frac{2Ac^2x^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{11}{2}}}{11}+\frac{4Bbcx^{\frac{15}{2}}}{15}+\frac{2Bc^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(3/2),x)
```

```
[Out] 2*A*b**2*x**(7/2)/7 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(15/2)/15 + 2*B*b*
*2*x**(11/2)/11 + 4*B*b*c*x**(15/2)/15 + 2*B*c**2*x**(19/2)/19
```

3.173 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{5}Ab^2x^{5/2} + \frac{2}{13}cx^{13/2}(Ac+2bB) + \frac{2}{9}bx^{9/2}(2Ac+bB) + \frac{2}{17}Bc^2x^{17/2}$$

```
[Out] 2/5*A*b^2*x^(5/2)+2/9*b*(2*A*c+B*b)*x^(9/2)+2/13*c*(A*c+2*B*b)*x^(13/2)+2/1
7*B*c^2*x^(17/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1584, 448}

$$\frac{2}{5}Ab^2x^{5/2} + \frac{2}{13}cx^{13/2}(Ac+2bB) + \frac{2}{9}bx^{9/2}(2Ac+bB) + \frac{2}{17}Bc^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

```
[Out] (2*A*b^2*x^(5/2))/5 + (2*b*(b*B + 2*A*c)*x^(9/2))/9 + (2*c*(2*b*B + A*c)*x^
(13/2))/13 + (2*B*c^2*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}} \, dx &= \int x^{3/2}(A+Bx^2)(b+cx^2)^2 \, dx \\
&= \int \left(Ab^2x^{3/2} + b(bB+2Ac)x^{7/2} + c(2bB+Ac)x^{11/2} + Bc^2x^{15/2}\right) dx \\
&= \frac{2}{5}Ab^2x^{5/2} + \frac{2}{9}b(bB+2Ac)x^{9/2} + \frac{2}{13}c(2bB+Ac)x^{13/2} + \frac{2}{17}Bc^2x^{17/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989Ab^2 + 765cx^4(Ac+2bB) + 1105bx^2(2Ac+bB) + 585Bc^2x^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

```
[Out] (2*x^(5/2)*(1989*A*b^2 + 1105*b*(b*B + 2*A*c)*x^2 + 765*c*(2*b*B + A*c)*x^4
 + 585*B*c^2*x^6))/9945
```

fricas [A] time = 0.80, size = 56, normalized size = 0.89

$$\frac{2}{9945}\left(585\,Bc^2x^8+765\left(2\,Bbc+Ac^2\right)x^6+1989\,Ab^2x^2+1105\left(Bb^2+2\,Abc\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*c^2*x^8 + 765*(2*B*b*c + A*c^2)*x^6 + 1989*A*b^2*x^2 + 1105*(
B*b^2 + 2*A*b*c)*x^4)*sqrt(x)
```

giac [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{2}{17}\,Bc^2x^{\frac{17}{2}}+\frac{4}{13}\,Bbcx^{\frac{13}{2}}+\frac{2}{13}\,Ac^2x^{\frac{13}{2}}+\frac{2}{9}\,Bb^2x^{\frac{9}{2}}+\frac{4}{9}\,Abcx^{\frac{9}{2}}+\frac{2}{5}\,Ab^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 4/13*B*b*c*x^(13/2) + 2/13*A*c^2*x^(13/2) + 2/9*B*b^2
*x^(9/2) + 4/9*A*b*c*x^(9/2) + 2/5*A*b^2*x^(5/2)
```

maple [A] time = 0.06, size = 56, normalized size = 0.89

$$\frac{2\left(585B\,c^2x^6+765A\,c^2x^4+1530Bbc\,x^4+2210Abc\,x^2+1105B\,b^2x^2+1989b^2A\right)x^{\frac{5}{2}}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x)
```

```
[Out] 2/9945*x^(5/2)*(585*B*c^2*x^6+765*A*c^2*x^4+1530*B*b*c*x^4+2210*A*b*c*x^2+1
105*B*b^2*x^2+1989*A*b^2)
```

maxima [A] time = 1.37, size = 51, normalized size = 0.81

$$\frac{2}{17}\,Bc^2x^{\frac{17}{2}}+\frac{2}{13}\left(2\,Bbc+Ac^2\right)x^{\frac{13}{2}}+\frac{2}{5}\,Ab^2x^{\frac{5}{2}}+\frac{2}{9}\left(Bb^2+2\,Abc\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 2/13*(2*B*b*c + A*c^2)*x^(13/2) + 2/5*A*b^2*x^(5/2) +
 2/9*(B*b^2 + 2*A*b*c)*x^(9/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,b^2}{9}+\frac{4\,A\,c\,b}{9}\right)+x^{13/2}\left(\frac{2\,A\,c^2}{13}+\frac{4\,B\,b\,c}{13}\right)+\frac{2\,A\,b^2\,x^{5/2}}{5}+\frac{2\,B\,c^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x)
```

```
[Out] x^(9/2)*((2*B*b^2)/9 + (4*A*b*c)/9) + x^(13/2)*((2*A*c^2)/13 + (4*B*b*c)/13
) + (2*A*b^2*x^(5/2))/5 + (2*B*c^2*x^(17/2))/17
```

sympy [A] time = 12.06, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{5}{2}}}{5}+\frac{4Abcx^{\frac{9}{2}}}{9}+\frac{2Ac^2x^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{9}{2}}}{9}+\frac{4Bbcx^{\frac{13}{2}}}{13}+\frac{2Bc^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(5/2),x)
```

```
[Out] 2*A*b**2*x**(5/2)/5 + 4*A*b*c*x**(9/2)/9 + 2*A*c**2*x**(13/2)/13 + 2*B*b**2
*x**(9/2)/9 + 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(17/2)/17
```

3.174 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{7}bx^{7/2}(2Ac+bB)+\frac{2}{15}Bc^2x^{15/2}$$

```
[Out] 2/3*A*b^2*x^(3/2)+2/7*b*(2*A*c+B*b)*x^(7/2)+2/11*c*(A*c+2*B*b)*x^(11/2)+2/1
5*B*c^2*x^(15/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{7}bx^{7/2}(2Ac+bB)+\frac{2}{15}Bc^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

```
[Out] (2*A*b^2*x^(3/2))/3 + (2*b*(b*B + 2*A*c)*x^(7/2))/7 + (2*c*(2*b*B + A*c)*x^
(11/2))/11 + (2*B*c^2*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx &= \int \sqrt{x}\,(A+Bx^2)(b+cx^2)^2\,dx\\ &= \int \left(Ab^2\sqrt{x}+b(bB+2Ac)x^{5/2}+c(2bB+Ac)x^{9/2}+Bc^2x^{13/2}\right)dx\\ &= \frac{2}{3}Ab^2x^{3/2}+\frac{2}{7}b(bB+2Ac)x^{7/2}+\frac{2}{11}c(2bB+Ac)x^{11/2}+\frac{2}{15}Bc^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385Ab^2+105cx^4(Ac+2bB)+165bx^2(2Ac+bB)+77Bc^2x^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

```
[Out] (2*x^(3/2)*(385*A*b^2 + 165*b*(b*B + 2*A*c)*x^2 + 105*c*(2*b*B + A*c)*x^4 +
 77*B*c^2*x^6))/1155
```

fricas [A] time = 0.96, size = 54, normalized size = 0.86

$$\frac{2}{1155}\left(77\,Bc^2x^7+105\left(2\,Bbc+Ac^2\right)x^5+385\,Ab^2x+165\left(Bb^2+2\,Abc\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*c^2*x^7 + 105*(2*B*b*c + A*c^2)*x^5 + 385*A*b^2*x + 165*(B*b^2
 + 2*A*b*c)*x^3)*sqrt(x)
```

giac [A] time = 0.17, size = 53, normalized size = 0.84

$$\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{4}{11}\,Bbcx^{\frac{11}{2}}+\frac{2}{11}\,Ac^2x^{\frac{11}{2}}+\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{4}{7}\,Abcx^{\frac{7}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 4/11*B*b*c*x^(11/2) + 2/11*A*c^2*x^(11/2) + 2/7*B*b^2
*x^(7/2) + 4/7*A*b*c*x^(7/2) + 2/3*A*b^2*x^(3/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(77B\,c^2x^6+105A\,c^2x^4+210Bbc\,x^4+330Abc\,x^2+165B\,b^2x^2+385b^2A\right)x^{\frac{3}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*B*c^2*x^6+105*A*c^2*x^4+210*B*b*c*x^4+330*A*b*c*x^2+165*
B*b^2*x^2+385*A*b^2)
```

maxima [A] time = 1.33, size = 51, normalized size = 0.81

$$\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{2}{11}\left(2\,Bbc+Ac^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}+\frac{2}{7}\left(Bb^2+2\,Abc\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 2/11*(2*B*b*c + A*c^2)*x^(11/2) + 2/3*A*b^2*x^(3/2) +
 2/7*(B*b^2 + 2*A*b*c)*x^(7/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,B\,b^2}{7}+\frac{4\,A\,c\,b}{7}\right)+x^{11/2}\left(\frac{2\,A\,c^2}{11}+\frac{4\,B\,b\,c}{11}\right)+\frac{2\,A\,b^2\,x^{3/2}}{3}+\frac{2\,B\,c^2\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*B*b^2)/7 + (4*A*b*c)/7) + x^(11/2)*((2*A*c^2)/11 + (4*B*b*c)/11
) + (2*A*b^2*x^(3/2))/3 + (2*B*c^2*x^(15/2))/15
```

sympy [A] time = 15.46, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Abcx^{\frac{7}{2}}}{7}+\frac{2Ac^2x^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{7}{2}}}{7}+\frac{4Bbcx^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*A*b**2*x**(3/2)/3 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(11/2)/11 + 2*B*b**2
*x**(7/2)/7 + 4*B*b*c*x**(11/2)/11 + 2*B*c**2*x**(15/2)/15
```

3.175 $\int x^{7/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=85

$$\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

```
[Out] 2/21*A*b^3*x^(21/2)+2/25*b^2*(3*A*c+B*b)*x^(25/2)+6/29*b*c*(A*c+B*b)*x^(29/
2)+2/33*c^2*(A*c+3*B*b)*x^(33/2)+2/37*B*c^3*x^(37/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{21}Ab^3x^{21/2}+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3 dx &= \int x^{19/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3 dx\\ &= \int \left(Ab^3x^{19/2}+b^2(bB+3Ac)x^{23/2}+3bc(bB+Ac)x^{27/2}+c^2(3bB+Ac)x^{31}\right.\\ &= \frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

fricas [A] time = 0.60, size = 78, normalized size = 0.92

$$\frac{2}{6196575}\left(167475\,Bc^3x^{18}+187775\left(3\,Bbc^2+Ac^3\right)x^{16}+641025\left(Bb^2c+Abc^2\right)x^{14}+295075\,Ab^3x^{10}+247863\left(B\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/6196575*(167475*B*c^3*x^18 + 187775*(3*B*b*c^2 + A*c^3)*x^16 + 641025*(B*
b^2*c + A*b*c^2)*x^14 + 295075*A*b^3*x^10 + 247863*(B*b^3 + 3*A*b^2*c)*x^12
)*sqrt(x)
```

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{11}\,Bbc^2x^{\frac{33}{2}}+\frac{2}{33}\,Ac^3x^{\frac{33}{2}}+\frac{6}{29}\,Bb^2cx^{\frac{29}{2}}+\frac{6}{29}\,Abc^2x^{\frac{29}{2}}+\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{6}{25}\,Ab^2cx^{\frac{25}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/11*B*b*c^2*x^(33/2) + 2/33*A*c^3*x^(33/2) + 6/29*B*
b^2*c*x^(29/2) + 6/29*A*b*c^2*x^(29/2) + 2/25*B*b^3*x^(25/2) + 6/25*A*b^2*c
*x^(25/2) + 2/21*A*b^3*x^(21/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(167475B\,c^3x^8+187775A\,c^3x^6+563325Bb\,c^2x^6+641025Ab\,c^2x^4+641025B\,b^2c\,x^4+743589A\,b^2c\,x^2+2478\right.}{6196575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/6196575*x^(21/2)*(167475*B*c^3*x^8+187775*A*c^3*x^6+563325*B*b*c^2*x^6+64
1025*A*b*c^2*x^4+641025*B*b^2*c*x^4+743589*A*b^2*c*x^2+247863*B*b^3*x^2+295
075*A*b^3)
```

maxima [A] time = 1.32, size = 73, normalized size = 0.86

$$\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{33}\left(3\,Bbc^2+Ac^3\right)x^{\frac{33}{2}}+\frac{6}{29}\left(Bb^2c+Abc^2\right)x^{\frac{29}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{2}{25}\left(Bb^3+3\,Ab^2c\right)x^{\frac{25}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/33*(3*B*b*c^2 + A*c^3)*x^(33/2) + 6/29*(B*b^2*c + A
*b*c^2)*x^(29/2) + 2/21*A*b^3*x^(21/2) + 2/25*(B*b^3 + 3*A*b^2*c)*x^(25/2)
```

mupad [B] time = 0.10, size = 69, normalized size = 0.81

$$x^{25/2}\left(\frac{2\,B\,b^3}{25}+\frac{6\,A\,c\,b^2}{25}\right)+x^{33/2}\left(\frac{2\,A\,c^3}{33}+\frac{2\,B\,b\,c^2}{11}\right)+\frac{2\,A\,b^3\,x^{21/2}}{21}+\frac{2\,B\,c^3\,x^{37/2}}{37}+\frac{6\,b\,c\,x^{29/2}\,\left(A\,c+B\,b\right)}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(25/2)*((2*B*b^3)/25 + (6*A*b^2*c)/25) + x^(33/2)*((2*A*c^3)/33 + (2*B*b*
c^2)/11) + (2*A*b^3*x^(21/2))/21 + (2*B*c^3*x^(37/2))/37 + (6*b*c*x^(29/2)*
(A*c + B*b))/29
```

<u>**sympy**</u> **[A]** time = 134.77, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{6Ab^2cx^{\frac{25}{2}}}{25}+\frac{6Abc^2x^{\frac{29}{2}}}{29}+\frac{2Ac^3x^{\frac{33}{2}}}{33}+\frac{2Bb^3x^{\frac{25}{2}}}{25}+\frac{6Bb^2cx^{\frac{29}{2}}}{29}+\frac{2Bbc^2x^{\frac{33}{2}}}{11}+\frac{2Bc^3x^{\frac{37}{2}}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(21/2)/21 + 6*A*b**2*c*x**(25/2)/25 + 6*A*b*c**2*x**(29/2)/29 +
 2*A*c**3*x**(33/2)/33 + 2*B*b**3*x**(25/2)/25 + 6*B*b**2*c*x**(29/2)/29 +
2*B*b*c**2*x**(33/2)/11 + 2*B*c**3*x**(37/2)/37
```

3.176 $\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=85

$$\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

```
[Out] 2/19*A*b^3*x^(19/2)+2/23*b^2*(3*A*c+B*b)*x^(23/2)+2/9*b*c*(A*c+B*b)*x^(27/2
)+2/31*c^2*(A*c+3*B*b)*x^(31/2)+2/35*B*c^3*x^(35/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{19}Ab^3x^{19/2}+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx &= \int x^{17/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{17/2}+b^2(bB+3Ac)x^{21/2}+3bc(bB+Ac)x^{25/2}+c^2(3bB+Ac)x^{29/2}\right.\\ &= \frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

fricas [A] time = 0.76, size = 78, normalized size = 0.92

$$\frac{2}{4267305}\left(121923\,Bc^3x^{17} + 137655\left(3\,Bbc^2 + Ac^3\right)x^{15} + 474145\left(Bb^2c + Abc^2\right)x^{13} + 224595\,Ab^3x^9 + 185535\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/4267305*(121923*B*c^3*x^17 + 137655*(3*B*b*c^2 + A*c^3)*x^15 + 474145*(B*
b^2*c + A*b*c^2)*x^13 + 224595*A*b^3*x^9 + 185535*(B*b^3 + 3*A*b^2*c)*x^11)
*sqrt(x)
```

giac [A] time = 0.17, size = 77, normalized size = 0.91

$$\frac{2}{35}\,Bc^3x^{\frac{35}{2}} + \frac{6}{31}\,Bbc^2x^{\frac{31}{2}} + \frac{2}{31}\,Ac^3x^{\frac{31}{2}} + \frac{2}{9}\,Bb^2cx^{\frac{27}{2}} + \frac{2}{9}\,Abc^2x^{\frac{27}{2}} + \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{6}{23}\,Ab^2cx^{\frac{23}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 6/31*B*b*c^2*x^(31/2) + 2/31*A*c^3*x^(31/2) + 2/9*B*b
^2*c*x^(27/2) + 2/9*A*b*c^2*x^(27/2) + 2/23*B*b^3*x^(23/2) + 6/23*A*b^2*c*x
^(23/2) + 2/19*A*b^3*x^(19/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(121923B\,c^3x^8 + 137655A\,c^3x^6 + 412965Bb\,c^2x^6 + 474145Ab\,c^2x^4 + 474145B\,b^2c\,x^4 + 556605A\,b^2c\,x^2 + 18\right.}{4267305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/4267305*x^(19/2)*(121923*B*c^3*x^8+137655*A*c^3*x^6+412965*B*b*c^2*x^6+47
4145*A*b*c^2*x^4+474145*B*b^2*c*x^4+556605*A*b^2*c*x^2+185535*B*b^3*x^2+224
595*A*b^3)
```

maxima [A] time = 1.36, size = 73, normalized size = 0.86

$$\frac{2}{35}\,Bc^3x^{\frac{35}{2}} + \frac{2}{31}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{31}{2}} + \frac{2}{9}\left(Bb^2c + Abc^2\right)x^{\frac{27}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{2}{23}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{23}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 2/31*(3*B*b*c^2 + A*c^3)*x^(31/2) + 2/9*(B*b^2*c + A*
b*c^2)*x^(27/2) + 2/19*A*b^3*x^(19/2) + 2/23*(B*b^3 + 3*A*b^2*c)*x^(23/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{23/2}\left(\frac{2\,B\,b^3}{23} + \frac{6\,A\,c\,b^2}{23}\right) + x^{31/2}\left(\frac{2\,A\,c^3}{31} + \frac{6\,B\,b\,c^2}{31}\right) + \frac{2\,A\,b^3\,x^{19/2}}{19} + \frac{2\,B\,c^3\,x^{35/2}}{35} + \frac{2\,b\,c\,x^{27/2}\,\left(A\,c + B\,b\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(23/2)*((2*B*b^3)/23 + (6*A*b^2*c)/23) + x^(31/2)*((2*A*c^3)/31 + (6*B*b*
c^2)/31) + (2*A*b^3*x^(19/2))/19 + (2*B*c^3*x^(35/2))/35 + (2*b*c*x^(27/2)*
(A*c + B*b))/9
```

sympy [A] time = 92.32, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{19}{2}}}{19}+\frac{6Ab^2cx^{\frac{23}{2}}}{23}+\frac{2Abc^2x^{\frac{27}{2}}}{9}+\frac{2Ac^3x^{\frac{31}{2}}}{31}+\frac{2Bb^3x^{\frac{23}{2}}}{23}+\frac{2Bb^2cx^{\frac{27}{2}}}{9}+\frac{6Bbc^2x^{\frac{31}{2}}}{31}+\frac{2Bc^3x^{\frac{35}{2}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(19/2)/19 + 6*A*b**2*c*x**(23/2)/23 + 2*A*b*c**2*x**(27/2)/9 +
2*A*c**3*x**(31/2)/31 + 2*B*b**3*x**(23/2)/23 + 2*B*b**2*c*x**(27/2)/9 + 6*
B*b*c**2*x**(31/2)/31 + 2*B*c**3*x**(35/2)/35
```

3.177 $\int x^{3/2} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=85

$$\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

```
[Out] 2/17*A*b^3*x^(17/2)+2/21*b^2*(3*A*c+B*b)*x^(21/2)+6/25*b*c*(A*c+B*b)*x^(25/
2)+2/29*c^2*(A*c+3*B*b)*x^(29/2)+2/33*B*c^3*x^(33/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{17}Ab^3x^{17/2}+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B + A
*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))/3
3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx &= \int x^{15/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{15/2}+b^2(bB+3Ac)x^{19/2}+3bc(bB+Ac)x^{23/2}+c^2(3bB+Ac)x^{27}\right.\\ &= \frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2(bB+3Ac)x^{21/2}+\frac{6}{25}bc(bB+Ac)x^{25/2}+\frac{2}{29}c^2(3bB+\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B + A
*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))/3
3
```

fricas [A] time = 0.97, size = 78, normalized size = 0.92

$$\frac{2}{2847075}\left(86275\,Bc^3x^{16} + 98175\left(3\,Bbc^2 + Ac^3\right)x^{14} + 341649\left(Bb^2c + Abc^2\right)x^{12} + 167475\,Ab^3x^8 + 135575\left(Bb^3 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/2847075*(86275*B*c^3*x^16 + 98175*(3*B*b*c^2 + A*c^3)*x^14 + 341649*(B*b^
2*c + A*b*c^2)*x^12 + 167475*A*b^3*x^8 + 135575*(B*b^3 + 3*A*b^2*c)*x^10)*s
qrt(x)
```

giac [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{33}\,Bc^3x^{\frac{33}{2}} + \frac{6}{29}\,Bbc^2x^{\frac{29}{2}} + \frac{2}{29}\,Ac^3x^{\frac{29}{2}} + \frac{6}{25}\,Bb^2cx^{\frac{25}{2}} + \frac{6}{25}\,Abc^2x^{\frac{25}{2}} + \frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{7}\,Ab^2cx^{\frac{21}{2}} + \frac{2}{17}\,Ab^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 6/29*B*b*c^2*x^(29/2) + 2/29*A*c^3*x^(29/2) + 6/25*B*
b^2*c*x^(25/2) + 6/25*A*b*c^2*x^(25/2) + 2/21*B*b^3*x^(21/2) + 2/7*A*b^2*c*
x^(21/2) + 2/17*A*b^3*x^(17/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(86275B\,c^3x^8 + 98175A\,c^3x^6 + 294525Bb\,c^2x^6 + 341649Ab\,c^2x^4 + 341649B\,b^2c\,x^4 + 406725A\,b^2c\,x^2 + 135575\right.}{2847075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/2847075*x^(17/2)*(86275*B*c^3*x^8+98175*A*c^3*x^6+294525*B*b*c^2*x^6+3416
49*A*b*c^2*x^4+341649*B*b^2*c*x^4+406725*A*b^2*c*x^2+135575*B*b^3*x^2+16747
5*A*b^3)
```

maxima [A] time = 1.38, size = 73, normalized size = 0.86

$$\frac{2}{33}\,Bc^3x^{\frac{33}{2}} + \frac{2}{29}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{29}{2}} + \frac{6}{25}\left(Bb^2c + Abc^2\right)x^{\frac{25}{2}} + \frac{2}{17}\,Ab^3x^{\frac{17}{2}} + \frac{2}{21}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 2/29*(3*B*b*c^2 + A*c^3)*x^(29/2) + 6/25*(B*b^2*c + A
*b*c^2)*x^(25/2) + 2/17*A*b^3*x^(17/2) + 2/21*(B*b^3 + 3*A*b^2*c)*x^(21/2)
```

mupad [B] time = 0.04, size = 69, normalized size = 0.81

$$x^{21/2}\left(\frac{2\,B\,b^3}{21} + \frac{2\,A\,c\,b^2}{7}\right)+x^{29/2}\left(\frac{2\,A\,c^3}{29} + \frac{6\,B\,b\,c^2}{29}\right)+\frac{2\,A\,b^3\,x^{17/2}}{17}+\frac{2\,B\,c^3\,x^{33/2}}{33}+\frac{6\,b\,c\,x^{25/2}\,\left(A\,c + B\,b\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(21/2)*((2*B*b^3)/21 + (2*A*b^2*c)/7) + x^(29/2)*((2*A*c^3)/29 + (6*B*b*c
^2)/29) + (2*A*b^3*x^(17/2))/17 + (2*B*c^3*x^(33/2))/33 + (6*b*c*x^(25/2)*(
A*c + B*b))/25
```

sympy [**A**] time = 54.01, size = 114, normalized size = 1.34

$$\frac{2Ab^{3}x^{\frac{17}{2}}}{17}+\frac{2Ab^{2}cx^{\frac{21}{2}}}{7}+\frac{6Abc^{2}x^{\frac{25}{2}}}{25}+\frac{2Ac^{3}x^{\frac{29}{2}}}{29}+\frac{2Bb^{3}x^{\frac{21}{2}}}{21}+\frac{6Bb^{2}cx^{\frac{25}{2}}}{25}+\frac{6Bbc^{2}x^{\frac{29}{2}}}{29}+\frac{2Bc^{3}x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(17/2)/17 + 2*A*b**2*c*x**(21/2)/7 + 6*A*b*c**2*x**(25/2)/25 +
2*A*c**3*x**(29/2)/29 + 2*B*b**3*x**(21/2)/21 + 6*B*b**2*c*x**(25/2)/25 + 6
*B*b*c**2*x**(29/2)/29 + 2*B*c**3*x**(33/2)/33
```

3.178 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=85

$$\frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

```
[Out] 2/15*A*b^3*x^(15/2)+2/19*b^2*(3*A*c+B*b)*x^(19/2)+6/23*b*c*(A*c+B*b)*x^(23/
2)+2/27*c^2*(A*c+3*B*b)*x^(27/2)+2/31*B*c^3*x^(31/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{15}Ab^3x^{15/2}+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B + A
*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))/3
1
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx &= \int x^{13/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{13/2}+b^2(bB+3Ac)x^{17/2}+3bc(bB+Ac)x^{21/2}+c^2(3bB+Ac)x^{25/2}+\right.\\ &= \frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2(bB+3Ac)x^{19/2}+\frac{6}{23}bc(bB+Ac)x^{23/2}+\frac{2}{27}c^2(3bB+Ac\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B + A
*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))/3
1
```

fricas [A] time = 0.84, size = 78, normalized size = 0.92

$$\frac{2}{1828845}\left(58995\,Bc^3x^{15} + 67735\left(3\,Bbc^2 + Ac^3\right)x^{13} + 238545\left(Bb^2c + Abc^2\right)x^{11} + 121923\,Ab^3x^7 + 96255\left(Bb\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1828845*(58995*B*c^3*x^15 + 67735*(3*B*b*c^2 + A*c^3)*x^13 + 238545*(B*b^
2*c + A*b*c^2)*x^11 + 121923*A*b^3*x^7 + 96255*(B*b^3 + 3*A*b^2*c)*x^9)*sqr
t(x)
```

giac [A] time = 0.17, size = 77, normalized size = 0.91

$$\frac{2}{31}\,Bc^3x^{\frac{31}{2}} + \frac{2}{9}\,Bbc^2x^{\frac{27}{2}} + \frac{2}{27}\,Ac^3x^{\frac{27}{2}} + \frac{6}{23}\,Bb^2cx^{\frac{23}{2}} + \frac{6}{23}\,Abc^2x^{\frac{23}{2}} + \frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{6}{19}\,Ab^2cx^{\frac{19}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/9*B*b*c^2*x^(27/2) + 2/27*A*c^3*x^(27/2) + 6/23*B*b
^2*c*x^(23/2) + 6/23*A*b*c^2*x^(23/2) + 2/19*B*b^3*x^(19/2) + 6/19*A*b^2*c*
x^(19/2) + 2/15*A*b^3*x^(15/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(58995B\,c^3x^8 + 67735A\,c^3x^6 + 203205Bb\,c^2x^6 + 238545Ab\,c^2x^4 + 238545B\,b^2c\,x^4 + 288765A\,b^2c\,x^2 + 9625\right.}{1828845}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x)
```

```
[Out] 2/1828845*x^(15/2)*(58995*B*c^3*x^8+67735*A*c^3*x^6+203205*B*b*c^2*x^6+2385
45*A*b*c^2*x^4+238545*B*b^2*c*x^4+288765*A*b^2*c*x^2+96255*B*b^3*x^2+121923
*A*b^3)
```

maxima [A] time = 1.33, size = 73, normalized size = 0.86

$$\frac{2}{31}\,Bc^3x^{\frac{31}{2}} + \frac{2}{27}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{27}{2}} + \frac{6}{23}\left(Bb^2c + Abc^2\right)x^{\frac{23}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{2}{19}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/27*(3*B*b*c^2 + A*c^3)*x^(27/2) + 6/23*(B*b^2*c + A
*b*c^2)*x^(23/2) + 2/15*A*b^3*x^(15/2) + 2/19*(B*b^3 + 3*A*b^2*c)*x^(19/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{19/2}\left(\frac{2\,B\,b^3}{19}+\frac{6\,A\,c\,b^2}{19}\right)+x^{27/2}\left(\frac{2\,A\,c^3}{27}+\frac{2\,B\,b\,c^2}{9}\right)+\frac{2\,A\,b^3\,x^{15/2}}{15}+\frac{2\,B\,c^3\,x^{31/2}}{31}+\frac{6\,b\,c\,x^{23/2}\,\left(A\,c+B\,b\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(19/2)*((2*B*b^3)/19 + (6*A*b^2*c)/19) + x^(27/2)*((2*A*c^3)/27 + (2*B*b*
c^2)/9) + (2*A*b^3*x^(15/2))/15 + (2*B*c^3*x^(31/2))/31 + (6*b*c*x^(23/2)*(
A*c + B*b))/23
```

sympy [A] time = 5.51, size = 95, normalized size = 1.12

$$\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bc^3x^{\frac{31}{2}}}{31}+\frac{2x^{\frac{27}{2}}\left(Ac^3+3Bbc^2\right)}{27}+\frac{2x^{\frac{23}{2}}\left(3Abc^2+3Bb^2c\right)}{23}+\frac{2x^{\frac{19}{2}}\left(3Ab^2c+Bb^3\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3*x**(1/2),x)
```

```
[Out] 2*A*b**3*x**(15/2)/15 + 2*B*c**3*x**(31/2)/31 + 2*x**(27/2)*(A*c**3 + 3*B*b
*c**2)/27 + 2*x**(23/2)*(3*A*b*c**2 + 3*B*b**2*c)/23 + 2*x**(19/2)*(3*A*b**
2*c + B*b**3)/19
```

3.179 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2x^{17/2}(3Ac+bB)+\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

```
[Out] 2/13*A*b^3*x^(13/2)+2/17*b^2*(3*A*c+B*b)*x^(17/2)+2/7*b*c*(A*c+B*b)*x^(21/2
)+2/25*c^2*(A*c+3*B*b)*x^(25/2)+2/29*B*c^3*x^(29/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{17}b^2x^{17/2}(3Ac+bB)+\frac{2}{13}Ab^3x^{13/2}+\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B + A
*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/29
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx &= \int x^{11/2}(A+Bx^2)(b+cx^2)^3\,dx\\ &= \int \left(Ab^3x^{11/2}+b^2(bB+3Ac)x^{15/2}+3bc(bB+Ac)x^{19/2}+c^2(3bB+Ac)x^{23/2}+\right.\\ &= \frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2(bB+3Ac)x^{17/2}+\frac{2}{7}bc(bB+Ac)x^{21/2}+\frac{2}{25}c^2(3bB+Ac)x^{2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2x^{17/2}(3Ac+bB)+\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B + A
*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/29
```

fricas [A] time = 0.89, size = 78, normalized size = 0.92

$$\frac{2}{1121575}\left(38675\,Bc^3x^{14}+44863\left(3\,Bbc^2+Ac^3\right)x^{12}+160225\left(Bb^2c+Abc^2\right)x^{10}+86275\,Ab^3x^6+65975\left(Bb^3+3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1121575*(38675*B*c^3*x^14 + 44863*(3*B*b*c^2 + A*c^3)*x^12 + 160225*(B*b^
2*c + A*b*c^2)*x^10 + 86275*A*b^3*x^6 + 65975*(B*b^3 + 3*A*b^2*c)*x^8)*sqrt
(x)
```

giac [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{29}\,Bc^3x^{\frac{29}{2}}+\frac{6}{25}\,Bbc^2x^{\frac{25}{2}}+\frac{2}{25}\,Ac^3x^{\frac{25}{2}}+\frac{2}{7}\,Bb^2cx^{\frac{21}{2}}+\frac{2}{7}\,Abc^2x^{\frac{21}{2}}+\frac{2}{17}\,Bb^3x^{\frac{17}{2}}+\frac{6}{17}\,Ab^2cx^{\frac{17}{2}}+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 6/25*B*b*c^2*x^(25/2) + 2/25*A*c^3*x^(25/2) + 2/7*B*b
^2*c*x^(21/2) + 2/7*A*b*c^2*x^(21/2) + 2/17*B*b^3*x^(17/2) + 6/17*A*b^2*c*x
^(17/2) + 2/13*A*b^3*x^(13/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(38675B\,c^3x^8+44863A\,c^3x^6+134589Bb\,c^2x^6+160225Ab\,c^2x^4+160225B\,b^2c\,x^4+197925A\,b^2c\,x^2+65975B\right.}{1121575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] 2/1121575*x^(13/2)*(38675*B*c^3*x^8+44863*A*c^3*x^6+134589*B*b*c^2*x^6+1602
25*A*b*c^2*x^4+160225*B*b^2*c*x^4+197925*A*b^2*c*x^2+65975*B*b^3*x^2+86275*
A*b^3)
```

maxima [A] time = 1.36, size = 73, normalized size = 0.86

$$\frac{2}{29}\,Bc^3x^{\frac{29}{2}}+\frac{2}{25}\left(3\,Bbc^2+Ac^3\right)x^{\frac{25}{2}}+\frac{2}{7}\left(Bb^2c+Abc^2\right)x^{\frac{21}{2}}+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}+\frac{2}{17}\left(Bb^3+3\,Ab^2c\right)x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 2/25*(3*B*b*c^2 + A*c^3)*x^(25/2) + 2/7*(B*b^2*c + A*
b*c^2)*x^(21/2) + 2/13*A*b^3*x^(13/2) + 2/17*(B*b^3 + 3*A*b^2*c)*x^(17/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{17/2}\left(\frac{2\,B\,b^3}{17}+\frac{6\,A\,c\,b^2}{17}\right)+x^{25/2}\left(\frac{2\,A\,c^3}{25}+\frac{6\,B\,b\,c^2}{25}\right)+\frac{2\,A\,b^3\,x^{13/2}}{13}+\frac{2\,B\,c^3\,x^{29/2}}{29}+\frac{2\,b\,c\,x^{21/2}\,\left(A\,c+B\,b\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(1/2),x)
```

```
[Out] x^(17/2)*((2*B*b^3)/17 + (6*A*b^2*c)/17) + x^(25/2)*((2*A*c^3)/25 + (6*B*b*
c^2)/25) + (2*A*b^3*x^(13/2))/13 + (2*B*c^3*x^(29/2))/29 + (2*b*c*x^(21/2)*
(A*c + B*b))/7
```

sympy [**A**] time = 29.88, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{13}{2}}}{13} + \frac{6Ab^2cx^{\frac{17}{2}}}{17} + \frac{2Abc^2x^{\frac{21}{2}}}{7} + \frac{2Ac^3x^{\frac{25}{2}}}{25} + \frac{2Bb^3x^{\frac{17}{2}}}{17} + \frac{2Bb^2cx^{\frac{21}{2}}}{7} + \frac{6Bbc^2x^{\frac{25}{2}}}{25} + \frac{2Bc^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*A*b**3*x**(13/2)/13 + 6*A*b**2*c*x**(17/2)/17 + 2*A*b*c**2*x**(21/2)/7 +
2*A*c**3*x**(25/2)/25 + 2*B*b**3*x**(17/2)/17 + 2*B*b**2*c*x**(21/2)/7 + 6*
B*b*c**2*x**(25/2)/25 + 2*B*c**3*x**(29/2)/29
```

3.180 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{11}Ab^3x^{11/2}+\frac{2}{15}b^2x^{15/2}(3Ac+bB)+\frac{2}{23}c^2x^{23/2}(Ac+3bB)+\frac{6}{19}bcx^{19/2}(Ac+bB)+\frac{2}{27}Bc^3x^{27/2}$$

```
[Out] 2/11*A*b^3*x^(11/2)+2/15*b^2*(3*A*c+B*b)*x^(15/2)+6/19*b*c*(A*c+B*b)*x^(19/
2)+2/23*c^2*(A*c+3*B*b)*x^(23/2)+2/27*B*c^3*x^(27/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{15}b^2x^{15/2}(3Ac+bB)+\frac{2}{11}Ab^3x^{11/2}+\frac{2}{23}c^2x^{23/2}(Ac+3bB)+\frac{6}{19}bcx^{19/2}(Ac+bB)+\frac{2}{27}Bc^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

```
[Out] (2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B + A
*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))/2
7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx &= \int x^{9/2}(A+Bx^2)(b+cx^2)^3\,dx\\ &= \int \left(Ab^3x^{9/2}+b^2(bB+3Ac)x^{13/2}+3bc(bB+Ac)x^{17/2}+c^2(3bB+Ac)x^{21/2}+Bc^3\right.\\ &= \frac{2}{11}Ab^3x^{11/2}+\frac{2}{15}b^2(bB+3Ac)x^{15/2}+\frac{6}{19}bc(bB+Ac)x^{19/2}+\frac{2}{23}c^2(3bB+Ac)x^{23/}\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{11}Ab^3x^{11/2}+\frac{2}{15}b^2x^{15/2}(3Ac+bB)+\frac{2}{23}c^2x^{23/2}(Ac+3bB)+\frac{6}{19}bcx^{19/2}(Ac+bB)+\frac{2}{27}Bc^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

```
[Out] (2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B + A
*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))/2
7
```

fricas [A] time = 0.89, size = 78, normalized size = 0.92

$$\frac{2}{648945}\left(24035\,Bc^3x^{13} + 28215\left(3\,Bbc^2 + Ac^3\right)x^{11} + 102465\left(Bb^2c + Abc^2\right)x^9 + 58995\,Ab^3x^5 + 43263\left(Bb^3 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/648945*(24035*B*c^3*x^13 + 28215*(3*B*b*c^2 + A*c^3)*x^11 + 102465*(B*b^2
*c + A*b*c^2)*x^9 + 58995*A*b^3*x^5 + 43263*(B*b^3 + 3*A*b^2*c)*x^7)*sqrt(x
)
```

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{27}\,Bc^3x^{\frac{27}{2}} + \frac{6}{23}\,Bbc^2x^{\frac{23}{2}} + \frac{2}{23}\,Ac^3x^{\frac{23}{2}} + \frac{6}{19}\,Bb^2cx^{\frac{19}{2}} + \frac{6}{19}\,Abc^2x^{\frac{19}{2}} + \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{2}{5}\,Ab^2cx^{\frac{15}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 6/23*B*b*c^2*x^(23/2) + 2/23*A*c^3*x^(23/2) + 6/19*B*
b^2*c*x^(19/2) + 6/19*A*b*c^2*x^(19/2) + 2/15*B*b^3*x^(15/2) + 2/5*A*b^2*c*
x^(15/2) + 2/11*A*b^3*x^(11/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(24035B\,c^3x^8 + 28215A\,c^3x^6 + 84645Bb\,c^2x^6 + 102465Ab\,c^2x^4 + 102465B\,b^2c\,x^4 + 129789A\,b^2c\,x^2 + 4326\right.}{648945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x)
```

```
[Out] 2/648945*x^(11/2)*(24035*B*c^3*x^8+28215*A*c^3*x^6+84645*B*b*c^2*x^6+102465
*A*b*c^2*x^4+102465*B*b^2*c*x^4+129789*A*b^2*c*x^2+43263*B*b^3*x^2+58995*A*
b^3)
```

maxima [A] time = 1.32, size = 73, normalized size = 0.86

$$\frac{2}{27}\,Bc^3x^{\frac{27}{2}} + \frac{2}{23}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{23}{2}} + \frac{6}{19}\left(Bb^2c + Abc^2\right)x^{\frac{19}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}} + \frac{2}{15}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 2/23*(3*B*b*c^2 + A*c^3)*x^(23/2) + 6/19*(B*b^2*c + A
*b*c^2)*x^(19/2) + 2/11*A*b^3*x^(11/2) + 2/15*(B*b^3 + 3*A*b^2*c)*x^(15/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,b^3}{15} + \frac{2\,A\,c\,b^2}{5}\right) + x^{23/2}\left(\frac{2\,A\,c^3}{23} + \frac{6\,B\,b\,c^2}{23}\right) + \frac{2\,A\,b^3\,x^{11/2}}{11} + \frac{2\,B\,c^3\,x^{27/2}}{27} + \frac{6\,b\,c\,x^{19/2}\,\left(A\,c + B\,b\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x)
```

```
[Out] x^(15/2)*((2*B*b^3)/15 + (2*A*b^2*c)/5) + x^(23/2)*((2*A*c^3)/23 + (6*B*b*c
^2)/23) + (2*A*b^3*x^(11/2))/11 + (2*B*c^3*x^(27/2))/27 + (6*b*c*x^(19/2)*(
A*c + B*b))/19
```

sympy [**A**] time = 31.77, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{2Ab^2cx^{\frac{15}{2}}}{5}+\frac{6Abc^2x^{\frac{19}{2}}}{19}+\frac{2Ac^3x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{15}{2}}}{15}+\frac{6Bb^2cx^{\frac{19}{2}}}{19}+\frac{6Bbc^2x^{\frac{23}{2}}}{23}+\frac{2Bc^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*A*b**3*x**(11/2)/11 + 2*A*b**2*c*x**(15/2)/5 + 6*A*b*c**2*x**(19/2)/19 +
2*A*c**3*x**(23/2)/23 + 2*B*b**3*x**(15/2)/15 + 6*B*b**2*c*x**(19/2)/19 + 6
*B*b*c**2*x**(23/2)/23 + 2*B*c**3*x**(27/2)/27
```

3.181 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} \, dx$

Optimal. Leaf size=85

$$\frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2x^{13/2}(3Ac + bB) + \frac{2}{21}c^2x^{21/2}(Ac + 3bB) + \frac{6}{17}bcx^{17/2}(Ac + bB) + \frac{2}{25}Bc^3x^{25/2}$$

```
[Out] 2/9*A*b^3*x^(9/2)+2/13*b^2*(3*A*c+B*b)*x^(13/2)+6/17*b*c*(A*c+B*b)*x^(17/2)
+2/21*c^2*(A*c+3*B*b)*x^(21/2)+2/25*B*c^3*x^(25/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{13}b^2x^{13/2}(3Ac + bB) + \frac{2}{9}Ab^3x^{9/2} + \frac{2}{21}c^2x^{21/2}(Ac + 3bB) + \frac{6}{17}bcx^{17/2}(Ac + bB) + \frac{2}{25}Bc^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

```
[Out] (2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*c
)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{5/2}} \, dx &= \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3 \, dx \\
&= \int \left(Ab^3x^{7/2} + b^2(bB+3Ac)x^{11/2} + 3bc(bB+Ac)x^{15/2} + c^2(3bB+Ac)x^{19/2} + \right. \\
&= \frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2(bB+3Ac)x^{13/2} + \frac{6}{17}bc(bB+Ac)x^{17/2} + \frac{2}{21}c^2(3bB+Ac)x^{21}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2x^{13/2}(3Ac + bB) + \frac{2}{21}c^2x^{21/2}(Ac + 3bB) + \frac{6}{17}bcx^{17/2}(Ac + bB) + \frac{2}{25}Bc^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

```
[Out] (2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*c
)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/25
```

fricas [A] time = 0.77, size = 78, normalized size = 0.92

$$\frac{2}{348075}\left(13923\,Bc^3x^{12} + 16575\left(3\,Bbc^2 + Ac^3\right)x^{10} + 61425\left(Bb^2c + Abc^2\right)x^8 + 38675\,Ab^3x^4 + 26775\left(Bb^3 + 3\,Al\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*c^3*x^12 + 16575*(3*B*b*c^2 + A*c^3)*x^10 + 61425*(B*b^2*
c + A*b*c^2)*x^8 + 38675*A*b^3*x^4 + 26775*(B*b^3 + 3*A*b^2*c)*x^6)*sqrt(x)
```

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{25}\,Bc^3x^{\frac{25}{2}} + \frac{2}{7}\,Bbc^2x^{\frac{21}{2}} + \frac{2}{21}\,Ac^3x^{\frac{21}{2}} + \frac{6}{17}\,Bb^2cx^{\frac{17}{2}} + \frac{6}{17}\,Abc^2x^{\frac{17}{2}} + \frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{6}{13}\,Ab^2cx^{\frac{13}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/7*B*b*c^2*x^(21/2) + 2/21*A*c^3*x^(21/2) + 6/17*B*b
^2*c*x^(17/2) + 6/17*A*b*c^2*x^(17/2) + 2/13*B*b^3*x^(13/2) + 6/13*A*b^2*c*
x^(13/2) + 2/9*A*b^3*x^(9/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(13923B\,c^3x^8 + 16575A\,c^3x^6 + 49725Bb\,c^2x^6 + 61425Ab\,c^2x^4 + 61425B\,b^2c\,x^4 + 80325A\,b^2c\,x^2 + 26775B\,b^3x^2\right.}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x)
```

```
[Out] 2/348075*x^(9/2)*(13923*B*c^3*x^8+16575*A*c^3*x^6+49725*B*b*c^2*x^6+61425*A
*b*c^2*x^4+61425*B*b^2*c*x^4+80325*A*b^2*c*x^2+26775*B*b^3*x^2+38675*A*b^3)
```

maxima [A] time = 1.40, size = 73, normalized size = 0.86

$$\frac{2}{25}\,Bc^3x^{\frac{25}{2}} + \frac{2}{21}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{21}{2}} + \frac{6}{17}\left(Bb^2c + Abc^2\right)x^{\frac{17}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{2}{13}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/21*(3*B*b*c^2 + A*c^3)*x^(21/2) + 6/17*(B*b^2*c + A
*b*c^2)*x^(17/2) + 2/9*A*b^3*x^(9/2) + 2/13*(B*b^3 + 3*A*b^2*c)*x^(13/2)
```

mupad [B] time = 0.04, size = 69, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,b^3}{13} + \frac{6\,A\,c\,b^2}{13}\right) + x^{21/2}\left(\frac{2\,A\,c^3}{21} + \frac{2\,B\,b\,c^2}{7}\right) + \frac{2\,A\,b^3\,x^{9/2}}{9} + \frac{2\,B\,c^3\,x^{25/2}}{25} + \frac{6\,b\,c\,x^{17/2}\,\left(A\,c + B\,b\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x)
```

```
[Out] x^(13/2)*((2*B*b^3)/13 + (6*A*b^2*c)/13) + x^(21/2)*((2*A*c^3)/21 + (2*B*b*
c^2)/7) + (2*A*b^3*x^(9/2))/9 + (2*B*c^3*x^(25/2))/25 + (6*b*c*x^(17/2)*(A*
c + B*b))/17
```

sympy [A] time = 35.91, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{9}{2}}}{9}+\frac{6Ab^2cx^{\frac{13}{2}}}{13}+\frac{6Abc^2x^{\frac{17}{2}}}{17}+\frac{2Ac^3x^{\frac{21}{2}}}{21}+\frac{2Bb^3x^{\frac{13}{2}}}{13}+\frac{6Bb^2cx^{\frac{17}{2}}}{17}+\frac{2Bbc^2x^{\frac{21}{2}}}{7}+\frac{2Bc^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*A*b**3*x**(9/2)/9 + 6*A*b**2*c*x**(13/2)/13 + 6*A*b*c**2*x**(17/2)/17 + 2
*A*c**3*x**(21/2)/21 + 2*B*b**3*x**(13/2)/13 + 6*B*b**2*c*x**(17/2)/17 + 2*
B*b*c**2*x**(21/2)/7 + 2*B*c**3*x**(25/2)/25
```

3.182 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}} dx$

Optimal. Leaf size=85

$$\frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

```
[Out] 2/7*A*b^3*x^(7/2)+2/11*b^2*(3*A*c+B*b)*x^(11/2)+2/5*b*c*(A*c+B*b)*x^(15/2)+
2/19*c^2*(A*c+3*B*b)*x^(19/2)+2/23*B*c^3*x^(23/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 448}

$$\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{7}Ab^3x^{7/2}+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

```
[Out] (2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*c
)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx &= \int x^{5/2}(A+Bx^2)(b+cx^2)^3\,dx\\ &= \int \left(Ab^3x^{5/2}+b^2(bB+3Ac)x^{9/2}+3bc(bB+Ac)x^{13/2}+c^2(3bB+Ac)x^{17/2}+Bc^3x\right.\\ &= \frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2(bB+3Ac)x^{11/2}+\frac{2}{5}bc(bB+Ac)x^{15/2}+\frac{2}{19}c^2(3bB+Ac)x^{19/2}+\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.00

$$\frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

```
[Out] (2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*c
)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```

fricas [A] time = 0.91, size = 78, normalized size = 0.92

$$\frac{2}{168245}\left(7315\,Bc^3x^{11}+8855\left(3\,Bbc^2+Ac^3\right)x^9+33649\left(Bb^2c+Abc^2\right)x^7+24035\,Ab^3x^3+15295\left(Bb^3+3\,Ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*c^3*x^11 + 8855*(3*B*b*c^2 + A*c^3)*x^9 + 33649*(B*b^2*c +
 A*b*c^2)*x^7 + 24035*A*b^3*x^3 + 15295*(B*b^3 + 3*A*b^2*c)*x^5)*sqrt(x)
```

giac [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{23}\,Bc^3x^{\frac{23}{2}}+\frac{6}{19}\,Bbc^2x^{\frac{19}{2}}+\frac{2}{19}\,Ac^3x^{\frac{19}{2}}+\frac{2}{5}\,Bb^2cx^{\frac{15}{2}}+\frac{2}{5}\,Abc^2x^{\frac{15}{2}}+\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{6}{11}\,Ab^2cx^{\frac{11}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 6/19*B*b*c^2*x^(19/2) + 2/19*A*c^3*x^(19/2) + 2/5*B*b
^2*c*x^(15/2) + 2/5*A*b*c^2*x^(15/2) + 2/11*B*b^3*x^(11/2) + 6/11*A*b^2*c*x
^(11/2) + 2/7*A*b^3*x^(7/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(7315B\,c^3x^8+8855A\,c^3x^6+26565Bb\,c^2x^6+33649Ab\,c^2x^4+33649B\,b^2c\,x^4+45885A\,b^2c\,x^2+15295B\,b^3x^2\right.}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x)
```

```
[Out] 2/168245*x^(7/2)*(7315*B*c^3*x^8+8855*A*c^3*x^6+26565*B*b*c^2*x^6+33649*A*b
*c^2*x^4+33649*B*b^2*c*x^4+45885*A*b^2*c*x^2+15295*B*b^3*x^2+24035*A*b^3)
```

maxima [A] time = 1.34, size = 73, normalized size = 0.86

$$\frac{2}{23}\,Bc^3x^{\frac{23}{2}}+\frac{2}{19}\left(3\,Bbc^2+Ac^3\right)x^{\frac{19}{2}}+\frac{2}{5}\left(Bb^2c+Abc^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}+\frac{2}{11}\left(Bb^3+3\,Ab^2c\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 2/19*(3*B*b*c^2 + A*c^3)*x^(19/2) + 2/5*(B*b^2*c + A*
b*c^2)*x^(15/2) + 2/7*A*b^3*x^(7/2) + 2/11*(B*b^3 + 3*A*b^2*c)*x^(11/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,b^3}{11}+\frac{6\,A\,c\,b^2}{11}\right)+x^{19/2}\left(\frac{2\,A\,c^3}{19}+\frac{6\,B\,b\,c^2}{19}\right)+\frac{2\,A\,b^3\,x^{7/2}}{7}+\frac{2\,B\,c^3\,x^{23/2}}{23}+\frac{2\,b\,c\,x^{15/2}\,\left(A\,c+B\,b\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x)
```

```
[Out] x^(11/2)*((2*B*b^3)/11 + (6*A*b^2*c)/11) + x^(19/2)*((2*A*c^3)/19 + (6*B*b*
c^2)/19) + (2*A*b^3*x^(7/2))/7 + (2*B*c^3*x^(23/2))/23 + (2*b*c*x^(15/2)*(A
*c + B*b))/5
```

sympy [A] time = 53.84, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{7}{2}}}{7} + \frac{6Ab^2cx^{\frac{11}{2}}}{11} + \frac{2Abc^2x^{\frac{15}{2}}}{5} + \frac{2Ac^3x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{11}{2}}}{11} + \frac{2Bb^2cx^{\frac{15}{2}}}{5} + \frac{6Bbc^2x^{\frac{19}{2}}}{19} + \frac{2Bc^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*A*b**3*x**(7/2)/7 + 6*A*b**2*c*x**(11/2)/11 + 2*A*b*c**2*x**(15/2)/5 + 2*
A*c**3*x**(19/2)/19 + 2*B*b**3*x**(11/2)/11 + 2*B*b**2*c*x**(15/2)/5 + 6*B*
b*c**2*x**(19/2)/19 + 2*B*c**3*x**(23/2)/23
```

3.183 $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=278

$$-\frac{b^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB}{}$$

```
[Out] 2/3*b*(-A*c+B*b)*x^(3/2)/c^3-2/7*(-A*c+B*b)*x^(7/2)/c^2+2/11*B*x^(11/2)/c+1
/2*b^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^
(1/2)-1/2*b^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(1
5/4)*2^(1/2)-1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/c^(15/4)*2^(1/2)+1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*(b*B - A*c)*x^(3/2))/(3*c^3) - (2*(b*B - A*c)*x^(7/2))/(7*c^2) + (2*B*
x^(11/2))/(11*c) + (b^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*Log[Sqr
t[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(15/4)) +
 (b^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*c^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{x^{9/2}(A+Bx^2)}{b+cx^2}\,dx$$
$$= \frac{2Bx^{11/2}}{11c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{11c}$$
$$= -\frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{(b(bB-Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{c^2}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{\left(b^2(bB-Ac)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^3}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{\left(2b^2(bB-Ac)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \cdots\right.}{c^3}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{\left(b^2(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, \cdots\right.}{c^{7/2}}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{\left(b^2(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2} \cdots\right.}{2c^4}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{\cdots}\right.}{2\sqrt{2}c^{15/4}}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{b^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} - \cdots$$

Mathematica [A] time = 0.26, size = 133, normalized size = 0.48

$$\frac{2x^{3/2}\left(-11bc\left(7A+3Bx^2\right)+3c^2x^2\left(11A+7Bx^2\right)+77b^2B\right)}{231c^3}+\frac{b(-b)^{3/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{15/4}}+\frac{(-b)^{7/4}(bB-A\cdots}{c\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*b^2*B - 11*b*c*(7*A + 3*B*x^2) + 3*c^2*x^2*(11*A + 7*B*x^2))
)/(231*c^3) + ((-b)^(3/4)*b*(b*B - A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)
])/c^(15/4) + ((-b)^(7/4)*(b*B - A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)]
)/c^(15/4)
```

fricas [B] time = 0.91, size = 920, normalized size = 3.31

$$-\frac{924\,c^3\left(-\frac{B^4b^{11}-4\,AB^3b^{10}c+6\,A^2B^2b^9c^2-4\,A^3Bb^8c^3+A^4b^7c^4}{c^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^{16}-6\,AB^5b^{15}c+15\,A^2B^4b^{14}c^2-20\,A^3B^3b^{13}c^3+15\,A^4B^2b^{12}c\cdots\right.}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/462*(924*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*
b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*arctan((sqrt((B^6*b^16 - 6*A*B^5*b^15*c
```

```
+ 15*A^2*B^4*b^14*c^2 - 20*A^3*B^3*b^13*c^3 + 15*A^4*B^2*b^12*c^4 - 6*A^5*B
*b^11*c^5 + A^6*b^10*c^6)*x - (B^4*b^11*c^7 - 4*A*B^3*b^10*c^8 + 6*A^2*B^2*
b^9*c^9 - 4*A^3*B*b^8*c^10 + A^4*b^7*c^11)*sqrt(-(B^4*b^11 - 4*A*B^3*b^10*c
 + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15))*c^4*(-(B^4*b^1
1 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^1
5)^(1/4) + (B^3*b^8*c^4 - 3*A*B^2*b^7*c^5 + 3*A^2*B*b^6*c^6 - A^3*b^5*c^7)*
sqrt(x)*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3
+ A^4*b^7*c^4)/c^15)^(1/4))/(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2
- 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)) - 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c +
6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(c^11*(-(
B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c
^4)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)
*sqrt(x)) + 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^
3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(-c^11*(-(B^4*b^11 - 4*A*B^3*b^10
*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*
b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) - 4*(21*B*c^2
*x^5 - 33*(B*b*c - A*c^2)*x^3 + 77*(B*b^2 - A*b*c)*x)*sqrt(x))/c^3
```

giac [**A**] time = 0.24, size = 298, normalized size = 1.07

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{3}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{6}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{3}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{2c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 - 1/2*sqrt(2)*((b*c^3)^
(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)
^(3/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 - 1/4*sq
rt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^6 + 2/231*(21*B*c^10*x^(11/2) - 33*B*b*c^9*x^(7/
2) + 33*A*c^10*x^(7/2) + 77*B*b^2*c^8*x^(3/2) - 77*A*b*c^9*x^(3/2))/c^11
```

maple [**A**] time = 0.08, size = 336, normalized size = 1.21

$$\frac{2Bx^{\frac{11}{2}}}{11c}+\frac{2Ax^{\frac{7}{2}}}{7c}-\frac{2Bbx^{\frac{7}{2}}}{7c^{2}}-\frac{2Abx^{\frac{3}{2}}}{3c^{2}}+\frac{2Bb^{2}x^{\frac{3}{2}}}{3c^{3}}+\frac{\sqrt{2}\,Ab^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{\sqrt{2}\,Ab^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{\sqrt{2}\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/11*B*x^(11/2)/c+2/7/c*x^(7/2)*A-2/7/c^2*x^(7/2)*b*B-2/3/c^2*x^(3/2)*A*b+2
/3/c^3*x^(3/2)*B*b^2+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2
*b^2/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*b^
2/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4*b^3/c
^4/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+
(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*b^3/c^4/(b/c)^(1/4)*2^(1/2)*B
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b^3/c^4/(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.07, size = 237, normalized size = 0.85

$$-\frac{\left(Bb^3-Ab^2c\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\ldots\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(B*b^3 - A*b^2*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(
c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log
(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) +
 sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/
4)*c^(3/4)))/c^3 + 2/231*(21*B*c^2*x^(11/2) - 33*(B*b*c - A*c^2)*x^(7/2) +
77*(B*b^2 - A*b*c)*x^(3/2))/c^3
```

mupad [B] time = 0.23, size = 115, normalized size = 0.41

$$x^{7/2}\left(\frac{2A}{7c}-\frac{2Bb}{7c^2}\right)+\frac{2Bx^{11/2}}{11c}+\frac{(-b)^{7/4}\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac-Bb)}{c^{15/4}}-\frac{bx^{3/2}\left(\frac{2A}{c}-\frac{2Bb}{c^2}\right)}{3c}+\frac{(-b)^{7/4}\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)(\ldots}{c^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(7/2)*((2*A)/(7*c) - (2*B*b)/(7*c^2)) + (2*B*x^(11/2))/(11*c) + ((-b)^(7/
4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(15/4) + ((-b)^(7/4)*a
tan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(15/4) - (b*x^(3/2)*
((2*A)/c - (2*B*b)/c^2))/(3*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.184 $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=276

$$\frac{b^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}-\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}+\frac{b^{5/4}(bB-Ac}{}$$

```
[Out] -2/5*(-A*c+B*b)*x^(5/2)/c^2+2/9*B*x^(9/2)/c+1/2*b^(5/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-1/2*b^(5/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+1/4*b^(5/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2
)-1/4*b^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/c^(13/4)*2^(1/2)+2*b*(-A*c+B*b)*x^(1/2)/c^3
```

Rubi [A] time = 0.25, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}-\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}+\frac{b^{5/4}(bB-Ac}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*(b*B - A*c)*Sqrt[x])/c^3 - (2*(b*B - A*c)*x^(5/2))/(5*c^2) + (2*B*x^(9
/2))/(9*c) + (b^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(13/4)) + (b^(5/4)*(b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(13/4)) - (b^(5
/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(2*Sqrt[2]*c^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx=\int \frac{x^{7/2}\left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$=\frac{2Bx^{9/2}}{9c}-\frac{\left(2\left(\frac{9bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{9c}$$
$$=-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}+\frac{(b(bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{c^2}$$
$$=\frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}-\frac{\left(b^2(bB-Ac)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c^3}$$
$$=\frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}-\frac{\left(2b^2(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^3}$$
$$=\frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}-\frac{\left(b^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\right.}{c^3}$$
$$=\frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}-\frac{\left(b^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,d\right.}{2c^{7/2}}$$
$$=\frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}+\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{2\sqrt{2}c^{13/4}}$$
$$=\frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}+\frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}}-\frac{b^{5/4}}{}$$

Mathematica [A] time = 0.31, size = 227, normalized size = 0.82

$$\frac{\frac{45\sqrt{2}b^{5/4}(bB-Ac)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)\right)}{\sqrt[4]{c}}+\frac{90\sqrt{2}b^{5/4}(bB-Ac)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+\right.\right.}{\sqrt[4]{c}}}{180c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (360*b*(b*B - A*c)*Sqrt[x] + 72*c*(-(b*B) + A*c)*x^(5/2) + 40*B*c^2*x^(9/2)
 + (90*Sqrt[2]*b^(5/4)*(b*B - A*c)*(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)] - ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]))/c^(1/4) + (45*Sqrt
[2]*b^(5/4)*(b*B - A*c)*(Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/c^(
1/4))/(180*c^3)
```

fricas [B] time = 0.89, size = 714, normalized size = 2.59

$$\frac{180c^3\left(-\frac{B^4b^9-4AB^3b^8c+6A^2B^2b^7c^2-4A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{B^4b^9-4AB^3b^8c+6A^2B^2b^7c^2-4A^3Bb^6c^3+A^4b^5c^4}{c^{13}}}+\left(B^2b^4-2ABb^3c+A\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*
c^3 + A^4*b^5*c^4)/c^13)^(1/4)*arctan((sqrt(c^6*sqrt(-(B^4*b^9 - 4*A*B^3*b^
8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13) + (B^2*b^4 -
 2*A*B*b^3*c + A^2*b^2*c^2)*x)*c^10*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*
b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(3/4) + (B*b^2*c^10 - A*b*c^
11)*sqrt(x)*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^
3 + A^4*b^5*c^4)/c^13)^(3/4))/(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2
- 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)) + 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A
^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(c^3*(-(B^4*
b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^
13)^(1/4) - (B*b^2 - A*b*c)*sqrt(x)) - 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c +
6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(-c^3*(-(
B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4
)/c^13)^(1/4) - (B*b^2 - A*b*c)*sqrt(x)) + 4*(5*B*c^2*x^4 + 45*B*b^2 - 45*A
*b*c - 9*(B*b*c - A*c^2)*x^2)*sqrt(x))/c^3
```

giac [A] time = 0.20, size = 298, normalized size = 1.08

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{4}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)\right.}{2\left(\frac{b}{c}\right)}\right.}{2c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*((b*c^3)^
(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)
^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 1/4*sq
rt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^4 + 2/45*(5*B*c^8*x^(9/2) - 9*B*b*c^7*x^(5/2) +
9*A*c^8*x^(5/2) + 45*B*b^2*c^6*sqrt(x) - 45*A*b*c^7*sqrt(x))/c^9
```

maple [A] time = 0.05, size = 330, normalized size = 1.20

$$\frac{2Bx^{\frac{9}{2}}}{9c}+\frac{2Ax^{\frac{5}{2}}}{5c}-\frac{2Bbx^{\frac{5}{2}}}{5c^{2}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2c^{2}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c^{2}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/9*B*x^(9/2)/c+2/5/c*A*x^(5/2)-2/5/c^2*B*x^(5/2)*b-2/c^2*A*b*x^(1/2)+2/c^3
*B*b^2*x^(1/2)+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)-1)+1/4*b/c^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)
+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*b/c^2*(b/c)^
(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b^2/c^3*(b/c)^(1/
4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4*b^2/c^3*(b/c)^(1/4)*
2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(
1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*b^2/c^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)
/(b/c)^(1/4)*x^(1/2)+1)
```

maxima [A] time = 3.01, size = 259, normalized size = 0.94

$$-\frac{\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) +
2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(
2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
 + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^3 + 2/45*(5*B*c^2*x^(9/2)
- 9*(B*b*c - A*c^2)*x^(5/2) + 45*(B*b^2 - A*b*c)*sqrt(x))/c^3
```

mupad [B] time = 0.25, size = 788, normalized size = 2.86

$$x^{5/2}\left(\frac{2A}{5c}-\frac{2Bb}{5c^2}\right)+\frac{2Bx^{9/2}}{9c}-\frac{(-b)^{5/4}\operatorname{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2b^4c^2-2ABb^5c+B^2b^6\right)}{c^3}-\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^4-32Ab^3c\right)1i}{2c^{13/4}}\right)(Ac-Bb)}{2c^{13/4}}+\frac{(-b)^{5/4}\left(\right.}{}}{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2b^4c^2-2ABb^5c+B^2b^6\right)}{c^3}-\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^4-32Ab^3c\right)1i}{2c^{13/4}}\right)(Ac-Bb)1i}{2c^{13/4}}-\frac{(-b)^{5/4}\left(\right.}{}}\right)}{c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(5/2)*((2*A)/(5*c) - (2*B*b)/(5*c^2)) + (2*B*x^(9/2))/(9*c) - ((-b)^(5/4)
*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3
- ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b
)*1i)/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*
B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4
)))*(A*c - B*b)*1i)/(2*c^(13/4)))/(((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*
b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*
c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) - ((-b)^(5/4)*((16*x^(1/2)*(B^2
*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4
- 32*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4))))*(A*c - B*b)*1i)/c^
(13/4) - ((-b)^(5/4)*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2
- 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(
2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6
+ A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*
A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)))/(((-b)^(5/4)*((16*x^(
1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(
32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b)*1i)/(2*c^(13/4)) - ((-
b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5
/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b)*1i)/(
2*c^(13/4))))*(A*c - B*b))/c^(13/4) - (b*x^(1/2)*((2*A)/c - (2*B*b)/c^2))/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.185 $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=257

$$\frac{b^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac}{}$$

```
[Out] -2/3*(-A*c+B*b)*x^(3/2)/c^2+2/7*B*x^(7/2)/c-1/2*b^(3/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/2*b^(3/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/4*b^(3/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2
)-1/4*b^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/c^(11/4)*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{3/4}(bB-Ac}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*(b*B - A*c)*x^(3/2))/(3*c^2) + (2*B*x^(7/2))/(7*c) - (b^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^
(11/4)) + (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{x^{5/2}\left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$= \frac{2Bx^{7/2}}{7c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{b+cx^2}\,dx}{7c}$$
$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(2b(bB-Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{(b(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(b(bB-Ac))\,\text{S}}{}$$
$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{(b(bB-}{}$$
$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}} - \frac{b^{3/4}(bB-}{}$$
$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}\,c^{11/4}}$$

Mathematica [A] time = 0.15, size = 110, normalized size = 0.43

$$\frac{2c^{3/4}x^{3/2}\left(7Ac-7bB+3Bcx^2\right)-21(-b)^{3/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+21(-b)^{3/4}(bB-Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{21c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*c^(3/4)*x^(3/2)*(-7*b*B + 7*A*c + 3*B*c*x^2) - 21*(-b)^(3/4)*(b*B - A*c)
*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + 21*(-b)^(3/4)*(b*B - A*c)*ArcTanh[(
c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(21*c^(11/4))
```

fricas [B] time = 1.02, size = 899, normalized size = 3.50

$$84\,c^2\left(-\frac{B^4b^7-4\,AB^3b^6c+6\,A^2B^2b^5c^2-4\,A^3Bb^4c^3+A^4b^3c^4}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^{10}-6\,AB^5b^9c+15\,A^2B^4b^8c^2-20\,A^3B^3b^7c^3+15\,A^4B^2b^6c^4-6\,A^5Bb^5c^5\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c
^3 + A^4*b^3*c^4)/c^11)^(1/4)*arctan((sqrt((B^6*b^10 - 6*A*B^5*b^9*c + 15*A
^2*B^4*b^8*c^2 - 20*A^3*B^3*b^7*c^3 + 15*A^4*B^2*b^6*c^4 - 6*A^5*B*b^5*c^5
+ A^6*b^4*c^6)*x - (B^4*b^7*c^5 - 4*A*B^3*b^6*c^6 + 6*A^2*B^2*b^5*c^7 - 4*A
^3*B*b^4*c^8 + A^4*b^3*c^9)*sqrt(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*
c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11))*c^3*(-(B^4*b^7 - 4*A*B^3*b^6*c
```

```
+ 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4) + (B^3*b^5
*c^3 - 3*A*B^2*b^4*c^4 + 3*A^2*B*b^3*c^5 - A^3*b^2*c^6)*sqrt(x)*(-(B^4*b^7
- 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^
(1/4))/(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4
*b^3*c^4)) - 21*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*
B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6
*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 -
3*A*B^2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) + 21*c^2*(-(B^4*b^7
 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)
^(1/4)*log(-c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^
4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^2*b^4*c + 3*A^2*B*b^3*c
^2 - A^3*b^2*c^3)*sqrt(x)) + 4*(3*B*c*x^3 - 7*(B*b - A*c)*x)*sqrt(x))/c^2
```

giac [A] time = 0.21, size = 264, normalized size = 1.03

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{5}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*((b*c^3)^(3/4)
*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^5 + 2/21*(3*B*c^6*x^(7/2) - 7*B*b*c^5*x^(3/2) + 7*A*c^6*x^(3/2))/
c^7
```

maple [A] time = 0.05, size = 308, normalized size = 1.20

$$\frac{2Bx^{\frac{7}{2}}}{7c}+\frac{2Ax^{\frac{3}{2}}}{3c}-\frac{2Bbx^{\frac{3}{2}}}{3c^{2}}-\frac{\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}-\frac{\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}-\frac{\sqrt{2}\,Ab\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/7*B*x^(7/2)/c+2/3/c*x^(3/2)*A-2/3/c^2*x^(3/2)*b*B-1/2*b/c^2/(b/c)^(1/4)*2
^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4*b/c^2/(b/c)^(1/4)*2^(1/2
)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x
^(1/2)+(b/c)^(1/2)))-1/2*b/c^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)+1)+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)-1)+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*b^2/c^3
/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

maxima [A] time = 3.13, size = 214, normalized size = 0.83

$$\frac{\left(Bb^2 - Abc\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\right.}{}\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b^2 - A*b*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
+ 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sq
rt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*
c^(3/4)))/c^2 + 2/21*(3*B*c*x^(7/2) - 7*(B*b - A*c)*x^(3/2))/c^2
```

mupad [B] time = 0.24, size = 92, normalized size = 0.36

$$x^{3/2}\left(\frac{2A}{3c} - \frac{2Bb}{3c^2}\right) + \frac{2Bx^{7/2}}{7c} + \frac{(-b)^{3/4}\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac - Bb)}{c^{11/4}} + \frac{(-b)^{3/4}\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)(Ac - Bb)\,1\mathrm{i}}{c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(3/2)*((2*A)/(3*c) - (2*B*b)/(3*c^2)) + (2*B*x^(7/2))/(7*c) + ((-b)^(3/4)
*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(11/4) + ((-b)^(3/4)*ata
n((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(11/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.186 $\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=255

$$-\frac{\sqrt[4]{b}\,(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{\sqrt[4]{b}\,(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}-\frac{\sqrt[4]{b}\,(bB\ldots}{}$$

```
[Out] 2/5*B*x^(5/2)/c-1/2*b^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(
1/4))/c^(9/4)*2^(1/2)+1/2*b^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/c^(9/4)*2^(1/2)-1/4*b^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(
1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)+1/4*b^(1/4)*(-A*c+B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)-2*(-A*c+B*b
)*x^(1/2)/c^2
```

Rubi [A] time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\sqrt{x}\,(bB-Ac)}{c^2}-\frac{\sqrt[4]{b}\,(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{\sqrt[4]{b}\,(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\ldots\right)}{2\sqrt{2}\,c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*(b*B - A*c)*Sqrt[x])/c^2 + (2*B*x^(5/2))/(5*c) - (b^(1/4)*(b*B - A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(1/4)*
(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)
) - (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx=\int \frac{x^{3/2}\left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$=\frac{2Bx^{5/2}}{5c}-\frac{\left(2\left(\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{5c}$$
$$=-\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}+\frac{(b(bB-Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c^2}$$
$$=-\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}+\frac{(2b(bB-Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^2}$$
$$=-\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}+\frac{\left(\sqrt{b}\,(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^2}+\frac{\left(\sqrt{b}\,(bB\right.}{}$$
$$=-\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}+\frac{\left(\sqrt{b}\,(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2c^{5/2}}+\frac{\left(\sqrt{b}\right.}{}$$
$$=-\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}-\frac{\sqrt[4]{b}\,(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{\sqrt[4]{b}\,(b}{}$$
$$=-\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}-\frac{\sqrt[4]{b}\,(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{\sqrt[4]{b}\,(bB-Ac)\tan^{-1}}{\sqrt{2}\,c^{9}}$$

Mathematica [A] time = 0.22, size = 208, normalized size = 0.82

$$\frac{-40\sqrt{x}\,(bB-Ac)+\frac{5\sqrt{2}\,\sqrt[4]{b}\,(Ac-bB)\left(\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)\right)}{\sqrt[4]{c}}-\frac{10\sqrt{2}\,\sqrt[4]{b}\,(bB-Ac)\left(\tan^{-1}\left(1-\right.\right.}{}}{20c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-40*(b*B - A*c)*Sqrt[x] + 8*B*c*x^(5/2) - (10*Sqrt[2]*b^(1/4)*(b*B - A*c)*
(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)]))/c^(1/4) + (5*Sqrt[2]*b^(1/4)*(-(b*B) + A*c)*(Log[Sqrt
[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/c^(1/4))/(20*c^2)
```

fricas [B] time = 1.01, size = 660, normalized size = 2.59

$$-\frac{20\,c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^4\sqrt{-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}}+\left(B^2b^2-2\,ABbc+\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*
c^3 + A^4*b*c^4)/c^9)^(1/4)*arctan((sqrt(c^4*sqrt(-(B^4*b^5 - 4*A*B^3*b^4*c
```

```
 + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9) + (B^2*b^2 - 2*A*B
*b*c + A^2*c^2)*x)*c^7*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A
^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(3/4) + (B*b*c^7 - A*c^8)*sqrt(x)*(-(B^4*b^5
 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(3
/4))/(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b
*c^4)) + 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2
*c^3 + A^4*b*c^4)/c^9)^(1/4)*log(c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2
*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) -
 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A
^4*b*c^4)/c^9)^(1/4)*log(-c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^
2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) - 4*(B*c
*x^2 - 5*B*b + 5*A*c)*sqrt(x))/c^2
```

giac [A] time = 0.20, size = 263, normalized size = 1.03

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{3}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*((b*c^3)^(1/4)
*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*((b*c^3
)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^3 + 2/5*(B*c^4*x^(5/2) - 5*B*b*c^3*sqrt(x) + 5*A*c^4*sqrt(x))/c^5
```

maple [A] time = 0.06, size = 299, normalized size = 1.17

$$\frac{2Bx^{\frac{5}{2}}}{5c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4c}+\frac{\left(\frac{b}{c}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/5*B*x^(5/2)/c+2/c*A*x^(1/2)-2/c^2*b*B*x^(1/2)-1/2/c*(b/c)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c*(b/c)^(1/4)*2^(1/2)*A*arctan(2
^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4/c*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)
*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+
1/2/c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*b+1/2/c
^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*b+1/4/c^2*(b
/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)
^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))*b
```

maxima [A] time = 2.99, size = 235, normalized size = 0.92

$$\frac{\left(\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\cdots\right)}{4c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2
*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^2 + 2/5*(B*c*x^(5/2) - 5*(B*b
 - A*c)*sqrt(x))/c^2
```

mupad [B] time = 0.27, size = 789, normalized size = 3.09

$$\sqrt{x}\,\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)+\frac{2\,B\,x^{5/2}}{5\,c}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)(A\,c-B\,b)}{2\,c^{9/4}}\right)1\mathrm{i}}{2\,c^{9/4}}+\frac{(-b)\cdots}{\cdots}}{\frac{(-b)^{1/4}\,(A\,c-B\,b)\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)(A\,c-B\,b)}{2\,c^{9/4}}\right)}{2\,c^{9/4}}-\frac{(-b)\cdots}{\cdots}}\right)\cdots}{c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(1/2)*((2*A)/c - (2*B*b)/c^2) + (2*B*x^(5/2))/(5*c) - ((-b)^(1/4)*atan(((
(-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/
c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c^(9/4)))*1i)/(
2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 -
2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c
^(9/4)))*1i)/(2*c^(9/4)))/(((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 +
A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*
c - B*b))/(2*c^(9/4))))/(2*c^(9/4)) - ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*
(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B
*b^3*c)*(A*c - B*b))/(2*c^(9/4))))/(2*c^(9/4))))*(A*c - B*b)*1i)/c^(9/4) -
((-b)^(1/4)*atan((((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c
^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*
1i)/(2*c^(9/4))))/(2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b
^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c
)*(A*c - B*b)*1i)/(2*c^(9/4))))/(2*c^(9/4)))/(((-b)^(1/4)*(A*c - B*b)*((16*
x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^
2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4)))*1i)/(2*c^(9/4)) - ((-b)^(1/4)*
(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(
1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4)))*1i)/(2*c^(9/4
))))*(A*c - B*b))/c^(9/4)
```

sympy [A] time = 164.46, size = 393, normalized size = 1.54

$$\begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{c} \\ \frac{\sqrt[4]{-1}\,A\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}\,\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c}-\frac{\sqrt[4]{-1}\,A\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}\,\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c}+\frac{\sqrt[4]{-1}\,A\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{c}+\frac{2A\sqrt{x}}{c}-\frac{\sqrt[4]{-1}\,B\cdots}{\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(b, 0) & Eq(c, 0)), ((2*A*
x**(5/2)/5 + 2*B*x**(9/2)/9)/b, Eq(c, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/
c, Eq(b, 0)), ((-1)**(1/4)*A*b**(1/4)*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/4
)*(1/c)**(1/4) + sqrt(x))/(2*c) - (-1)**(1/4)*A*b**(1/4)*(1/c)**(1/4)*log((
-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c) + (-1)**(1/4)*A*b**(1/4)*
(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/c + 2*A*sqrt
(x)/c - (-1)**(1/4)*B*b**(5/4)*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)
**(1/4) + sqrt(x))/(2*c**2) + (-1)**(1/4)*B*b**(5/4)*(1/c)**(1/4)*log((-1)*
*(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c**2) - (-1)**(1/4)*B*b**(5/4)*(
1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/c**2 - 2*B*b*
sqrt(x)/c**2 + 2*B*x**(5/2)/(5*c), True))
```

3.187 $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=237

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\tan^{-}}{\sqrt{2}\sqrt[4]{}}$$

```
[Out] 2/3*B*x^(3/2)/c+1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^
(1/4)/c^(7/4)*2^(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/b^(1/4)/c^(7/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(
1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 459, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{(bB-Ac)\tan^{-}}{\sqrt{2}\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(1/4)*c^(7/4)) +
 ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
2*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{\sqrt{x}\left(A+Bx^2\right)}{b+cx^2}\,dx$$
$$= \frac{2Bx^{3/2}}{3c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{3c}$$
$$= \frac{2Bx^{3/2}}{3c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{3c}$$
$$= \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x\right.}{c^{3/2}}$$
$$= \frac{2Bx^{3/2}}{3c} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}\right.}{2c^2}$$
$$= \frac{2Bx^{3/2}}{3c} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{7/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\right.}{2\sqrt{2}\sqrt[4]{b}\,c^{7/4}}$$
$$= \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{7/4}} - \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{7/4}} - \frac{(bB-A}{}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 0.40

$$\frac{(3Ac-3bB)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+3(bB-Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+2\sqrt[4]{-b}\,Bc^{3/4}x^{3/2}}{3\sqrt[4]{-b}\,c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*(-b)^(1/4)*B*c^(3/4)*x^(3/2) + (-3*b*B + 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])
/(-b)^(1/4)] + 3*(b*B - A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(3*(-b)
^(1/4)*c^(7/4))
```

fricas [B] time = 1.13, size = 834, normalized size = 3.52

$$\frac{4\,Bx^{\frac{3}{2}} - 12\,c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{bc^7}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^6-6\,AB^5b^5c+15\,A^2B^4b^4c^2-20\,A^3B^3b^3c^3+15\,A^4B^2b^2c^4-6\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) - 12*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*arctan((sqrt((B^6*b^6 - 6*A*B^5*b^5*c
 + 15*A^2*B^4*b^4*c^2 - 20*A^3*B^3*b^3*c^3 + 15*A^4*B^2*b^2*c^4 - 6*A^5*B*b
*c^5 + A^6*c^6)*x - (B^4*b^5*c^3 - 4*A*B^3*b^4*c^4 + 6*A^2*B^2*b^3*c^5 - 4*
A^3*B*b^2*c^6 + A^4*b*c^7)*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c
^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7)))*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*
A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4) + (B^3*b^3*c^2 -
3*A*B^2*b^2*c^3 + 3*A^2*B*b*c^4 - A^3*c^5)*sqrt(x)*(-(B^4*b^4 - 4*A*B^3*b^3
```

```
*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4))/(B^4*b^4
 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)) + 3*c*(-(B^
4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7
))^(1/4)*log(b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B
*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2
 - A^3*c^3)*sqrt(x)) - 3*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(-b*c^5*(-(B^4*b^4 - 4*A*B^3*b^
3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^
3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)))/c
```

giac [A] time = 0.23, size = 251, normalized size = 1.06

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{4}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)}{2\,bc^{4}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c - 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 1/2*
sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 1/4*sqrt(2)*((b*c^3)^(3/4
)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b*c^4) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4)
```

maple [A] time = 0.05, size = 280, normalized size = 1.18

$$\frac{2B\,x^{\frac{3}{2}}}{3c}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,Bb\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{b}{c}\right)\cdots}\cdots\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2/3*B*x^(3/2)/c+1/2/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)+1)+1/2/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/
4/c/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/c^2/(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*b-1/2/c^2/(b/c)^(1/4)*2^(1/2)*B*arctan(
2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*b-1/4/c^2/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^
(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/
2)))*b
```

maxima [A] time = 3.02, size = 194, normalized size = 0.82

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c}-\frac{(Bb-Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\cdots}{\cdots}\cdots\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/c - 1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^
(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqr
t(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) -
sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)
*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt
(b))/(b^(1/4)*c^(3/4)))/c
```

mupad [B] time = 0.15, size = 71, normalized size = 0.30

$$\frac{2\,B\,x^{3/2}}{3\,c}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(A\,c-B\,b\right)}{{(-b)}^{1/4}\,c^{7/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(A\,c-B\,b\right)}{{(-b)}^{1/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (2*B*x^(3/2))/(3*c) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b
)^(1/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^
(1/4)*c^(7/4))
```

sympy [A] time = 65.60, size = 459, normalized size = 1.94

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{c} \\ -\frac{(-1)^{\frac{3}{4}}A\left(\frac{1}{c}\right)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2\sqrt[4]{b}}+\frac{(-1)^{\frac{3}{4}}A\left(\frac{1}{c}\right)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2\sqrt[4]{b}}-\frac{(-1)^{\frac{3}{4}}A\left(\frac{1}{c}\right)^{\frac{3}{4}}\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{\sqrt[4]{b}}+\frac{2(-1)^{\frac{3}{4}}A\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\cdots}{\sqrt[4]{b}\cdots}\right)}{\sqrt[4]{b}\,c\sqrt[4]{\frac{1}{c}}}\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(3/2)/3), Eq(b, 0) & Eq(c, 0)), ((2*A
*x**(3/2)/3 + 2*B*x**(7/2)/7)/b, Eq(c, 0)), ((-2*A/sqrt(x) + 2*B*x**(3/2)/3
)/c, Eq(b, 0)), (-(-1)**(3/4)*A*(1/c)**(3/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c
)**(1/4) + sqrt(x))/(2*b**(1/4)) + (-1)**(3/4)*A*(1/c)**(3/4)*log((-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(1/4)) - (-1)**(3/4)*A*(1/c)**(3/
4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(1/4) + 2*(-1)**(3/
4)*A*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(1/4)*c*(1/c)**(
1/4)) + (-1)**(3/4)*B*b**(3/4)*(1/c)**(3/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)
**(1/4) + sqrt(x))/(2*c) - (-1)**(3/4)*B*b**(3/4)*(1/c)**(3/4)*log((-1)**(1
/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c) + (-1)**(3/4)*B*b**(3/4)*(1/c)**
(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/c - 2*(-1)**(3/4)*B
*b**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(c**2*(1/c)**(1
/4)) + 2*B*x**(3/2)/(3*c), True))
```

3.188 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=235

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}\,b^{3/4}c^{5/}}$$

```
[Out] 1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2^
(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5
/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(
1/2))/b^(3/4)/c^(5/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2^(1/2)+2*B*x^(1/2)/c
```

Rubi [A] time = 0.18, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 459, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}\,b^{3/4}c^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(Sqrt[2]*b^(3/4)*c^(5/4)) + ((b*B - A*c)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(3/4)*c^(5/4)) - ((b
*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sq
rt[2]*b^(3/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\left(b+cx^2\right)}\,dx$$
$$= \frac{2B\sqrt{x}}{c} - \frac{\left(2\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c}$$
$$= \frac{2B\sqrt{x}}{c} - \frac{\left(4\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2B\sqrt{x}}{c} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,c} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{\right.}}{\sqrt{b}\,c}$$
$$= \frac{2B\sqrt{x}}{c} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\,c^{3/2}} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x\right.}}{2\sqrt{b}\,c^{3/2}}$$
$$= \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{2\sqrt{2}\,b^{3/4}c^{5/4}}$$
$$= \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}} - \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}} + \frac{(bB-Ac)\,\mathrm{l}}{}$$

Mathematica [**A**] time = 0.15, size = 166, normalized size = 0.71

$$\frac{(bB-Ac)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\,\mathrm{t}\right.}{2\sqrt{2}\,b^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*(2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)] - 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x]))/(2*Sqrt[2]*b^(3/4)*c^(5/4))
```

fricas [**B**] time = 0.72, size = 645, normalized size = 2.74

$$\frac{4\,c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2c^2\sqrt{-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}}+\left(B^2b^2-2\,ABbc+A^2c^2\right)x\,b^2c^4}}{B^4b^4-4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A
^4*c^4)/(b^3*c^5))^(1/4)*arctan((sqrt(b^2*c^2*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*
c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5)) + (B^2*b^2 - 2*
A*B*b*c + A^2*c^2)*x)*b^2*c^4*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^
2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(3/4) + (B*b^3*c^4 - A*b^2*c^5)*sqr
t(x)*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c
```

```
^4)/(b^3*c^5))^(3/4))/(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)) + c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4)*log(b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c
+ 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*
c)*sqrt(x)) - c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*
c^3 + A^4*c^4)/(b^3*c^5))^(1/4)*log(-b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2
*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt
(x)) + 4*B*sqrt(x))/c
```

giac [A] time = 0.18, size = 251, normalized size = 1.07

$$\frac{2\,B\sqrt{x}}{c}-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^2}-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{2}\right.}{2\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c - 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(
1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/2*sq
rt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/4*sqrt(2)*((b*c^3)^(1/4)*
B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b*c^2) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
```

maple [A] time = 0.06, size = 277, normalized size = 1.18

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] 2*B*x^(1/2)/c+1/2*(b/c)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)+1)+1/2*(b/c)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/4*
(b/c)^(1/4)/b*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(
b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/c*(b/c)^(1/4)*2^(1/2)*B*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)-1)-1/4/c*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2
)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))
```

maxima [A] time = 3.16, size = 218, normalized size = 0.93

$$\frac{2\,B\sqrt{x}}{c}-\frac{\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/c - 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1
/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(
b)*sqrt(c))) + 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sq
rt(c))) + sqrt(2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*
c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c
```

mupad [B] time = 0.29, size = 739, normalized size = 3.14

$$\frac{2\,B\,\sqrt{x}}{c}-\frac{\mathrm{atan}\left(\frac{\frac{\left(A\,c-B\,b\right)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)\left(A\,c-B\,b\right)}{2\,{\left(-b\right)}^{3/4}\,c^{5/4}}\right)1\mathrm{i}}{2\,{\left(-b\right)}^{3/4}\,c^{5/4}}+\frac{\left(A\,c-B\,b\right)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{\left(32\,B\,b^2\,c^2-32\,\cdots\right.}{2\,{\left(-b\right)}^{3}\cdots}\right.}{2\,{\left(-b\right)}^{3/4}\,c^{5/4}}}{\frac{\left(A\,c-B\,b\right)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)\left(A\,c-B\,b\right)}{2\,{\left(-b\right)}^{3/4}\,c^{5/4}}\right)}{2\,{\left(-b\right)}^{3/4}\,c^{5/4}}-\frac{\left(A\,c-B\,b\right)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{\left(32\,B\,b^2\,c^2-32\,\cdots\right.}{2\,{\left(-b\right)}^{3}\cdots}\right.}{2\,{\left(-b\right)}^{3/4}\,c^{5/4}}}\right.\cdots}{{\left(-b\right)}^{3/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (2*B*x^(1/2))/c - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c -
 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/(2*(-b)^(3/4)*c^
(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16
*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/(2*(
-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)))/(((A*c - B*b)*(x^(1/2)*(16*
A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c
- B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) - ((A*c - B*b)*(x^(
1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c
^3)*(A*c - B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4))))*(A*c - B
*b)*1i)/((-b)^(3/4)*c^(5/4)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 1
6*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/
(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*(x^(1/2)*(16
*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c
 - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)))/(((A*c - B*b)*
(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A
*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4))
- ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B
*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^
(3/4)*c^(5/4))))*(A*c - B*b))/((-b)^(3/4)*c^(5/4))
```

sympy [A] time = 27.83, size = 355, normalized size = 1.51

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) & \cdots \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{c} & \cdots \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} & \cdots \\ -\frac{\sqrt[4]{-1}\,A\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}\,A\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}}-\frac{\sqrt[4]{-1}\,A\sqrt[4]{\frac{1}{c}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}\,B\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\cdots\right.}{2c}\cdots & \cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-
2*A/(3*x**(3/2)) + 2*B*sqrt(x))/c, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/
5)/b, Eq(c, 0)), (-(-1)**(1/4)*A*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/
c)**(1/4) + sqrt(x))/(2*b**(3/4)) + (-1)**(1/4)*A*(1/c)**(1/4)*log((-1)**(1
/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)) - (-1)**(1/4)*A*(1/c)**(1
/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(3/4) + (-1)**(1/4
)*B*b**(1/4)*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))
/(2*c) - (-1)**(1/4)*B*b**(1/4)*(1/c)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)
**(1/4) + sqrt(x))/(2*c) + (-1)**(1/4)*B*b**(1/4)*(1/c)**(1/4)*atan((-1)**(
3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/c + 2*B*sqrt(x)/c, True))
```

3.189 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=235

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\tan^{-1}\left(1-\right.}{\sqrt{2}b^{5/4}c^{3/}}$$

```
[Out] -1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(3/4)*2
^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(
3/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-2*A/b/x^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 453, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}-\frac{(bB-Ac)\tan^{-1}\left(1-\right.}{\sqrt{2}b^{5/4}c^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)*c^(3/4)) -
((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{bx^2+cx^4}\,dx=\int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)}\,dx$$
$$=-\frac{2A}{b\sqrt{x}}-\frac{\left(2\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b}$$
$$=-\frac{2A}{b\sqrt{x}}-\frac{\left(4\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b}$$
$$=-\frac{2A}{b\sqrt{x}}-\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b\sqrt{c}}+\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{\right.}}{b\sqrt{c}}$$
$$=-\frac{2A}{b\sqrt{x}}+\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2bc}+\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x\right.}{2bc}$$
$$=-\frac{2A}{b\sqrt{x}}+\frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{2\sqrt{2}\,b^{5/4}c^{3/4}}$$
$$=-\frac{2A}{b\sqrt{x}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{(bB-Ac)\,\mathrm{l}}{}$$

Mathematica [A] time = 0.10, size = 74, normalized size = 0.31

$$\frac{\frac{(bB-Ac)\left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)\right)}{\sqrt[4]{-b}\,c^{3/4}}-\frac{2A}{\sqrt{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] ((-2*A)/Sqrt[x] + ((b*B - A*c)*(ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + ArcT
anh[(b*c^(1/4)*Sqrt[x])/(-b)^(5/4)]))/((-b)^(1/4)*c^(3/4)))/b
```

fricas [B] time = 1.11, size = 843, normalized size = 3.59

$$\frac{4\,bx\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^6-6\,AB^5b^5c+15\,A^2B^4b^4c^2-20\,A^3B^3b^3c^3+15\,A^4B^2b^2c^4-6\,A^5Bbc^5+A^6c^6\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
 A^4*c^4)/(b^5*c^3))^(1/4)*arctan((sqrt((B^6*b^6 - 6*A*B^5*b^5*c + 15*A^2*B
^4*b^4*c^2 - 20*A^3*B^3*b^3*c^3 + 15*A^4*B^2*b^2*c^4 - 6*A^5*B*b*c^5 + A^6*
c^6)*x - (B^4*b^7*c - 4*A*B^3*b^6*c^2 + 6*A^2*B^2*b^5*c^3 - 4*A^3*B*b^4*c^4
 + A^4*b^3*c^5)*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^3)))*b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b
^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4) + (B^3*b^4*c - 3*A*B^2*b
```

```
^3*c^2 + 3*A^2*B*b^2*c^3 - A^3*b*c^4)*sqrt(x)*(-(B^4*b^4 - 4*A*B^3*b^3*c +
6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4))/(B^4*b^4 - 4
*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)) - b*x*(-(B^4*b
^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3)
)^(1/4)*log(b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*
c^2 - A^3*c^3)*sqrt(x)) + b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^
2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*log(-b^4*c^2*(-(B^4*b^4 - 4*A
*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4)
- (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - 4*A*sqrt(x
))/(b*x)
```

giac [**A**] time = 0.19, size = 251, normalized size = 1.07

$$-\frac{2\,A}{b\sqrt{x}}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^3}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{2}\right.}{2\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2*A/(b*sqrt(x)) + 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1
/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) - 1/4*sqrt(2)*((b*c^3)
^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(
b/c))/(b^2*c^3) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-
sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

maple [**A**] time = 0.06, size = 277, normalized size = 1.18

$$-\frac{\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\;A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\;B\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/2/b/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4/b/(b
/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)
^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/b/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)-1)+1/4/c/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2/c/(b/c)
^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-2*A/b/x^(1/2)
```

maxima [**A**] time = 3.01, size = 194, normalized size = 0.83

$$\frac{(Bb-Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\right.}{}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/
sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3
/4)))/b - 2*A/(b*sqrt(x))
```

mupad [B] time = 0.22, size = 71, normalized size = 0.30

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(A\,c-B\,b)}{(-b)^{5/4}\,c^{3/4}}-\frac{2\,A}{b\,\sqrt{x}}-\frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(A\,c-B\,b)}{(-b)^{5/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^(5/4)*c^(3/4)) - (2*
A)/(b*x^(1/2)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^(5
/4)*c^(3/4))
```

sympy [A] time = 18.45, size = 456, normalized size = 1.94

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}}{c} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ -\frac{2A}{b\sqrt{x}}+\frac{(-1)^{\frac{3}{4}}Ac\left(\frac{1}{c}\right)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{5}{4}}}-\frac{(-1)^{\frac{3}{4}}Ac\left(\frac{1}{c}\right)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{5}{4}}}+\frac{(-1)^{\frac{3}{4}}Ac\left(\frac{1}{c}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{5}{4}}}-\frac{2(-1)^{\frac{3}{4}}A\operatorname{atan}\cdots}{b^{\frac{5}{4}}\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-
2*A/(5*x**(5/2)) - 2*B/sqrt(x))/c, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(3/2)
/3)/b, Eq(c, 0)), (-2*A/(b*sqrt(x)) + (-1)**(3/4)*A*c*(1/c)**(3/4)*log(-(-1
)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)) - (-1)**(3/4)*A*c*(1
/c)**(3/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)) +
(-1)**(3/4)*A*c*(1/c)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4
)))/b**(5/4) - 2*(-1)**(3/4)*A*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1
/4)))/(b**(5/4)*(1/c)**(1/4)) - (-1)**(3/4)*B*(1/c)**(3/4)*log(-(-1)**(1/4)
*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(1/4)) + (-1)**(3/4)*B*(1/c)**(3/4)
*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(1/4)) - (-1)**(3/4
)*B*(1/c)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(1/4)
 + 2*(-1)**(3/4)*B*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(1
/4)*c*(1/c)**(1/4)), True))
```

3.190 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=237

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{(bB-Ac)\tan^{-}}{\sqrt{2}b}$$

```
[Out] -2/3*A/b/x^(3/2)-1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(7/4)/c^(1/4)*2^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1
/4))/b^(7/4)/c^(1/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^
(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*
c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 453, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{(bB-Ac)\tan^{-}}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) - ((b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)*c^(1/4))
+ ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx=\int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)}\,dx$$

$$=-\frac{2A}{3bx^{3/2}}-\frac{\left(2\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{3b}$$

$$=-\frac{2A}{3bx^{3/2}}-\frac{\left(4\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{3b}$$

$$=-\frac{2A}{3bx^{3/2}}+\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^{3/2}}+\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx\right.}{b^{3/2}}$$

$$=-\frac{2A}{3bx^{3/2}}+\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{3/2}\sqrt{c}}+\frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}}{}}\right.}{2b^{3/2}\sqrt{c}}$$

$$=-\frac{2A}{3bx^{3/2}}-\frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{}\right.}{2\sqrt{2}\,b^{7/4}\sqrt[4]{c}}$$

$$=-\frac{2A}{3bx^{3/2}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{(bB-}{}$$

Mathematica [A] time = 0.15, size = 168, normalized size = 0.71

$$-\frac{(bB-Ac)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{2\sqrt{2}\,b^{7/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*(2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

fricas [B] time = 0.93, size = 653, normalized size = 2.76

$$-\frac{12\,bx^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4\sqrt{-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}}+\left(B^2b^2-2\,ABbc+A^2\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c
^3 + A^4*c^4)/(b^7*c))^(1/4)*arctan((sqrt(b^4*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*
c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c)) + (B^2*b^2 - 2*A*
B*b*c + A^2*c^2)*x)*b^5*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(3/4) + (B*b^6*c - A*b^5*c^2)*sqrt(x)*(-(
B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7
```

```
*c))^(3/4))/(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
A^4*c^4)) + 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4)*log(b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^
2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(
x)) - 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^
3 + A^4*c^4)/(b^7*c))^(1/4)*log(-b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2
*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) +
 4*A*sqrt(x))/(b*x^2)
```

giac [A] time = 0.23, size = 251, normalized size = 1.06

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{2}c}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{2}c}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^2*c) - 2/3*A/(b*x^(3/2))
```

maple [A] time = 0.06, size = 280, normalized size = 1.18

$$-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b^{2}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] -2/3*A/b/x^(3/2)-1/4/b^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))*c-1/2/b^2*(
b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*c-1/2/b^2*(b/c)^
(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*c+1/4/b*(b/c)^(1/4)*2
^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1
/2)*x^(1/2)+(b/c)^(1/2)))+1/2/b*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^
(1/4)*x^(1/2)+1)+1/2/b*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 3.07, size = 218, normalized size = 0.92

$$\frac{\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\cdots}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2
*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b - 2/3*A/(b*x^(3/2))
```

mupad [B] time = 0.30, size = 811, normalized size = 3.42

$$-\frac{2\,A}{3\,b\,x^{3/2}}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)}{2\,(-b)^{7/4}\,c^{1/4}}\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3 \cdots\right.\right.}{2\,(-b)^{7/4}\,c^{1/4}}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)}{2\,(-b)^{7/4}\,c^{1/4}}\right)}{2\,(-b)^{7/4}\,c^{1/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3 \cdots\right.\right.}{2\,(-b)^{7/4}\,c^{1/4}}}\right.\cdots}{(-b)^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] - (2*A)/(3*b*x^(3/2)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B
^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/
(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*
(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5
*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)))/(
((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) -
((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4))))/(2*(-b
)^(7/4)*c^(1/4)) - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 -
 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4
)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b)*1i)/((-b)^(7/4)*c^(1/4))
- (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^
4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/
4))))/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B
^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1
i)/(2*(-b)^(7/4)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4)))/(((A*c - B*b)*(x^(1/2)*
(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5
*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4))
 - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4)
 + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1
i)/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b))/((-b)^(7/4)*c^(1/4))
```

sympy [A] time = 27.10, size = 364, normalized size = 1.54

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) & \cdots \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{c} & \cdots \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} & \cdots \\ -\frac{2A}{3bx^{\frac{3}{2}}}+\frac{\sqrt[4]{-1}\,Ac\sqrt[4]{\frac{1}{c}}\,\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}\,Ac\sqrt[4]{\frac{1}{c}}\,\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}}+\frac{\sqrt[4]{-1}\,Ac\sqrt[4]{\frac{1}{c}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}\,B\sqrt[4]{\frac{1}{c}}\,\log \cdots}{\cdots} & \cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0))
, ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/c, Eq(b, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*sqrt(x))/b, Eq(c, 0)), (-2*A/(3*b*x**(3/2)) + (-1)**(1/4)*A*c*(1/c)*
*(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)) - (-1
)**(1/4)*A*c*(1/c)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/
(2*b**(7/4)) + (-1)**(1/4)*A*c*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1
/4)*(1/c)**(1/4)))/b**(7/4) - (-1)**(1/4)*B*(1/c)**(1/4)*log(-(-1)**(1/4)*b
**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)) + (-1)**(1/4)*B*(1/c)**(1/4)*l
og((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)) - (-1)**(1/4)*
B*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(3/4),
True))
```

3.191 $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)} dx$

Optimal. Leaf size=255

$$-\frac{\sqrt[4]{c}\,(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{\sqrt[4]{c}\,(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{\sqrt[4]{c}\,(bB-}{}$$

```
[Out] -2/5*A/b/x^(5/2)+1/2*c^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^
(1/4))/b^(9/4)*2^(1/2)-1/2*c^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1
/2)/b^(1/4))/b^(9/4)*2^(1/2)-1/4*c^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+1/4*c^(1/4)*(-A*c+B*b)*ln(b^
(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)-2*(-A*c+B*
b)/b^2/x^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2(bB-Ac)}{b^2\sqrt{x}}-\frac{\sqrt[4]{c}\,(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{\sqrt[4]{c}\,(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\right.}{2\sqrt{2}\,b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(5*b*x^(5/2)) - (2*(b*B - A*c))/(b^2*Sqrt[x]) + (c^(1/4)*(b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) - (c^(1/4
)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/
4)) - (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) + (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{5b}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(c(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(2c(bB-Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\left(\sqrt{c}\,(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} - \frac{\left(\sqrt{c}\,(bB-A\right.}{}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(bB-Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{(bB-Ac)\,\text{Su}}{}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{\sqrt[4]{c}\,(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}} + \frac{\sqrt[4]{c}\,(bB-}{}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\sqrt[4]{c}\,(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}} - \frac{\sqrt[4]{c}\,(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}\,b^{9/4}}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.18

$$-\frac{2\left(5x^2(bB-Ac)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{cx^2}{b}\right)+Ab\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(A*b + 5*(b*B - A*c)*x^2*Hypergeometric2F1[-1/4, 1, 3/4, -((c*x^2)/b)])
)/(5*b^2*x^(5/2))
```

fricas [B] time = 0.91, size = 883, normalized size = 3.46

$$-\frac{20\,b^2x^3\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^6c^2-6\,AB^5b^5c^3+15\,A^2B^4b^4c^4-20\,A^3B^3b^3c^5+15\,A^4B^2b^2c^6-\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^
3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*arctan((sqrt((B^6*b^6*c^2 - 6*A*B^5*b^5*c^3
 + 15*A^2*B^4*b^4*c^4 - 20*A^3*B^3*b^3*c^5 + 15*A^4*B^2*b^2*c^6 - 6*A^5*B*b
*c^7 + A^6*c^8)*x - (B^4*b^9*c - 4*A*B^3*b^8*c^2 + 6*A^2*B^2*b^7*c^3 - 4*A^
3*B*b^6*c^4 + A^4*b^5*c^5)*sqrt(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b
```

```
^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9))*b^2*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2
+ 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4) + (B^3*b^5*c - 3*
A*B^2*b^4*c^2 + 3*A^2*B*b^3*c^3 - A^3*b^2*c^4)*sqrt(x)*(-(B^4*b^4*c - 4*A*B
^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4))/(B^4*
b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)) - 5
*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4
 + A^4*c^5)/b^9)^(1/4)*log(b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b
^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2
 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) + 5*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3
*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(-b^7*(-(
B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/
b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x
)) + 4*(5*(B*b - A*c)*x^2 + A*b)*sqrt(x))/(b^2*x^3)
```

giac [A] time = 0.24, size = 268, normalized size = 1.05

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}} Bb-\left(bc^{3}\right)^{\frac{3}{4}} Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}c^{2}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}} Bb-\left(bc^{3}\right)^{\frac{3}{4}} Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{2\,b^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) - 1/2*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^
3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) -
1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 2/5*(5*B*b*x^2 - 5*A*c*x^2 + A*b)/(b
^2*x^(5/2))
```

maple [A] time = 0.06, size = 299, normalized size = 1.17

$$\frac{\sqrt{2}\ Ac\arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2}}+\frac{\sqrt{2}\ Ac\arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2}}+\frac{\sqrt{2}\ Ac\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2}}-\frac{\sqrt{2}\ B\arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/2/b^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*c+1/4/b
^2/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+
(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))*c+1/2/b^2/(b/c)^(1/4)*2^(1/2)*A*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)*c-1/2/b/(b/c)^(1/4)*2^(1/2)*B*arctan(2
^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4/b/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)
*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-
1/2/b/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-2/5*A/b/x
^(5/2)+2/b^2/x^(1/2)*A*c-2/b/x^(1/2)*B
```

maxima [A] time = 3.08, size = 213, normalized size = 0.84

$$-\frac{\left(Bbc-Ac^{2}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}}{}\right.}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(B*b*c - A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt
(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + s
qrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)
*c^(3/4)))/b^2 - 2/5*(5*(B*b - A*c)*x^2 + A*b)/(b^2*x^(5/2))
```

mupad [B] time = 0.23, size = 90, normalized size = 0.35

$$\frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(A\,c-B\,b)}{b^{9/4}}-\frac{\frac{2\,A}{5\,b}-\frac{2\,x^{2}\,(A\,c-B\,b)}{b^{2}}}{x^{5/2}}-\frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(A\,c-B\,b)}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4) - ((2*A
)/(5*b) - (2*x^2*(A*c - B*b))/b^2)/x^(5/2) - ((-c)^(1/4)*atanh(((-c)^(1/4)*
x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4)
```

sympy [A] time = 108.60, size = 366, normalized size = 1.44

$$A\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\ b=0\wedge \\ -\frac{2}{9cx^{\frac{9}{2}}} & \text{for}\ b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\ c=0 \\ -\frac{2}{5bx^{\frac{5}{2}}}+\frac{2c}{b^{2}\sqrt{x}}-\frac{(-1)^{\frac{3}{4}}c\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}}+\frac{(-1)^{\frac{3}{4}}c\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}}+\frac{(-1)^{\frac{3}{4}}c\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] A*Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(b,
0)), (-2/(5*b*x**(5/2)), Eq(c, 0)), (-2/(5*b*x**(5/2)) + 2*c/(b**2*sqrt(x))
 - (-1)**(3/4)*c*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(9
/4)*(1/c)**(1/4)) + (-1)**(3/4)*c*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + s
qrt(x))/(2*b**(9/4)*(1/c)**(1/4)) + (-1)**(3/4)*c*atan((-1)**(3/4)*sqrt(x)/
```

```
(b**(1/4)*(1/c)**(1/4)))/(b**(9/4)*(1/c)**(1/4)), True)) + B*Piecewise((zoo
/x**(5/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2)), Eq(b, 0)), (-2/(b*sqrt
(x)), Eq(c, 0)), (-2/(b*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/4)*b**(1/4)*(1
/c)**(1/4) + sqrt(x))/(2*b**(5/4)*(1/c)**(1/4)) - (-1)**(3/4)*log((-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*(1/c)**(1/4)) - (-1)**(3/4)
*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(5/4)*(1/c)**(1/4)),
 True))
```

3.192 $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=257

$$\frac{c^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{c^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{3/4}(bB-}{}$$

```
[Out] -2/7*A/b/x^(7/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)+1/2*c^(3/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)-1/2*c^(3/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)+1/4*c^(3/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2
)-1/4*c^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(11/4)*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{c^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{3/4}(bB-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(7*b*x^(7/2)) - (2*(b*B - A*c))/(3*b^2*x^(3/2)) + (c^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^
(11/4)) + (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) - (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{7b}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(2c(bB-Ac))\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} - \frac{(c(bB-Ac))\,\mathrm{Su}\ldots}{\ldots}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{\left(\sqrt{c}\,(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} - \frac{\left(\sqrt{c}\,(bB\ldots\right.}{\ldots}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\,b^{11/4}} - \frac{c^{3/4}(bB-\ldots}{\ldots}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}} - \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1\ldots\right.}{\sqrt{2}\,b^{11/4}}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.18

$$\frac{14x^2(Ac-bB)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^2}{b}\right)-6Ab}{21b^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-6*A*b + 14*(-(b*B) + A*c)*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^2)/b
)])/(21*b^2*x^(7/2))
```

fricas [B] time = 1.15, size = 707, normalized size = 2.75

$$84\,b^2x^4\left(-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B^2b^2c^5-4\,A^3Bbc^6+A^4c^7}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6\sqrt{-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B^2b^2c^5-4\,A^3Bbc^6+A^4c^7}{b^{11}}}+\left(B^2b^2c^2-2\,A\ldots\right.}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A
^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*arctan((sqrt(b^6*sqrt(-(B^4*b^4*c^3 - 4*A
*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11) + (B^2*b^
2*c^2 - 2*A*B*b*c^3 + A^2*c^4)*x)*b^8*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*
A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(3/4) + (B*b^9*c - A*b^8*c
^2)*sqrt(x)*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*
```

```
b*c^6 + A^4*c^7)/b^11)^(3/4))/(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^
2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)) + 21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3
*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(b^3*(-(
B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7
)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*
B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(
-b^3*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 +
 A^4*c^7)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 4*(7*(B*b - A*c)*x^2 + 3
*A*b)*sqrt(x))/(b^2*x^4)
```

giac [A] time = 0.19, size = 257, normalized size = 1.00

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{3}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^3 - 1/2*sqrt(2)*((b*c^3)^(1/4
)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/b^3 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 1/4*sqrt(2)*((b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sq
rt(b/c))/b^3 - 2/21*(7*B*b*x^2 - 7*A*c*x^2 + 3*A*b)/(b^2*x^(7/2))
```

maple [A] time = 0.06, size = 308, normalized size = 1.20

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^{3}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^{3}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac^{2}\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b^{3}}-\frac{\left(\frac{b}{c}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/4
*c^2/b^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2
))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*c^2/b^3*(b/c)^(1/4)*2^(
1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*c/b^2*(b/c)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/4*c/b^2*(b/c)^(1/4)*2^(1/2)*B*ln(
(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+
(b/c)^(1/2)))-1/2*c/b^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^
(1/2)+1)-2/7*A/b/x^(7/2)+2/3/b^2/x^(3/2)*A*c-2/3/b/x^(3/2)*B
```

maxima [A] time = 2.93, size = 247, normalized size = 0.96

$$-\frac{\frac{2\sqrt{2}\left(Bbc-Ac^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\left(Bbc-Ac^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\left(Bbc-Ac^{2}\right)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*b*c - A*c^2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))
) + 2*sqrt(2)*(B*b*c - A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
- 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)))
 + sqrt(2)*(B*b*c - A*c^2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
+ sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c - A*c^2)*log(-sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2 - 2/21*(7*(B
*b - A*c)*x^2 + 3*A*b)/(b^2*x^(7/2))
```

mupad [B] time = 0.32, size = 555, normalized size = 2.16

$$-\frac{\frac{2\,A}{7\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{3\,b^2}}{x^{7/2}}+\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{3/4}\,(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)-\frac{(-c)^{3/4}\,(A\,c-B\,b)\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)1\mathrm{i}}{2\,b^{11/4}}\right)}{2\,b^{11/4}}+\frac{(-c)^{3/4}\cdots}{\cdots}}{\frac{(-c)^{3/4}\,(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)-\frac{(-c)^{3/4}\,(A\,c-B\,b)\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)1\mathrm{i}}{2\,b^{11/4}}\right)1\mathrm{i}}{2\,b^{11/4}}-\frac{(-c)^{3/4}\cdots}{\cdots}}\right)\cdots}{b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(3/4)*atan((((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^2
*b^8*c^5 - 32*A*B*b^7*c^6) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b
^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)) + ((-c)^(3/4)*(A*c - B*b)*(x^(1/2)
*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c - B*
b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)))/(((-c)^(
3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6
) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4))
)*1i)/(2*b^(11/4)) - ((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*
B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*
B*b^10*c^4)*1i)/(2*b^(11/4)))*1i)/(2*b^(11/4))))*(A*c - B*b))/b^(11/4) - ((
2*A)/(7*b) - (2*x^2*(A*c - B*b))/(3*b^2))/x^(7/2) - ((-c)^(3/4)*atan((A^3*c
^8*x^(1/2)*1i - B^3*b^3*c^5*x^(1/2)*1i - A^2*B*b*c^7*x^(1/2)*3i + A*B^2*b^2
*c^6*x^(1/2)*3i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(A^3*c - 3*A^2*B*b) + 3*A*B^2*b
^2) - B^3*b^3)))*(A*c - B*b)*1i)/b^(11/4)
```

sympy [A] time = 109.30, size = 405, normalized size = 1.58

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}\right) \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{c} \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ -\frac{2A}{7bx^{\frac{7}{2}}}+\frac{2Ac}{3b^2x^{\frac{3}{2}}}-\frac{\sqrt[4]{-1}\,Ac^2\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}}+\frac{\sqrt[4]{-1}\,Ac^2\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}}-\frac{\sqrt[4]{-1}\,Ac^2\sqrt[4]{\frac{1}{c}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{11}{4}}}-\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(b, 0) & Eq(c, 0
)), ((-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2)))/c, Eq(b, 0)), ((-2*A/(7*x**(7
```

```
/2)) - 2*B/(3*x**(3/2)))/b, Eq(c, 0)), (-2*A/(7*b*x**(7/2)) + 2*A*c/(3*b**2
*x**(3/2)) - (-1)**(1/4)*A*c**2*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c
)**(1/4) + sqrt(x))/(2*b**(11/4)) + (-1)**(1/4)*A*c**2*(1/c)**(1/4)*log((-1
)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(11/4)) - (-1)**(1/4)*A*c**
2*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(11/4)
- 2*B/(3*b*x**(3/2)) + (-1)**(1/4)*B*c*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/
4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)) - (-1)**(1/4)*B*c*(1/c)**(1/4)*log(
(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)) + (-1)**(1/4)*B*c
*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(7/4), T
rue))
```

3.193 $\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=276

$$\frac{c^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}-\frac{c^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}-\frac{c^{5/4}(bB-}{}$$

```
[Out] -2/9*A/b/x^(9/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-1/2*c^(5/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/2*c^(5/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/4*c^(5/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2
)-1/4*c^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(13/4)*2^(1/2)+2*c*(-A*c+B*b)/b^3/x^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}-\frac{c^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}-\frac{c^{5/4}(bB-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(9*b*x^(9/2)) - (2*(b*B - A*c))/(5*b^2*x^(5/2)) + (2*c*(b*B - A*c))/
(b^3*Sqrt[x]) - (c^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*Log[Sqrt[b
] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(13/4)) - (c
^(5/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(2*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{9b}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} - \frac{(c(bB-Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{\left(c^2(bB-Ac)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{\left(2c^2(bB-Ac)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{\left(c^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(c(bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{\right.}{2b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{2\sqrt{2}b^{13/4}}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} + \frac{c^{5/4}(bB}{}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.17

$$\frac{2\left(9x^2(Ac-bB)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^2}{b}\right)-5Ab\right)}{45b^2x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (2*(-5*A*b + 9*(-(b*B) + A*c)*x^2*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^2
)/b)]))/(45*b^2*x^(9/2))
```

fricas [B] time = 0.98, size = 931, normalized size = 3.37

$$\frac{180\,b^3x^5\left(-\frac{B^4b^4c^5-4\,AB^3b^3c^6+6\,A^2B^2b^2c^7-4\,A^3Bbc^8+A^4c^9}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^6c^8-6\,AB^5b^5c^9+15\,A^2B^4b^4c^{10}-20\,A^3B^3b^3c^{11}+15\,A^4B^2b^2c^{\cdots}\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*
A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*arctan((sqrt((B^6*b^6*c^8 - 6*A*B^5*b^5*
```

```
c^9 + 15*A^2*B^4*b^4*c^10 - 20*A^3*B^3*b^3*c^11 + 15*A^4*B^2*b^2*c^12 - 6*A
^5*B*b*c^13 + A^6*c^14)*x - (B^4*b^11*c^5 - 4*A*B^3*b^10*c^6 + 6*A^2*B^2*b^
9*c^7 - 4*A^3*B*b^8*c^8 + A^4*b^7*c^9)*sqrt(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6
 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13))*b^3*(-(B^4*b^4*c^5 -
 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)
 + (B^3*b^6*c^4 - 3*A*B^2*b^5*c^5 + 3*A^2*B*b^4*c^6 - A^3*b^3*c^7)*sqrt(x)*
(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*
c^9)/b^13)^(1/4))/(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^
3*B*b*c^8 + A^4*c^9)) - 45*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2
*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(b^10*(-(B^4*b^4*c^5
 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/
4) - (B^3*b^3*c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 4
5*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*
c^8 + A^4*c^9)/b^13)^(1/4)*log(-b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A
^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4 - 3*A*
B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 4*(45*(B*b*c - A*c^2)*x^4
 - 5*A*b^2 - 9*(B*b^2 - A*b*c)*x^2)*sqrt(x))/(b^3*x^5)
```

giac [A] time = 0.20, size = 291, normalized size = 1.05

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{4}c}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{4}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 1/2*sqrt(2)*((b*c^3)^(
3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^4*c) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 1/4*sqr
t(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^4*c) + 2/45*(45*B*b*c*x^4 - 45*A*c^2*x^4 - 9*B*b^2*x
^2 + 9*A*b*c*x^2 - 5*A*b^2)/(b^3*x^(9/2))
```

maple [A] time = 0.06, size = 330, normalized size = 1.20

$$-\frac{\sqrt{2}\,A\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}-\frac{\sqrt{2}\,A\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}-\frac{\sqrt{2}\,A\,c^{2}\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}+\frac{\sqrt{2}\,Bc\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{b}{c}\cdots\right.}\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/2*c^2/b^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/
4*c^2/b^3/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/
2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*c^2/b^3/(b/c)^(1/4)*2^
(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c/b^2/(b/c)^(1/4)*2^(1/2)
*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/4*c/b^2/(b/c)^(1/4)*2^(1/2)*B*ln
((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)
+(b/c)^(1/2)))+1/2*c/b^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)-2/9*A/b/x^(9/2)+2/5/b^2/x^(5/2)*A*c-2/5/b/x^(5/2)*B-2/b^3*c^2/x^(
1/2)*A+2/b^2*c/x^(1/2)*B
```

maxima [A] time = 3.11, size = 237, normalized size = 0.86

$$\frac{\left(Bbc^2-Ac^3\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\mathrm{l}}{}\right.}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*c^2 - A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) +
sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4
)*c^(3/4)))/b^3 + 2/45*(45*(B*b*c - A*c^2)*x^4 - 5*A*b^2 - 9*(B*b^2 - A*b*c
)*x^2)/(b^3*x^(9/2))
```

mupad [B] time = 0.23, size = 107, normalized size = 0.39

$$\frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(A\,c-B\,b)}{b^{13/4}}-\frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{5\,b^2}+\frac{2\,c\,x^4\,(A\,c-B\,b)}{b^3}}{x^{9/2}}-\frac{(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(A\,c-B\,b)}{b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4) - ((2*
A)/(9*b) - (2*x^2*(A*c - B*b))/(5*b^2) + (2*c*x^4*(A*c - B*b))/b^3)/x^(9/2)
 - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.194 $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=278

$$-\frac{c^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}+\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}-\frac{c^{7/4}(bB-A}{}$$

```
[Out] -2/11*A/b/x^(11/2)-2/7*(-A*c+B*b)/b^2/x^(7/2)+2/3*c*(-A*c+B*b)/b^3/x^(3/2)-
1/2*c^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2
^(1/2)+1/2*c^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(
15/4)*2^(1/2)-1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)+1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 453, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}+\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}-\frac{c^{7/4}(bB-A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(11*b*x^(11/2)) - (2*(b*B - A*c))/(7*b^2*x^(7/2)) + (2*c*(b*B - A*c)
)/(3*b^3*x^(3/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(15/4)) - (c^(7/4)*(b*B - A*c)*Log[Sq
rt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(15/4))
 + (c^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(2*Sqrt[2]*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx=\int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)}\,dx$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{\left(2\left(-\frac{11bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{11b}$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}-\frac{(c(bB-Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{b^2}$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}+\frac{\left(c^2(bB-Ac)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b^3}$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}+\frac{\left(2c^2(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^3}$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}+\frac{\left(c^2(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^{7/2}}+$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}+\frac{\left(c^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\right.}{2b^{7/2}}$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}-\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right.}{2\sqrt{2}\,b^{15/4}}$$
$$=-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}-\frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{15/4}}+\frac{c^{7/4}(bB}{}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.17

$$\frac{-22x^2(bB-Ac)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\frac{cx^2}{b}\right)-14Ab}{77b^2x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-14*A*b - 22*(b*B - A*c)*x^2*Hypergeometric2F1[-7/4, 1, -3/4, -((c*x^2)/b)
])/(77*b^2*x^(11/2))
```

fricas [B] time = 0.76, size = 734, normalized size = 2.64

$$-\frac{924\,b^3x^6\left(-\frac{B^4b^4c^7-4\,AB^3b^3c^8+6\,A^2B^2b^2c^9-4\,A^3Bbc^{10}+A^4c^{11}}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^8\sqrt{-\frac{B^4b^4c^7-4\,AB^3b^3c^8+6\,A^2B^2b^2c^9-4\,A^3Bbc^{10}+A^4c^{11}}{b^{15}}}+\left(B^2b^2c^4-\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/462*(924*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 -
4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*arctan((sqrt(b^8*sqrt(-(B^4*b^4*c^7
- 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15) +
```

```
(B^2*b^2*c^4 - 2*A*B*b*c^5 + A^2*c^6)*x)*b^11*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*
c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(3/4) + (B*b^12*
c^2 - A*b^11*c^3)*sqrt(x)*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*
c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(3/4))/(B^4*b^4*c^7 - 4*A*B^3*b^3*c^
8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)) + 231*b^3*x^6*(-(B^4*b^
4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^
15)^(1/4)*log(b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*
A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 231*b^3
*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10
+ A^4*c^11)/b^15)^(1/4)*log(-b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B
^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqr
t(x)) - 4*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)*sqrt
(x))/(b^3*x^6)
```

giac [A] time = 0.19, size = 291, normalized size = 1.05

$$\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bbc-\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bbc-\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-\cdots\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^
(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/b^4 + 2/231*(77*B*b*c*x^4 - 77*A*c^2*x^4 - 33*B*b^2
*x^2 + 33*A*b*c*x^2 - 21*A*b^2)/(b^3*x^(11/2))
```

maple [A] time = 0.06, size = 336, normalized size = 1.21

$$-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^4}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^4}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\,c^3\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/2*c^3/b^4*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-1/
4*c^3/b^4*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/
2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*c^3/b^4*(b/c)^(1/4)*2^
(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/
2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/4*c^2/b^3*(b/c)^(1/4)*2^(1/2)*
B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(
1/2)+(b/c)^(1/2)))+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)+1)-2/11*A/b/x^(11/2)+2/7/b^2/x^(7/2)*A*c-2/7/b/x^(7/2)*B-2/3/b
^3*c^2/x^(3/2)*A+2/3/b^2*c/x^(3/2)*B
```

maxima [A] time = 3.12, size = 276, normalized size = 0.99

$$\frac{\frac{2\sqrt{2}\left(Bbc^2-Ac^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\left(Bbc^2-Ac^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\left(Bbc^2-Ac^3\right)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)
)) + 2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c
))) + sqrt(2)*(B*b*c^2 - A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(
c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c^2 - A*c^3)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3 + 2/2
31*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)/(b^3*x^(11/
2))
```

mupad [B] time = 0.35, size = 563, normalized size = 2.03

$$\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{A^3c^{10}\sqrt{x}-B^3b^3c^7\sqrt{x}-3A^2Bbc^9\sqrt{x}+3AB^2b^2c^8\sqrt{x}}{b^{1/4}(-c)^{27/4}\left(c\left(c\left(A^3c-3A^2Bb\right)+3AB^2b^2\right)-B^3b^3\right)}\right)(Ac-Bb)}{b^{15/4}}-\frac{\frac{2A}{11b}-\frac{2x^2(Ac-Bb)}{7b^2}+\frac{2cx^4(Ac-Bb)}{3b^3}}{x^{11/2}}-\frac{(-c)^{7/4}\,\mathrm{a}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(7/4)*atan((A^3*c^10*x^(1/2) - B^3*b^3*c^7*x^(1/2) - 3*A^2*B*b*c^9*x^
(1/2) + 3*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(c*(c*(A^3*c - 3*A^2*
B*b) + 3*A*B^2*b^2) - B^3*b^3)))*(A*c - B*b))/b^(15/4) - ((-c)^(7/4)*atan((
((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B
*b^10*c^8) - ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^
(15/4)))*1i)/(2*b^(15/4)) + ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^
9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13
*c^6 - 32*B*b^14*c^5))/(2*b^(15/4)))*1i)/(2*b^(15/4)))/(((-c)^(7/4)*(A*c -
B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) - ((-c)^
(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^(15/4))))/(2*b^(15/
4)) - ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 -
32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))
/(2*b^(15/4))))/(2*b^(15/4))))*(A*c - B*b)*1i)/b^(15/4) - ((2*A)/(11*b) - (
2*x^2*(A*c - B*b))/(7*b^2) + (2*c*x^4*(A*c - B*b))/(3*b^3))/x^(11/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.195 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=332

$$\frac{b^{5/4}(13bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{17/4}}-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{17/4}}+\frac{b^5}{}$$

```
[Out] -1/10*(-9*A*c+13*B*b)*x^(5/2)/c^3+1/18*(-9*A*c+13*B*b)*x^(9/2)/b/c^2-1/2*(-
A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)+1/8*b^(5/4)*(-9*A*c+13*B*b)*arctan(1-c^(1/4
)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-1/8*b^(5/4)*(-9*A*c+13*B*b)*arc
tan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+1/16*b^(5/4)*(-9*A*
c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)*2^
(1/2)-1/16*b^(5/4)*(-9*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/c^(17/4)*2^(1/2)+1/2*b*(-9*A*c+13*B*b)*x^(1/2)/c^4
```

Rubi [A] time = 0.28, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}(13bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{17/4}}-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{17/4}}+\frac{b^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(13*b*B - 9*A*c)*Sqrt[x])/(2*c^4) - ((13*b*B - 9*A*c)*x^(5/2))/(10*c^3)
+ ((13*b*B - 9*A*c)*x^(9/2))/(18*b*c^2) - ((b*B - A*c)*x^(13/2))/(2*b*c*(b
+ c*x^2)) + (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(4*Sqrt[2]*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(17/4)) + (b^(5/4)*(13*b*B - 9*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{13bB}{2}-\frac{9Ac}{2}\right)\int \frac{x^{11/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{(13bB-9Ac)\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{(b(13bB-9Ac))\int \frac{x^{3/2}}{b+cx^2}\,d}{4c^3}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^2(1\right.}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^2(1\right.}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^{3/2}(\right.}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} - \frac{\left(b^{3/2}(\right.}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{b^{5/4}(1}{}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)} + \frac{b^{5/4}(1}{}$$

Mathematica [A] time = 0.67, size = 417, normalized size = 1.26

$$\frac{90\sqrt{2}\,b^{5/4}(13bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) - 90\sqrt{2}\,b^{5/4}(13bB-9Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right) - 405\sqrt{2}\,Ab^{5/4}c\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (4320*b^2*B*c^(1/4)*Sqrt[x] - 2880*A*b*c^(5/4)*Sqrt[x] - 576*b*B*c^(5/4)*x^
(5/2) + 288*A*c^(9/4)*x^(5/2) + 160*B*c^(9/4)*x^(9/2) + (360*b^3*B*c^(1/4)*
Sqrt[x])/(b + c*x^2) - (360*A*b^2*c^(5/4)*Sqrt[x])/(b + c*x^2) + 90*Sqrt[2]
*b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 9
0*Sqrt[2]*b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)] + 585*Sqrt[2]*b^(9/4)*B*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x] - 405*Sqrt[2]*A*b^(5/4)*c*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x] - 585*Sqrt[2]*b^(9/4)*B*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 405*Sqrt[2]*A*b^(5/4)*c*Log[Sqrt[b] + Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(720*c^(17/4))
```

fricas [B] time = 0.78, size = 804, normalized size = 2.42

$$\frac{180\left(c^5x^2+bc^4\right)\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^8\sqrt{-\frac{28561\,B^4b^9-79092\,AB^3b^8c+}{}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A
^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*arctan
((sqrt(c^8*sqrt(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2
 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17) + (169*B^2*b^4 - 234*A*B*b
^3*c + 81*A^2*b^2*c^2)*x)*c^13*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134
*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(3/4) + (1
3*B*b^2*c^13 - 9*A*b*c^14)*sqrt(x)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 8
2134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(3/4))
/(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b
^6*c^3 + 6561*A^4*b^5*c^4)) + 45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092
*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c
^4)/c^17)^(1/4)*log(c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^
2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2
 - 9*A*b*c)*sqrt(x)) - 45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*
b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^1
7)^(1/4)*log(-c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*
c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A
*b*c)*sqrt(x)) + 4*(20*B*c^3*x^6 - 4*(13*B*b*c^2 - 9*A*c^3)*x^4 + 585*B*b^3
 - 405*A*b^2*c + 36*(13*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(x))/(c^5*x^2 + b*c^4
)
```

giac [A] time = 0.22, size = 335, normalized size = 1.01

$$-\frac{\sqrt{2}\left(13\left(bc^3\right)^{\frac{1}{4}}Bb^2-9\left(bc^3\right)^{\frac{1}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5}-\frac{\sqrt{2}\left(13\left(bc^3\right)^{\frac{1}{4}}Bb^2-9\left(bc^3\right)^{\frac{1}{4}}Abc\right)\arctan\left(-\frac{\sqrt{}}{}\right.}{8\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/8*sqrt(2)*(13*
(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B
*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/
c))/c^5 + 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log
(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/2*(B*b^3*sqrt(x) - A
*b^2*c*sqrt(x))/((c*x^2 + b)*c^4) + 2/45*(5*B*c^16*x^(9/2) - 18*B*b*c^15*x^
(5/2) + 9*A*c^16*x^(5/2) + 135*B*b^2*c^14*sqrt(x) - 90*A*b*c^15*sqrt(x))/c^
18
```

maple [A] time = 0.06, size = 372, normalized size = 1.12

$$\frac{2Bx^{\frac{9}{2}}}{9c^2}+\frac{2Ax^{\frac{5}{2}}}{5c^2}-\frac{4Bbx^{\frac{5}{2}}}{5c^3}-\frac{Ab^2\sqrt{x}}{2\left(cx^2+b\right)c^3}+\frac{Bb^3\sqrt{x}}{2\left(cx^2+b\right)c^4}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8c^3}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\,\mathrm{arc}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/9/c^2*B*x^(9/2)+2/5/c^2*A*x^(5/2)-4/5/c^3*B*x^(5/2)*b-4/c^3*A*b*x^(1/2)+6
/c^4*B*b^2*x^(1/2)-1/2*b^2/c^3*x^(1/2)/(c*x^2+b)*A+1/2*b^3/c^4*x^(1/2)/(c*x
^2+b)*B+9/8*b/c^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+
1)+9/8*b/c^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+9/
16*b/c^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2
))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-13/8*b^2/c^4*(b/c)^(1/4)*2^
(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-13/8*b^2/c^4*(b/c)^(1/4)*2^(1
/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-13/16*b^2/c^4*(b/c)^(1/4)*2^(1/
2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*
x^(1/2)+(b/c)^(1/2)))
```

maxima [A] time = 3.07, size = 298, normalized size = 0.90

$$\frac{\left(Bb^3-Ab^2c\right)\sqrt{x}}{2\left(c^5x^2+bc^4\right)}-\frac{\frac{2\sqrt{2}(13Bb-9Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(13Bb-9Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\sqrt{2}(13Bb}{16c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)*sqrt(x)/(c^5*x^2 + b*c^4) - 1/16*(2*sqrt(2)*(13*B*b -
 9*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b -
9*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b - 9*
A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^
(1/4)) - sqrt(2)*(13*B*b - 9*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^4 + 2/45*(5*B*c^2*x^(9/2) - 9*(
2*B*b*c - A*c^2)*x^(5/2) + 45*(3*B*b^2 - 2*A*b*c)*sqrt(x))/c^4
```

mupad [B] time = 0.22, size = 857, normalized size = 2.58

$$x^{5/2}\left(\frac{2A}{5c^2}-\frac{4Bb}{5c^3}\right)-\sqrt{x}\left(\frac{2b\left(\frac{2A}{c^2}-\frac{4Bb}{c^3}\right)}{c}+\frac{2Bb^2}{c^4}\right)+\frac{2Bx^{9/2}}{9c^2}+\frac{\sqrt{x}\left(\frac{Bb^3}{2}-\frac{Ab^2c}{2}\right)}{c^5x^2+bc^4}+\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{5/4}\left(\sqrt{x}\left(81A^2\right.\right.}{(-b)^{5/4}\left(\sqrt{x}\left(81A\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(5/2)*((2*A)/(5*c^2) - (4*B*b)/(5*c^3)) - x^(1/2)*((2*b*((2*A)/c^2 - (4*B
*b)/c^3))/c + (2*B*b^2)/c^4) + (2*B*x^(9/2))/(9*c^2) + (x^(1/2)*((B*b^3)/2
- (A*b^2*c)/2))/(b*c^4 + c^5*x^2) + ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1/2)
*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c -
 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)
) + ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c
^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c
- 13*B*b)*1i)/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^
4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*
b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4)) - ((-b)^(5/4)*((x^(1/2)*(1
69*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13
*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4))))*(9
*A*c - 13*B*b)*1i)/(4*c^(17/4)) - ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1/2)*(
169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 1
3*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4))
+ ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5
 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c
 - 13*B*b))/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*
c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^
3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)) - ((-b)^(5/4)*((x^(1/2
)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c
- 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(1
7/4))))*(9*A*c - 13*B*b))/(4*c^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.196 $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=310

$$\frac{b^{3/4}(11bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/}}{}$$

```
[Out] -1/6*(-7*A*c+11*B*b)*x^(3/2)/c^3+1/14*(-7*A*c+11*B*b)*x^(7/2)/b/c^2-1/2*(-A
*c+B*b)*x^(11/2)/b/c/(c*x^2+b)-1/8*b^(3/4)*(-7*A*c+11*B*b)*arctan(1-c^(1/4)
*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/8*b^(3/4)*(-7*A*c+11*B*b)*arct
an(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/16*b^(3/4)*(-7*A*c
+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)*2^(
1/2)-1/16*b^(3/4)*(-7*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1
/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{3/4}(11bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}-\frac{b^{3/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((11*b*B - 7*A*c)*x^(3/2))/(6*c^3) + ((11*b*B - 7*A*c)*x^(7/2))/(14*b*c^2)
 - ((b*B - A*c)*x^(11/2))/(2*b*c*(b + c*x^2)) - (b^(3/4)*(11*b*B - 7*A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(15/4)) + (b^(3/
4)*(11*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*c^(15/4)) + (b^(3/4)*(11*b*B - 7*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(15/4)) - (b^(3/4)*(11*b*B - 7*A*c)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(1
5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{11bB}{2}-\frac{7Ac}{2}\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} - \frac{(11bB-7Ac)\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{(b(11bB-7Ac))\int \frac{\sqrt{x}}{b+cx^2}\,d}{4c^3}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{(b(11bB-7Ac))\,\mathrm{Subst}\left(\int\right.}{2c^3}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} - \frac{(b(11bB-7Ac))\,\mathrm{Subst}\left(\int\right.}{4c^{7/2}}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{(b(11bB-7Ac))\,\mathrm{Subst}\left(\int\right.}{8}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} + \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}\right.}{8\sqrt{2}}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\left(b+cx^2\right)} - \frac{b^{3/4}(11bB-7Ac)\tan^{-1}\left(1\right.}{4\sqrt{2}\,c^{15/4}}$$

Mathematica [C] time = 0.30, size = 154, normalized size = 0.50

$$-\frac{(-b)^{3/4}(3bB-2Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{15/4}} + \frac{(-b)^{3/4}(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{15/4}} + \frac{2x^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3c^3} +$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(-2*b*B + A*c)*x^(3/2))/(3*c^3) + (2*B*x^(7/2))/(7*c^2) - ((-b)^(3/4)*(3
*b*B - 2*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(15/4) + ((-b)^(3/4)*
(3*b*B - 2*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(15/4) + (2*(-(b*B
) + A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*c^3)
```

fricas [B] time = 1.28, size = 989, normalized size = 3.19

$$84\left(c^4x^2+bc^3\right)\left(-\frac{14641\,B^4b^7-37268\,AB^3b^6c+35574\,A^2B^2b^5c^2-15092\,A^3Bb^4c^3+2401\,A^4b^3c^4}{c^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(1771561\,B^6b^{10}-6764142\,A\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^
2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*arctan(
(sqrt((1771561*B^6*b^10 - 6764142*A*B^5*b^9*c + 10761135*A^2*B^4*b^8*c^2 -
9130660*A^3*B^3*b^7*c^3 + 4357815*A^4*B^2*b^6*c^4 - 1109262*A^5*B*b^5*c^5 +
 117649*A^6*b^4*c^6)*x - (14641*B^4*b^7*c^7 - 37268*A*B^3*b^6*c^8 + 35574*A
^2*B^2*b^5*c^9 - 15092*A^3*B*b^4*c^10 + 2401*A^4*b^3*c^11)*sqrt(-(14641*B^4
*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 24
01*A^4*b^3*c^4)/c^15))*c^4*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2
*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4) + (1331*
B^3*b^5*c^4 - 2541*A*B^2*b^4*c^5 + 1617*A^2*B*b^3*c^6 - 343*A^3*b^2*c^7)*sq
rt(x)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*
A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4))/(14641*B^4*b^7 - 37268*A*B^3
*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)) -
 21*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*
b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(c^11*(-(1
4641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*
c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 16
17*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) + 21*(c^4*x^2 + b*c^3)*(-(1464
1*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3
 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(-c^11*(-(14641*B^4*b^7 - 37268*A*B^3*b
^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15
)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b
^2*c^3)*sqrt(x)) + 4*(12*B*c^2*x^5 - 4*(11*B*b*c - 7*A*c^2)*x^3 - 7*(11*B*b
^2 - 7*A*b*c)*x)*sqrt(x))/(c^4*x^2 + b*c^3)
```

giac [A] time = 0.27, size = 299, normalized size = 0.96

$$-\frac{Bb^2x^{\frac{3}{2}} - Abcx^{\frac{3}{2}}}{2\left(cx^2+b\right)c^3} + \frac{\sqrt{2}\left(11\left(bc^3\right)^{\frac{3}{4}}Bb - 7\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^6} + \frac{\sqrt{2}\left(11\left(bc^3\right)^{\frac{3}{4}}Bb - 7\left(bc^3\right)^{\frac{3}{4}}Ac}{8c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^(3/2) - A*b*c*x^(3/2))/((c*x^2 + b)*c^3) + 1/8*sqrt(2)*(11*(b
*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/8*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(
b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/
c)^(1/4))/c^6 - 1/16*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 + 1/16*sqrt(2)*(11*(b*c
^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/c^6 + 2/21*(3*B*c^12*x^(7/2) - 14*B*b*c^11*x^(3/2) + 7*A*c^12*x
^(3/2))/c^14
```

maple [A] time = 0.06, size = 348, normalized size = 1.12

$$\frac{2Bx^{\frac{7}{2}}}{7c^2} + \frac{Abx^{\frac{3}{2}}}{2\left(cx^2+b\right)c^2} - \frac{Bb^2x^{\frac{3}{2}}}{2\left(cx^2+b\right)c^3} + \frac{2Ax^{\frac{3}{2}}}{3c^2} - \frac{4Bbx^{\frac{3}{2}}}{3c^3} - \frac{7\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3} - \frac{7\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\right.}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/7/c^2*B*x^(7/2)+2/3/c^2*x^(3/2)*A-4/3/c^3*x^(3/2)*b*B+1/2*b/c^2*x^(3/2)/(
c*x^2+b)*A-1/2*b^2/c^3*x^(3/2)/(c*x^2+b)*B-7/16*b/c^3/(b/c)^(1/4)*2^(1/2)*A
*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1
/2)+(b/c)^(1/2)))-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)+1)-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)+11/16*b^2/c^4/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/
2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+11/8*b^2/c^4/(
b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+11/8*b^2/c^4/(b/
c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.11, size = 247, normalized size = 0.80

$$-\frac{\left(Bb^2-Abc\right)x^{\frac{3}{2}}}{2\left(c^4x^2+bc^3\right)}+\frac{\left(11\,Bb^2-7\,Abc\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b\right.}{}\right.}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*x^(3/2)/(c^4*x^2 + b*c^3) + 1/16*(11*B*b^2 - 7*A*b*c)*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3 + 2/21
*(3*B*c*x^(7/2) - 7*(2*B*b - A*c)*x^(3/2))/c^3
```

mupad [B] time = 0.17, size = 127, normalized size = 0.41

$$x^{3/2}\left(\frac{2\,A}{3\,c^2}-\frac{4\,B\,b}{3\,c^3}\right)+\frac{2\,B\,x^{7/2}}{7\,c^2}-\frac{x^{3/2}\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3}+\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(7\,A\,c-11\,B\,b)}{4\,c^{15/4}}+\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right.}{4\,c^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(3/2)*((2*A)/(3*c^2) - (4*B*b)/(3*c^3)) + (2*B*x^(7/2))/(7*c^2) - (x^(3/2
)*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(3/4)*atan((c^(1/4)*x^
(1/2))/(-b)^(1/4))*(7*A*c - 11*B*b))/(4*c^(15/4)) + ((-b)^(3/4)*atan((c^(1/
4)*x^(1/2)*1i)/(-b)^(1/4))*(7*A*c - 11*B*b)*1i)/(4*c^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.197 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}}+\frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}}-\frac{\sqrt[4]{b}\,(9\ldots}{}$$

```
[Out] 1/10*(-5*A*c+9*B*b)*x^(5/2)/b/c^2-1/2*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)-1/8*
b^(1/4)*(-5*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2
^(1/2)+1/8*b^(1/4)*(-5*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))
/c^(13/4)*2^(1/2)-1/16*b^(1/4)*(-5*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+1/16*b^(1/4)*(-5*A*c+9*B*b)*ln(b^
(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-1/2*(-5*A
*c+9*B*b)*x^(1/2)/c^3
```

Rubi [A] time = 0.25, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^{5/2}(9bB-5Ac)}{10bc^2}-\frac{\sqrt{x}\,(9bB-5Ac)}{2c^3}-\frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}}+\frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(\sqrt{\ldots}\right.}{8\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((9*b*B - 5*A*c)*Sqrt[x])/(2*c^3) + ((9*b*B - 5*A*c)*x^(5/2))/(10*b*c^2) -
 ((b*B - A*c)*x^(9/2))/(2*b*c*(b + c*x^2)) - (b^(1/4)*(9*b*B - 5*A*c)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13/4)) + (b^(1/4)*(
9*b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^
(13/4)) - (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4)) + (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{9bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} - \frac{(9bB-5Ac)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} + \frac{(b(9bB-5Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^3}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} + \frac{(b(9bB-5Ac))\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\right.}{2c^3}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\sqrt{b}\,(9bB-5Ac)\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}}{b}\right.}{4c^3}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\sqrt{b}\,(9bB-5Ac)\right)\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}}\right.}{8c^{7/2}}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} - \frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\right.}{8\sqrt{2}\,c^{13/4}}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\left(b+cx^2\right)} - \frac{\sqrt[4]{b}\,(9bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}\,c^{13/4}}$$

Mathematica [**A**] time = 0.55, size = 385, normalized size = 1.24

$$\frac{-10\sqrt{2}\,\sqrt[4]{b}\,(9bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)+10\sqrt{2}\,\sqrt[4]{b}\,(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)+\frac{40Abc^{5/4}\sqrt{x}}{b+cx^2}+25\sqrt{2}\,A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-320*b*B*c^(1/4)*Sqrt[x] + 160*A*c^(5/4)*Sqrt[x] + 32*B*c^(5/4)*x^(5/2) -
(40*b^2*B*c^(1/4)*Sqrt[x])/(b + c*x^2) + (40*A*b*c^(5/4)*Sqrt[x])/(b + c*x^
2) - 10*Sqrt[2]*b^(1/4)*(9*b*B - 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)] + 10*Sqrt[2]*b^(1/4)*(9*b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)] - 45*Sqrt[2]*b^(5/4)*B*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x] + 25*Sqrt[2]*A*b^(1/4)*c*Log[Sqrt[b] - Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[2]*b^(5/4)*B*Log[Sqrt[b] + Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 25*Sqrt[2]*A*b^(1/4)*c*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(80*c^(13/4))
```

fricas [B] time = 0.97, size = 748, normalized size = 2.41

$$-\frac{20\left(c^4x^2+bc^3\right)\left(-\frac{6561\,B^4b^5-14580\,AB^3b^4c+12150\,A^2B^2b^3c^2-4500\,A^3Bb^2c^3+625\,A^4bc^4}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{6561\,B^4b^5-14580\,AB^3b^4c+1\ldots}{}}}}{}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(c^4*x^2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2
*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*arctan((sqrt
(c^6*sqrt(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500
*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13) + (81*B^2*b^2 - 90*A*B*b*c + 25*A^2*c
^2)*x)*c^10*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4
500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(3/4) + (9*B*b*c^10 - 5*A*c^11)*sq
rt(x)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^
3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(3/4))/(6561*B^4*b^5 - 14580*A*B^3*b^4*c
 + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)) + 5*(c^4*x^
2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 45
00*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(c^3*(-(6561*B^4*b^5 - 145
80*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4
)/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) - 5*(c^4*x^2 + b*c^3)*(-(6561*B^4*
b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*
A^4*b*c^4)/c^13)^(1/4)*log(-c^3*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150
*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4) - (9*B*b
 - 5*A*c)*sqrt(x)) - 4*(4*B*c^2*x^4 - 45*B*b^2 + 25*A*b*c - 4*(9*B*b*c - 5*
A*c^2)*x^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

giac [A] time = 0.19, size = 298, normalized size = 0.96

$$\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^4}+\frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\ldots\right)}{2\ldots}\right)}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/8*sqrt(2)*(9*(b*c^3)
^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/16*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^
3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/16*s
qrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*(B*b^2*sqrt(x) - A*b*c*sqrt(x))/((c*x^2
 + b)*c^3) + 2/5*(B*c^8*x^(5/2) - 10*B*b*c^7*sqrt(x) + 5*A*c^8*sqrt(x))/c^1
0
```

maple [A] time = 0.07, size = 339, normalized size = 1.09

$$\frac{2Bx^{\frac{5}{2}}}{5c^2}+\frac{Ab\sqrt{x}}{2\left(cx^2+b\right)c^2}-\frac{Bb^2\sqrt{x}}{2\left(cx^2+b\right)c^3}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8c^2}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8c^2}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/5/c^2*B*x^(5/2)+2/c^2*A*x^(1/2)-4/c^3*b*B*x^(1/2)+1/2*b/c^2*x^(1/2)/(c*x^
2+b)*A-1/2*b^2/c^3*x^(1/2)/(c*x^2+b)*B-5/8/c^2*(b/c)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-5/16/c^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^
(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/
2)))-5/8/c^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/
8*b/c^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+9/16*b/
c^3*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+9/8*b/c^3*(b/c)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

maxima [A] time = 3.03, size = 271, normalized size = 0.87

$$-\frac{\left(Bb^2 - Abc\right)\sqrt{x}}{2\left(c^4x^2 + bc^3\right)} + \frac{\frac{2\sqrt{2}(9Bb-5Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(9Bb-5Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(9Bb-5Ac}{}}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*sqrt(x)/(c^4*x^2 + b*c^3) + 1/16*(2*sqrt(2)*(9*B*b - 5
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b - 5*A
*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - 5*A*c)
*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4
)) - sqrt(2)*(9*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^3 + 2/5*(B*c*x^(5/2) - 5*(2*B*b - A*c)
*sqrt(x))/c^3
```

mupad [B] time = 0.26, size = 823, normalized size = 2.65

$$\sqrt{x}\left(\frac{2A}{c^2} - \frac{4Bb}{c^3}\right) + \frac{2Bx^{5/2}}{5c^2} - \frac{\sqrt{x}\left(\frac{Bb^2}{2} - \frac{Abc}{2}\right)}{c^4x^2 + bc^3} + \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3} - \frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-\right.}{8c^{13/4}}\right.}{8c^{13/4}}}{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3} - \frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb\right.}{8c^{13/4}}\right.}{8c^{13/4}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(1/2)*((2*A)/c^2 - (4*B*b)/c^3) + (2*B*x^(5/2))/(5*c^2) - (x^(1/2)*((B*b^
2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(1/4)*atan((((-b)^(1/4)*((x^(1
/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c
- 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*c^(1
3/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))
/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(
5*A*c - 9*B*b)*1i)/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^
2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40
*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4)) - ((-b)^(1/4)*((x^(1
```

```
/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 + ((-b)^(1/4)*(5*A*c
- 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4
))))*(5*A*c - 9*B*b)*1i)/(4*c^(13/4)) + ((-b)^(1/4)*atan((((-b)^(1/4)*((x^(
1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c
 - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(
13/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)
)/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)
))*(5*A*c - 9*B*b))/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A
^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 4
0*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*c^(13/4)) - ((-b)^(1/4)
*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 + ((-b)^(1/4)*
(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b)*1
i)/(8*c^(13/4))))*(5*A*c - 9*B*b))/(4*c^(13/4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] Timed out
```

3.198 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{(7bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)}{4\sqrt{}}$$

```
[Out] 1/6*(-3*A*c+7*B*b)*x^(3/2)/b/c^2-1/2*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)+1/8*(
-3*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*2^
(1/2)-1/8*(-3*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/
c^(11/4)*2^(1/2)-1/16*(-3*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)+1/16*(-3*A*c+7*B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{3/2}(7bB-3Ac)}{6bc^2}-\frac{(7bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{(7bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((7*b*B - 3*A*c)*x^(3/2))/(6*b*c^2) - ((b*B - A*c)*x^(7/2))/(2*b*c*(b + c*x
^2)) + ((7*b*B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*S
qrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4
)) + ((7*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{7bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{5/2}}{b+cx^2}\,dx}{2bc}$$

$$=\frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}-\frac{(7bB-3Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$=\frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}-\frac{(7bB-3Ac)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2c^2}$$

$$=\frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}+\frac{(7bB-3Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4c^{5/2}}-\frac{(7bB-$$

$$=\frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}-\frac{(7bB-3Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^3}-\frac{(7$$

$$=\frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}-\frac{(7bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{(7b$$

$$=\frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}+\frac{(7bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{(7bB-3Ac)\tan}{4\sqrt{2}$$

Mathematica [C] time = 0.26, size = 136, normalized size = 0.47

$$\frac{(3Ac-6bB)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+(6bB-3Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+2\sqrt[4]{-b}\,Bc^{3/4}x^{3/2}}{3\sqrt[4]{-b}\,c^{11/4}}+\frac{2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(-b)^(1/4)*B*c^(3/4)*x^(3/2) + (-6*b*B + 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])
/(-b)^(1/4)] + (6*b*B - 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(3*(-
b)^(1/4)*c^(11/4)) + (2*(b*B - A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4,
-((c*x^2)/b)])/(3*b*c^2)
```

fricas [B] time = 1.05, size = 925, normalized size = 3.20

$$-\frac{12\left(c^3x^2+bc^2\right)\left(-\frac{2401\,B^4b^4-4116\,AB^3b^3c+2646\,A^2B^2b^2c^2-756\,A^3Bbc^3+81\,A^4c^4}{bc^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(117649\,B^6b^6-302526\,AB^5b^5c+324135\,A^2\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B
^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*arctan((sqrt((11
7649*B^6*b^6 - 302526*A*B^5*b^5*c + 324135*A^2*B^4*b^4*c^2 - 185220*A^3*B^3
*b^3*c^3 + 59535*A^4*B^2*b^2*c^4 - 10206*A^5*B*b*c^5 + 729*A^6*c^6)*x - (24
01*B^4*b^5*c^5 - 4116*A*B^3*b^4*c^6 + 2646*A^2*B^2*b^3*c^7 - 756*A^3*B*b^2*
c^8 + 81*A^4*b*c^9)*sqrt(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b
^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11)))*c^3*(-(2401*B^4*b^4 - 411
6*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
1))^(1/4) + (343*B^3*b^3*c^3 - 441*A*B^2*b^2*c^4 + 189*A^2*B*b*c^5 - 27*A^3
*c^6)*sqrt(x)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 7
56*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4))/(2401*B^4*b^4 - 4116*A*B^3*b^
3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)) - 3*(c^3*x^2 +
b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*
B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(b*c^8*(-(2401*B^4*b^4 - 4116*A*B^
3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(3
/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x
)) + 3*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*
b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(-b*c^8*(-(2401*
B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^
4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 -
 27*A^3*c^3)*sqrt(x)) - 4*(4*B*c*x^3 + (7*B*b - 3*A*c)*x)*sqrt(x))/(c^3*x^2
 + b*c^2)
```

giac [A] time = 0.21, size = 283, normalized size = 0.98

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c^2}+\frac{Bbx^{\frac{3}{2}}-Acx^{\frac{3}{2}}}{2\left(cx^2+b\right)c^2}-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{3}{4}}Bb-3\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5}-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{3}{4}}Bb-3\left(bc^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^2 + 1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*c^2) - 1/8
*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 1/8*sqrt(2)*(7*(b*c^3
)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) + 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*
(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
 - 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

maple [A] time = 0.06, size = 317, normalized size = 1.10

$$-\frac{A\,x^{\frac{3}{2}}}{2\left(c\,x^2+b\right)c}+\frac{Bb\,x^{\frac{3}{2}}}{2\left(c\,x^2+b\right)c^2}+\frac{2B\,x^{\frac{3}{2}}}{3c^2}+\frac{3\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}+\frac{3\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}+\frac{3\sqrt{2}\;A\ln\left(\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3*B*x^(3/2)/c^2-1/2/c*x^(3/2)/(c*x^2+b)*A+1/2/c^2*x^(3/2)/(c*x^2+b)*b*B-7
/16/c^3/(b/c)^(1/4)*2^(1/2)*b*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/
2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-7/8/c^3/(b/c)^(1/4)*2^(1/2
```

```
)*b*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-7/8/c^3/(b/c)^(1/4)*2^(1/2)*b*B
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/16/c^2/(b/c)^(1/4)*2^(1/2)*A*ln((x
-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b
/c)^(1/2)))+3/8/c^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)+1)+3/8/c^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.31, size = 223, normalized size = 0.77

$$\frac{(Bb-Ac)x^{\frac{3}{2}}}{2\left(c^3x^2+bc^2\right)}+\frac{2Bx^{\frac{3}{2}}}{3c^2}-\frac{(7Bb-3Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}\cdots\right)}{b\cdots}\cdots\right)}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)*x^(3/2)/(c^3*x^2 + b*c^2) + 2/3*B*x^(3/2)/c^2 - 1/16*(7*B*b
 - 3*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(
-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/
c^2
```

mupad [B] time = 0.25, size = 106, normalized size = 0.37

$$\frac{2\,B\,x^{3/2}}{3\,c^2}-\frac{x^{3/2}\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(3\,A\,c-7\,B\,b\right)}{4\,{(-b)}^{1/4}\,c^{11/4}}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{(-b)}^{1/4}}\right)\,\left(3\,A\,c-7\,B\,b\right)\,1{}\mathrm{i}}{4\,{(-b)}^{1/4}\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*c^2) - (x^(3/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (
atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 7*B*b))/(4*(-b)^(1/4)*c^(11/4))
 + (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 7*B*b)*1i)/(4*(-b)^(1/4)
*c^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.199 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(5bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}+\frac{(5bB-Ac)\tan}{4\sqrt{2}}$$

```
[Out] -1/2*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)+1/8*(-A*c+5*B*b)*arctan(1-c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)-1/8*(-A*c+5*B*b)*arctan(1+c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)+1/16*(-A*c+5*B*b)*ln(
b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(9/4)*2^(1/2)-
1/16*(-A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(
3/4)/c^(9/4)*2^(1/2)+1/2*(-A*c+5*B*b)*x^(1/2)/b/c^2
```

Rubi [A] time = 0.23, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(5bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}+\frac{(5bB-Ac)\tan}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((5*b*B - A*c)*Sqrt[x])/(2*b*c^2) - ((b*B - A*c)*x^(5/2))/(2*b*c*(b + c*x^2
)) + ((5*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(3/4)*c^(9/4)) - ((5*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(3/4)*c^(9/4)) + ((5*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(3/4)*c^(9/4)) - ((5*b*
B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqr
t[2]*b^(3/4)*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}+\frac{\left(\frac{5bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{2bc}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^2}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2c^2}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{b}\,c^2}-\frac{(5bB-$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}-\frac{(5bB-Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{b}\,c^{5/2}}-\frac{(5}{}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}+\frac{(5bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{(5}{}$$

$$=\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\left(b+cx^2\right)}+\frac{(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\tan}{4\sqrt{2}}$$

Mathematica [A] time = 0.53, size = 354, normalized size = 1.22

$$\frac{\frac{2\sqrt{2}(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}-\frac{2\sqrt{2}(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{3/4}}+\frac{\sqrt{2}Ac\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{b^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (32*B*c^(1/4)*Sqrt[x] + (8*b*B*c^(1/4)*Sqrt[x])/(b + c*x^2) - (8*A*c^(5/4)*
Sqrt[x])/(b + c*x^2) + (2*Sqrt[2]*(5*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/b^(3/4) - (2*Sqrt[2]*(5*b*B - A*c)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + 5*Sqrt[2]*b^(1/4)*B*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - (Sqrt[2]*A*c*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) - 5*Sqrt[2]*b^(1/4)*B*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + (Sqrt[2]*A*c*Log[S
qrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(16*c^(9/4)
)
```

fricas [B] time = 1.09, size = 725, normalized size = 2.51

$$\frac{4\left(c^3x^2+bc^2\right)\left(-\frac{625B^4b^4-500AB^3b^3c+150A^2B^2b^2c^2-20A^3Bbc^3+A^4c^4}{b^3c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2c^4\sqrt{-\frac{625B^4b^4-500AB^3b^3c+150A^2B^2b^2c^2-20A^3Bb}{b^3c^9}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2
*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*arctan((sqrt(b^2*c^4*sqrt
(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A
^4*c^4)/(b^3*c^9)) + (25*B^2*b^2 - 10*A*B*b*c + A^2*c^2)*x)*b^2*c^7*(-(625*
B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)
/(b^3*c^9))^(3/4) + (5*B*b^3*c^7 - A*b^2*c^8)*sqrt(x)*(-(625*B^4*b^4 - 500*
A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(3
/4))/(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3
+ A^4*c^4)) + (c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*
B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*log(b*c^2*(-(625*B
^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/
(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) - (c^3*x^2 + b*c^2)*(-(625*B^4*b^
4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*
c^9))^(1/4)*log(-b*c^2*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c
^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) +
4*(4*B*c*x^2 + 5*B*b - A*c)*sqrt(x))/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.19, size = 283, normalized size = 0.98

$$\frac{2B\sqrt{x}}{c^2} - \frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{1}{4}}Bb - \left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8bc^3} - \frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{1}{4}}Bb - \left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{\cdots}\right.}{\cdots}\right.}{8bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^2 - 1/8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/
8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/16*sqrt(2)*(5*(b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/(b*c^3) + 1/16*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/2*(B*b*sqrt(x)
- A*c*sqrt(x))/((c*x^2 + b)*c^2)
```

maple [A] time = 0.06, size = 323, normalized size = 1.12

$$-\frac{A\sqrt{x}}{2\left(cx^2+b\right)c} + \frac{Bb\sqrt{x}}{2\left(cx^2+b\right)c^2} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8bc} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8bc} + \frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2*B/c^2*x^(1/2)-1/2/c*x^(1/2)/(c*x^2+b)*A+1/2/c^2*x^(1/2)/(c*x^2+b)*b*B+1/8
/c*(b/c)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/16/c*(b/
c)^(1/4)/b*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c
)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/8/c*(b/c)^(1/4)/b*2^(1/2)*A*arctan(
2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)
/(b/c)^(1/4)*x^(1/2)-1)-5/16/c^2*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^
```

```
(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-5/8
/c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

maxima [A] time = 3.11, size = 250, normalized size = 0.87

$$\frac{(Bb-Ac)\sqrt{x}}{2\left(c^3x^2+bc^2\right)}+\frac{2B\sqrt{x}}{c^2}-\frac{\frac{2\sqrt{2}(5Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(5Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\sqrt{2}(5Bb\cdots}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)*sqrt(x)/(c^3*x^2 + b*c^2) + 2*B*sqrt(x)/c^2 - 1/16*(2*sqrt(
2)*(5*B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(
5*B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(5*B*b
 - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)
*c^(1/4)) - sqrt(2)*(5*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^2
```

mupad [B] time = 0.21, size = 744, normalized size = 2.57

$$\frac{2B\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(\frac{Ac}{2}-\frac{Bb}{2}\right)}{c^3x^2+bc^2}+\frac{\mathrm{atan}\left(\frac{\frac{(Ac-5Bb)\left(\frac{\sqrt{x}\left(A^2c^2-10ABbc+25B^2b^2\right)}{c}-\frac{(Ac-5Bb)\left(8Abc^2-40Bb^2c\right)}{8(-b)^{3/4}c^{9/4}}\right)1i}{8(-b)^{3/4}c^{9/4}}+\frac{(Ac-5Bb)\left(\frac{\sqrt{x}\left(A^2c^2-10ABbc+25\cdots\right)}{c}\cdots\right)}{8(-\cdots}}{\frac{(Ac-5Bb)\left(\frac{\sqrt{x}\left(A^2c^2-10ABbc+25B^2b^2\right)}{c}-\frac{(Ac-5Bb)\left(8Abc^2-40Bb^2c\right)}{8(-b)^{3/4}c^{9/4}}\right)}{8(-b)^{3/4}c^{9/4}}-\frac{(Ac-5Bb)\left(\frac{\sqrt{x}\left(A^2c^2-10ABbc+25\cdots\right)}{c}\cdots\right)}{8(-b\cdots}}\right)\cdots}{4(-b)^{3/4}c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*B*x^(1/2))/c^2 - (x^(1/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (atan
((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c - ((A*c -
 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)
*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c
 + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*
(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A
*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)
)))/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2
 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*
c^(9/4))))/(8*(-b)^(3/4)*c^(9/4))))*(A*c - 5*B*b)*1i)/(4*(-b)^(3/4)*c^(9/4)
) + (atan((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c
- ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(9/4))))/(8*(
-b)^(3/4)*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*
B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(9/
4))))/(8*(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b
^2 - 10*A*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(
3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)*((x^(1/2)*(A^2*c
^2 + 25*B^2*b^2 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*
1i)/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4))))*(A*c - 5*B*b))/(4*
(-b)^(3/4)*c^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.200 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=261

$$\frac{(Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\,\mathrm{ta}}{4\sqrt{}}$$

```
[Out] -1/2*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)-1/8*(A*c+3*B*b)*arctan(1-c^(1/4)*2^(1
/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/8*(A*c+3*B*b)*arctan(1+c^(1/
4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/16*(A*c+3*B*b)*ln(b^(
1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)-1/1
6*(A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)
/c^(7/4)*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\,\mathrm{ta}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*x^(3/2))/(2*b*c*(b + c*x^2)) - ((3*b*B + A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)) + ((3*b*B + A*c
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)
) + ((3*b*B + A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(8*Sqrt[2]*b^(5/4)*c^(7/4)) - ((3*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{2bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)} - \frac{(3bB+Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4bc^{3/2}} + \frac{(3bB+Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}\ldots}{}\right.}{4bc^{3/2}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)} + \frac{(3bB+Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc^2} + \frac{(3bB+Ac)\,\text{Subst}\ldots}{}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)} + \frac{(3bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{7/4}} - \frac{(3bB+Ac)\log\left(\sqrt{b}\ldots\right.}{8\sqrt{\ldots}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\left(b+cx^2\right)} - \frac{(3bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{7/4}}$$

Mathematica [C] time = 0.18, size = 95, normalized size = 0.36

$$\frac{2x^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^2c} + \frac{B\left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)\right)}{\sqrt[4]{-b}\,c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + ArcTanh[(b*c^(1/4)*Sqrt[x])/(-b)
^(5/4)]))/((-b)^(1/4)*c^(7/4)) + (2*(-(b*B) + A*c)*x^(3/2)*Hypergeometric2F
1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^2*c)
```

fricas [B] time = 1.03, size = 912, normalized size = 3.49

$$-\frac{4\,(Bb-Ac)x^{\frac{3}{2}} + 4\left(bc^2x^2+b^2c\right)\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(729\,B^6b^6+1458\,AB^5b^5c+\ldots\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*b - A*c)*x^(3/2) + 4*(b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B
^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*
arctan((sqrt((729*B^6*b^6 + 1458*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 + 540*A
^3*B^3*b^3*c^3 + 135*A^4*B^2*b^2*c^4 + 18*A^5*B*b*c^5 + A^6*c^6)*x - (81*B^
```

```
4*b^7*c^3 + 108*A*B^3*b^6*c^4 + 54*A^2*B^2*b^5*c^5 + 12*A^3*B*b^4*c^6 + A^4
*b^3*c^7)*sqrt(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3
*B*b*c^3 + A^4*c^4)/(b^5*c^7)))*b*c^2*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*
A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4) - (27*B^3*b^4*
c^2 + 27*A*B^2*b^3*c^3 + 9*A^2*B*b^2*c^4 + A^3*b*c^5)*sqrt(x)*(-(81*B^4*b^4
 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^
7))^(1/4))/(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*
c^3 + A^4*c^4)) - (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*
A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*log(b^4*c^5*(-
(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c
^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c
^3)*sqrt(x)) + (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2
*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-(8
1*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4
)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c^3
)*sqrt(x)))/(b*c^2*x^2 + b^2*c)
```

giac [A] time = 0.21, size = 273, normalized size = 1.05

$$-\frac{Bbx^{\frac{3}{2}} - Acx^{\frac{3}{2}}}{2\left(cx^2+b\right)bc} + \frac{\sqrt{2}\left(3\left(bc^3\right)^{\frac{3}{4}}Bb + \left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4} + \frac{\sqrt{2}\left(3\left(bc^3\right)^{\frac{3}{4}}Bb + \left(bc^3\right)^{\frac{3}{4}}Ac\right)\operatorname{arctar}}{8\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*b*c) + 1/8*sqrt(2)*(3*(b*c^3)
^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2
*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) + 1/8*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^2*c^4) - 1/16*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*lo
g(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 1/16*sqrt(2)*(3*
(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^4)
```

maple [A] time = 0.06, size = 305, normalized size = 1.17

$$\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc} + \frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc} + \frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}bc} + \frac{3\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(3/2)/(c*x^2+b)+1/8/b/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/
4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))
)+1/8/b/c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/c
^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/16/c^2/(b/
c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^
(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+3/8/c^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)+1)
```

maxima [A] time = 3.11, size = 217, normalized size = 0.83

$$-\frac{(Bb-Ac)x^{\frac{3}{2}}}{2\left(bc^2x^2+b^2c\right)}+\frac{(3\,Bb+Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{\cdot}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*x^(3/2)/(b*c^2*x^2 + b^2*c) + 1/16*(3*B*b + A*c)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
 + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^
(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c)
```

mupad [B] time = 0.23, size = 91, normalized size = 0.35

$$\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(A\,c+3\,B\,b\right)}{4\,{(-b)}^{5/4}\,c^{7/4}}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(A\,c+3\,B\,b\right)}{4\,{(-b)}^{5/4}\,c^{7/4}}+\frac{x^{3/2}\,\left(A\,c-B\,b\right)}{2\,b\,c\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4))
- (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4))
 + (x^(3/2)*(A*c - B*b))/(2*b*c*(b + c*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.201 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{(3Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{(3Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{(3Ac+bB)\tan}{4\sqrt{2}}$$

```
[Out] -1/8*(3*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)*
2^(1/2)+1/8*(3*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c
^(5/4)*2^(1/2)-1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2
)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)+1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)-1/2*(-A*c+B*b)*x^(1/
2)/b/c/(c*x^2+b)
```

Rubi [A] time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1584, 457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(3Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{(3Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{(3Ac+bB)\tan}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*Sqrt[x])/(2*b*c*(b + c*x^2)) - ((b*B + 3*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)
) - ((b*B + 3*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(8*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{2bc}$$
$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)} + \frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{bc}$$
$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)} + \frac{(bB+3Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}c} + \frac{(bB+3Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{}}{b+c}\right.}{4b^{3/2}c}$$
$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)} + \frac{(bB+3Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{3/2}c^{3/2}} + \frac{(bB+3Ac)\,\text{Subst}\left(\int\right.}{8b^{3}}$$
$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)} - \frac{(bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\log\left(\sqrt{b}+\right.}{8\sqrt{2}b}$$
$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\left(b+cx^2\right)} - \frac{(bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} -$$

Mathematica [A] time = 0.37, size = 203, normalized size = 0.78

$$\frac{\frac{(3Ac+bB)\left(8b^{3/4}\sqrt[4]{c}\sqrt{x}-3\sqrt{2}\left(b+cx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+\right.\right.\right.}{b^{7/4}\sqrt[4]{c}}}{48c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-32*B*Sqrt[x] + ((b*B + 3*A*c)*(8*b^(3/4)*c^(1/4)*Sqrt[x] - 3*Sqrt[2]*(b +
 c*x^2)*(2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 2*ArcTan[1 + (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])))/(b^(7/4)*c^(1/4)))/(48*c*(b + c*x^2))
```

fricas [B] time = 0.98, size = 717, normalized size = 2.75

$$\frac{4\left(bc^2x^2+b^2c\right)\left(-\frac{B^4b^4+12\,AB^3b^3c+54\,A^2B^2b^2c^2+108\,A^3Bbc^3+81\,A^4c^4}{b^7c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4c^2\sqrt{-\frac{B^4b^4+12\,AB^3b^3c+54\,A^2B^2b^2c^2+108\,A^3Bbc^3+81\,A^4c}{b^7c^5}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2
 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*arctan((sqrt(b^4*c^2*sqrt
```

```
(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4
*c^4)/(b^7*c^5)) + (B^2*b^2 + 6*A*B*b*c + 9*A^2*c^2)*x)*b^5*c^4*(-(B^4*b^4
+ 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*
c^5))^(3/4) - (B*b^6*c^4 + 3*A*b^5*c^5)*sqrt(x)*(-(B^4*b^4 + 12*A*B^3*b^3*c
 + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(3/4))/(B^
4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)
) + (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 +
108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(b^2*c*(-(B^4*b^4 + 12*A*
B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(
1/4) + (B*b + 3*A*c)*sqrt(x)) - (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b
^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*
log(-b^2*c*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c
^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - 4*(B*b - A*c)*
sqrt(x))/(b*c^2*x^2 + b^2*c)
```

giac [A] time = 0.21, size = 273, normalized size = 1.05

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb+3\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{2}c^{2}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb+3\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{8\,b^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/8*sqrt(2)*((b*c^
3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c
^2) - 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) - 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b*c)
```

maple [A] time = 0.06, size = 305, normalized size = 1.17

$$\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16b^{2}}+\frac{\left(\frac{b}{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(1/2)/(c*x^2+b)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b
/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)
^(1/2)))+3/8/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1
)+3/8/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+1/16/
b/c*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/8/b/c*(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/b/c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.11, size = 241, normalized size = 0.92

$$-\frac{(Bb-Ac)\sqrt{x}}{2\left(bc^2x^2+b^2c\right)}+\frac{\frac{2\sqrt{2}(Bb+3Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb+3Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb+3Ac)\log\left(\cdots\right.}{b\cdots}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*sqrt(x)/(b*c^2*x^2 + b^2*c) + 1/16*(2*sqrt(2)*(B*b + 3*A*c
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 3*A*c)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 3*A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(B*b + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(3/4)*c^(1/4)))/(b*c)
```

mupad [B] time = 0.33, size = 750, normalized size = 2.87

$$\frac{\operatorname{atan}\left(\frac{\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}-\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)1i}{8(-b)^{7/4}c^{5/4}}\right)}{8(-b)^{7/4}c^{5/4}}+\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}+\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)1i}{8(-b)^{7/4}c^{5/4}}\right)}{8(-b)^{7/4}c^{5/4}}}{\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}-\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)1i}{8(-b)^{7/4}c^{5/4}}\right)1i}{8(-b)^{7/4}c^{5/4}}-\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}+\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)1i}{8(-b)^{7/4}c^{5/4}}\right)1i}{8(-b)^{7/4}c^{5/4}}}\cdots\right.}{4(-b)^{7/4}c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2
- ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b
)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*
b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5/4))
)*1i)/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^
2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7
/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) - ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3
 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(
8*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4))))*(3*A*c + B*b)*1i)/(4*(-b)^
(7/4)*c^(5/4)) + (atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6
*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*
c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B
^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8
*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b)*((x^(1/2)*(9
*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b
*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b)^(7/4)*c^(5/4)) - ((3*A*c + B*
b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(2
4*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b)^(7/4)*c^(5/4))
))*(3*A*c + B*b))/(4*(-b)^(7/4)*c^(5/4)) + (x^(1/2)*(A*c - B*b))/(2*b*c*(b
+ c*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)

[Out] Timed out
```

3.202 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=284

$$\frac{(bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\tan^{-1}}{4\sqrt{2}b^{9}}$$

```
[Out] -1/8*(-5*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4)
*2^(1/2)+1/8*(-5*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)
/c^(3/4)*2^(1/2)+1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)-1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)+1/2*(-5*A*c+B*b)
/b^2/c/x^(1/2)+1/2*(A*c-B*b)/b/c/(c*x^2+b)/x^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\tan^{-1}}{4\sqrt{2}b^{9}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - 5*A*c)/(2*b^2*c*Sqrt[x]) - (b*B - A*c)/(2*b*c*Sqrt[x]*(b + c*x^2)) -
 ((b*B - 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b
^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(4*Sqrt[2]*b^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)*c^(3/4)) - ((b*B - 5*
A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*b^(9/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}+\frac{\left(-\frac{bB}{2}+\frac{5Ac}{2}\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$=\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}+\frac{(bB-5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$=\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}+\frac{(bB-5Ac)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b^2}$$

$$=\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}-\frac{(bB-5Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^2\sqrt{c}}+\frac{(bB-5Ac)\,\mathrm{S}}{}$$

$$=\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}+\frac{(bB-5Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^2c}+\frac{(bB-5}{}$$

$$=\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}+\frac{(bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{(bB-5A}{}$$

$$=\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\left(b+cx^2\right)}-\frac{(bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{(bB-5Ac)\tan^{-1}\left(1+\right.}{4\sqrt{2}\,b^{9/4}c^{3/}}$$

Mathematica [C] time = 0.32, size = 117, normalized size = 0.41

$$\frac{2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)+3A\left((-b)^{3/4}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)-(-b)^{3/4}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)-\frac{2b}{\sqrt{x}}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*A*((-2*b)/Sqrt[x] + (-b)^(3/4)*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/
4)] - (-b)^(3/4)*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)]) + 2*(b*B -
A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^3)
```

fricas [B] time = 1.16, size = 920, normalized size = 3.24

$$\frac{4\left(b^2cx^3+b^3x\right)\left(-\frac{B^4b^4-20\,AB^3b^3c+150\,A^2B^2b^2c^2-500\,A^3Bbc^3+625\,A^4c^4}{b^9c^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^6-30\,AB^5b^5c+375\,A^2B^4b^4c^2-2500\,A^3B^3b^3c^3+\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^
2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*arctan((sqrt((B^6*b^6 -
```

```
 30*A*B^5*b^5*c + 375*A^2*B^4*b^4*c^2 - 2500*A^3*B^3*b^3*c^3 + 9375*A^4*B^2
*b^2*c^4 - 18750*A^5*B*b*c^5 + 15625*A^6*c^6)*x - (B^4*b^9*c - 20*A*B^3*b^8
*c^2 + 150*A^2*B^2*b^7*c^3 - 500*A^3*B*b^6*c^4 + 625*A^4*b^5*c^5)*sqrt(-(B^
4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^
4)/(b^9*c^3)))*b^2*c*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 50
0*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4) + (B^3*b^5*c - 15*A*B^2*b^4*c
^2 + 75*A^2*B*b^3*c^3 - 125*A^3*b^2*c^4)*sqrt(x)*(-(B^4*b^4 - 20*A*B^3*b^3*
c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4))/
(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4
*c^4)) - (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*
c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(b^7*c^2*(-(B^4*b^
4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(
b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*c^3)
*sqrt(x)) + (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b
^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-(B^
4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^
4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*
c^3)*sqrt(x)) + 4*((B*b - 5*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^3 + b^3*x)
```

giac [A] time = 0.21, size = 278, normalized size = 0.98

$$\frac{Bbx^2 - 5\,Acx^2 - 4\,Ab}{2\left(cx^{\frac{5}{2}} + b\sqrt{x}\right)b^2} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}} Bb - 5\left(bc^3\right)^{\frac{3}{4}} Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}} Bb - 5\left(bc^3\right)^{\frac{3}{4}} A \cdots\right.}{8 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^2 - 5*A*c*x^2 - 4*A*b)/((c*x^(5/2) + b*sqrt(x))*b^2) + 1/8*sqrt(
2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) + 1/8*sqrt(2)*((b*c^3)^(3/4)*
B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c^3) - 1/16*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^
(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 1/1
6*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

maple [A] time = 0.06, size = 323, normalized size = 1.14

$$-\frac{Acx^{\frac{3}{2}}}{2\left(cx^2+b\right)b^2} + \frac{Bx^{\frac{3}{2}}}{2\left(cx^2+b\right)b} - \frac{5\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}} b^2} - \frac{5\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}} b^2} - \frac{5\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{\cdots}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{\cdots}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}} b^2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*x^(3/2)/(c*x^2+b)*A*c+1/2/b*x^(3/2)/(c*x^2+b)*B-5/16/b^2/(b/c)^(1/
4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*
2^(1/2)*x^(1/2)+(b/c)^(1/2)))-5/8/b^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)+1)-5/8/b^2/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)-1)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/8/b/c/(b/
```

```
c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/b/c/(b/c)^(1/4
)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2*A/b^2/x^(1/2)
```

maxima [A] time = 3.04, size = 222, normalized size = 0.78

$$\frac{(Bb-5\,Ac)x^2-4\,Ab}{2\left(b^2cx^{\frac{5}{2}}+b^3\sqrt{x}\right)}+\frac{(Bb-5\,Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\cdots\right.}{b^{\frac{1}{4}}c\cdots}\right.}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 5*A*c)*x^2 - 4*A*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) + 1/16*(B*b -
 5*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^
2
```

mupad [B] time = 0.23, size = 104, normalized size = 0.37

$$\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)(5\,A\,c-B\,b)}{4\,(-b)^{9/4}\,c^{3/4}}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)(5\,A\,c-B\,b)}{4\,(-b)^{9/4}\,c^{3/4}}-\frac{\frac{2\,A}{b}+\frac{x^2\,(5\,A\,c-B\,b)}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4))
- (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4))
 - ((2*A)/b + (x^2*(5*A*c - B*b))/(2*b^2))/(b*x^(1/2) + c*x^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.203 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{(3bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}-\frac{(3bB-7A}{}$$

```
[Out] 1/6*(-7*A*c+3*B*b)/b^2/c/x^(3/2)+1/2*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)-1/8*(-
7*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2^(
1/2)+1/8*(-7*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/
c^(1/4)*2^(1/2)-1/16*(-7*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)+1/16*(-7*A*c+3*B*b)*ln(b^(1/2)+x*c^
(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{(3bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*b*B - 7*A*c)/(6*b^2*c*x^(3/2)) - (b*B - A*c)/(2*b*c*x^(3/2)*(b + c*x^2))
 - ((3*b*B - 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(11/4)*c^(1/4)) + ((3*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(4*Sqrt[2]*b^(11/4)*c^(1/4)) - ((3*b*B - 7*A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11/4)*c^(1/4)) +
 ((3*b*B - 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}+\frac{\left(-\frac{3bB}{2}+\frac{7Ac}{2}\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}+\frac{(3bB-7Ac)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}+\frac{(3bB-7Ac)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b^2}$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}+\frac{(3bB-7Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{5/2}}+\frac{(3bB-$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}+\frac{(3bB-7Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{5/2}\sqrt{c}}+\frac{(3$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}-\frac{(3bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{(3b$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}-\frac{(3bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\tan}{4\sqrt{2}\,}$$

Mathematica [A] time = 0.58, size = 355, normalized size = 1.23

$$-\frac{24Ab^{3/4}c\sqrt{x}}{b+cx^2}-\frac{32Ab^{3/4}}{x^{3/2}}+\frac{6\sqrt{2}(7Ac-3bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}}+21\sqrt{2}\,Ac^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-32*A*b^(3/4))/x^(3/2) + (24*b^(7/4)*B*Sqrt[x])/(b + c*x^2) - (24*A*b^(3/
4)*c*Sqrt[x])/(b + c*x^2) + (6*Sqrt[2]*(-3*b*B + 7*A*c)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (6*Sqrt[2]*(3*b*B - 7*A*c)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (9*Sqrt[2]*b*B*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + 21*Sqrt[2]*A*c^(3/
4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + (9*Sqrt[2]*
b*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) - 2
1*Sqrt[2]*A*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(48*b^(11/4))
```

fricas [B] time = 1.09, size = 741, normalized size = 2.56

$$-\frac{12\left(b^2cx^4+b^3x^2\right)\left(-\frac{81B^4b^4-756AB^3b^3c+2646A^2B^2b^2c^2-4116A^3Bbc^3+2401A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6\sqrt{-\frac{81B^4b^4-756AB^3b^3c+2646A^2B^2}{b^{11}}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*
B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*arctan((sqrt
(b^6*sqrt(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*
B*b*c^3 + 2401*A^4*c^4)/(b^11*c)) + (9*B^2*b^2 - 42*A*B*b*c + 49*A^2*c^2)*x
)*b^8*c*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B
*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(3/4) + (3*B*b^9*c - 7*A*b^8*c^2)*sqrt(x)*
(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 +
 2401*A^4*c^4)/(b^11*c))^(3/4))/(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^
2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)) + 3*(b^2*c*x^4 + b^3*x^2)*(-(
81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 24
01*A^4*c^4)/(b^11*c))^(1/4)*log(b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*
A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b
 - 7*A*c)*sqrt(x)) - 3*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*
c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)
*log(-b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3
*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)) - 4*((3
*B*b - 7*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^4 + b^3*x^2)
```

giac [A] time = 0.19, size = 283, normalized size = 0.98

$$\frac{\sqrt{2}\left(3\left(bc^3\right)^{\frac{1}{4}}Bb-7\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c}+\frac{\sqrt{2}\left(3\left(bc^3\right)^{\frac{1}{4}}Bb-7\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\cdots}\right.}{2\left(\frac{b}{c}\right)\cdots}\right.}{8\,b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/8*sqrt(2)*(3*(b*
c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b -
 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3
*c) - 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b^2) - 2/3*A/(b^2*x^(3/2))
```

maple [A] time = 0.06, size = 317, normalized size = 1.10

$$-\frac{Ac\sqrt{x}}{2\left(c\,x^2+b\right)b^2}+\frac{B\sqrt{x}}{2\left(c\,x^2+b\right)b}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^3}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^3}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*x^(1/2)/(c*x^2+b)*A*c+1/2/b*x^(1/2)/(c*x^2+b)*B-7/16/b^3*(b/c)^(1/
4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*
2^(1/2)*x^(1/2)+(b/c)^(1/2)))*c-7/8/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2
)/(b/c)^(1/4)*x^(1/2)+1)*c-7/8/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/
c)^(1/4)*x^(1/2)-1)*c+3/16/b^2*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1
```

```
/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+3/8/b
^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/b^2*(b/c
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3*A/b^2/x^(3/2)
```

maxima [A] time = 3.11, size = 251, normalized size = 0.87

$$\frac{(3\,Bb-7\,Ac)x^2-4\,Ab}{6\left(b^2cx^{\frac{7}{2}}+b^3x^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}(3\,Bb-7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(3\,Bb-7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(3\cdots}{}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((3*B*b - 7*A*c)*x^2 - 4*A*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) + 1/16*(2*s
qrt(2)*(3*B*b - 7*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqr
t(2)*(3*B*b - 7*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(3*B*b - 7*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b)
)/(b^(3/4)*c^(1/4)) - sqrt(2)*(3*B*b - 7*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*
sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2
```

mupad [B] time = 0.37, size = 859, normalized size = 2.97

$$-\frac{\frac{2\,A}{3\,b}+\frac{x^2\,(7\,A\,c-3\,B\,b)}{6\,b^2}}{b\,x^{3/2}+c\,x^{7/2}}-\frac{\mathrm{atan}\left(\frac{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)}{8\,(-b)^{11/4}\,c^{1/4}}\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}+\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\cdots\right.}{}}{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)}{8\,(-b)^{11/4}\,c^{1/4}}\right)}{8\,(-b)^{11/4}\,c^{1/4}}-\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\cdots\right.}{}}\right)\cdots}{4\,(-b)^{11/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((2*A)/(3*b) + (x^2*(7*A*c - 3*B*b))/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) - (
atan((((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*
A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^
(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2)*(1
568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1
792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4
)*c^(1/4)))/(((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3
- 1344*A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(
8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c - 3*B*b)*(x^(1/2
)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b
)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/
4)*c^(1/4))))*(7*A*c - 3*B*b)*1i)/(4*(-b)^(11/4)*c^(1/4)) - (atan((((7*A*c
- 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) -
 ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1
/4))))/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^
5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1792*A*b^9*c^4
- 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)))/((
(7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7
*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/
4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c - 3*B*b)*(x^(1/2)*(1568*
A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1792*
```

```
A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)
*c^(1/4))))*(7*A*c - 3*B*b))/(4*(-b)^(11/4)*c^(1/4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.204 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{\sqrt[4]{c}}{}$$

```
[Out] 1/10*(-9*A*c+5*B*b)/b^2/c/x^(5/2)+1/2*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)+1/8*c
^(1/4)*(-9*A*c+5*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^
(1/2)-1/8*c^(1/4)*(-9*A*c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(13/4)*2^(1/2)-1/16*c^(1/4)*(-9*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/16*c^(1/4)*(-9*A*c+5*B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/2*(9*A*c
-5*B*b)/b^3/x^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}\,b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*b*B - 9*A*c)/(10*b^2*c*x^(5/2)) - (5*b*B - 9*A*c)/(2*b^3*Sqrt[x]) - (b*B
 - A*c)/(2*b*c*x^(5/2)*(b + c*x^2)) + (c^(1/4)*(5*b*B - 9*A*c)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (c^(1/4)*(5*b*B -
 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4))
 - (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4)) + (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] +
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\left(-\frac{5bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{(5bB-9Ac)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(c(5bB-9Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(c(5bB-9Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{\right.}{2b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\left(\sqrt{c}\,(5bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,d\right.}{4b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(5bB-9Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx\right.}{8b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}\,b^{13/4}}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\sqrt[4]{c}\,(5bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}} -$$

Mathematica [C] time = 0.64, size = 151, normalized size = 0.49

$$\frac{2cx^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^4} + \frac{4Ac-2bB}{b^3\sqrt{x}} - \frac{2A}{5b^2x^{5/2}} + \frac{\sqrt[4]{c}\,(bB-2Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{13/4}} + \frac{b\sqrt[4]{c}\,(bB-2Ac)\tanh}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*A)/(5*b^2*x^(5/2)) + (-2*b*B + 4*A*c)/(b^3*Sqrt[x]) + (c^(1/4)*(b*B - 2
*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(13/4) + (b*c^(1/4)*(b*B -
 2*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) + (2*c*(-(b*B) +
 A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^4)
```

fricas [B] time = 1.03, size = 974, normalized size = 3.14

$$-\frac{20\left(b^3cx^5+b^4x^3\right)\left(-\frac{625\,B^4b^4c-4500\,AB^3b^3c^2+12150\,A^2B^2b^2c^3-14580\,A^3Bbc^4+6561\,A^4c^5}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(15625\,B^6b^6c^2-168750\,AB^5\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 121
50*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*arctan((
sqrt((15625*B^6*b^6*c^2 - 168750*A*B^5*b^5*c^3 + 759375*A^2*B^4*b^4*c^4 - 1
822500*A^3*B^3*b^3*c^5 + 2460375*A^4*B^2*b^2*c^6 - 1771470*A^5*B*b*c^7 + 53
1441*A^6*c^8)*x - (625*B^4*b^11*c - 4500*A*B^3*b^10*c^2 + 12150*A^2*B^2*b^9
*c^3 - 14580*A^3*B*b^8*c^4 + 6561*A^4*b^7*c^5)*sqrt(-(625*B^4*b^4*c - 4500*
A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b
^13))*b^3*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 1
4580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4) + (125*B^3*b^6*c - 675*A*B^2*b
^5*c^2 + 1215*A^2*B*b^4*c^3 - 729*A^3*b^3*c^4)*sqrt(x)*(-(625*B^4*b^4*c - 4
500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^
5)/b^13)^(1/4))/(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3
 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)) - 5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*
b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 65
61*A^4*c^5)/b^13)^(1/4)*log(b^10*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12
150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3/4) - (125*
B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) +
5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B
^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(-b^10*(-(625
*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4
 + 6561*A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^
2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) + 4*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2
+ 4*(5*B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

giac [A] time = 0.21, size = 303, normalized size = 0.98

$$-\frac{Bbcx^{\frac{3}{2}} - Ac^{2}x^{\frac{3}{2}}}{2\left(cx^{2}+b\right)b^{3}} - \frac{\sqrt{2}\left(5\left(bc^{3}\right)^{\frac{3}{4}}Bb - 9\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^{4}c^{2}} - \frac{\sqrt{2}\left(5\left(bc^{3}\right)^{\frac{3}{4}}Bb - 9\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)a}{8b^{4}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c*x^(3/2) - A*c^2*x^(3/2))/((c*x^2 + b)*b^3) - 1/8*sqrt(2)*(5*(b*
c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 1/8*sqrt(2)*(5*(b*c^3)^(3/4)*B*b -
 9*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))
/(b/c)^(1/4))/(b^4*c^2) + 1/16*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/
4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*s
qrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 2/5*(5*B*b*x^2 - 10*A*c*x^2 + A*b)/(b
^3*x^(5/2))
```

maple [A] time = 0.06, size = 339, normalized size = 1.09

$$\frac{Ac^{2}x^{\frac{3}{2}}}{2\left(cx^{2}+b\right)b^{3}} - \frac{Bcx^{\frac{3}{2}}}{2\left(cx^{2}+b\right)b^{2}} + \frac{9\sqrt{2}\,Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}} + \frac{9\sqrt{2}\,Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}} + \frac{9\sqrt{2}\,Ac\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x^(3/2)/(c*x^2+b)*A-1/2/b^2*c*x^(3/2)/(c*x^2+b)*B+9/16/b^3*c/(b
/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)
^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1
/2)/(b/c)^(1/4)*x^(1/2)-1)-5/16/b^2/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)
*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-
5/8/b^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/b^2
/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b^2*A/x^(5
/2)+4/b^3/x^(1/2)*A*c-2/b^2/x^(1/2)*B
```

maxima [A] time = 3.03, size = 250, normalized size = 0.81

$$-\frac{5\left(5\,Bbc-9\,Ac^2\right)x^4+4\,Ab^2+4\left(5\,Bb^2-9\,Abc\right)x^2}{10\left(b^3cx^{\frac{9}{2}}+b^4x^{\frac{5}{2}}\right)}-\frac{\left(5\,Bbc-9\,Ac^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\text{arcta}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/10*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5*B*b^2 - 9*A*b*c)*x^2)/(b^
3*c*x^(9/2) + b^4*x^(5/2)) - 1/16*(5*B*b*c - 9*A*c^2)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)
))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*
sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x
) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

mupad [B] time = 0.25, size = 121, normalized size = 0.39

$$\frac{\frac{2x^2(9Ac-5Bb)}{5b^2}-\frac{2A}{5b}+\frac{cx^4(9Ac-5Bb)}{2b^3}}{b\,x^{5/2}+c\,x^{9/2}}+\frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(9\,Ac-5\,Bb)}{4\,b^{13/4}}-\frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)(9\,Ac-\cdots}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((2*x^2*(9*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (c*x^4*(9*A*c - 5*B*b))/(2
*b^3))/(b*x^(5/2) + c*x^(9/2)) + ((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1
/4))*(9*A*c - 5*B*b))/(4*b^(13/4)) - ((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))
/b^(1/4))*(9*A*c - 5*B*b))/(4*b^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.205 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=310

$$\frac{c^{3/4}(7bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{c^{3/4}(7}{}$$

```
[Out] 1/14*(-11*A*c+7*B*b)/b^2/c/x^(7/2)+1/6*(11*A*c-7*B*b)/b^3/x^(3/2)+1/2*(A*c-
B*b)/b/c/x^(7/2)/(c*x^2+b)+1/8*c^(3/4)*(-11*A*c+7*B*b)*arctan(1-c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-1/8*c^(3/4)*(-11*A*c+7*B*b)*arctan(1
+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+1/16*c^(3/4)*(-11*A*c+7*
B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
-1/16*c^(3/4)*(-11*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}(7bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{c^{3/4}(7}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (7*b*B - 11*A*c)/(14*b^2*c*x^(7/2)) - (7*b*B - 11*A*c)/(6*b^3*x^(3/2)) - (b
*B - A*c)/(2*b*c*x^(7/2)*(b + c*x^2)) + (c^(3/4)*(7*b*B - 11*A*c)*ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*
B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15
/4)) + (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{\left(-\frac{7bB}{2}+\frac{11Ac}{2}\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{(7bB-11Ac)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x,\right.}{2b^3}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx\right.}{4b^{7/2}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{\left(\sqrt{c}\,(7bB-11Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{8b^{7/2}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{}\right.}{8\sqrt{2}\,b^{15/4}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{c^{3/4}(7bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}$$

Mathematica [A] time = 0.73, size = 385, normalized size = 1.24

$$\frac{\frac{168Ab^{3/4}c^2\sqrt{x}}{b+cx^2}+\frac{448Ab^{3/4}c}{x^{3/2}}-\frac{96Ab^{7/4}}{x^{7/2}}+42\sqrt{2}\,c^{3/4}(7bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+42\sqrt{2}\,c^{3/4}(11Ac-7bB)\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-96*A*b^(7/4))/x^(7/2) - (224*b^(7/4)*B)/x^(3/2) + (448*A*b^(3/4)*c)/x^(3
/2) - (168*b^(7/4)*B*c*Sqrt[x])/(b + c*x^2) + (168*A*b^(3/4)*c^2*Sqrt[x])/(
b + c*x^2) + 42*Sqrt[2]*c^(3/4)*(7*b*B - 11*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)] + 42*Sqrt[2]*c^(3/4)*(-7*b*B + 11*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 147*Sqrt[2]*b*B*c^(3/4)*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 231*Sqrt[2]*A*c^(7/4)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 147*Sqrt[2]*b*B*c^(3/4)*L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 231*Sqrt[2]*A*c
^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(336*b^(
15/4))
```

fricas [B] time = 1.12, size = 795, normalized size = 2.56

$$\frac{84\left(b^3cx^6+b^4x^4\right)\left(-\frac{2401\,B^4b^4c^3-15092\,AB^3b^3c^4+35574\,A^2B^2b^2c^5-37268\,A^3Bbc^6+14641\,A^4c^7}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^8\sqrt{-\frac{2401\,B^4b^4c^3-15092\,A}{}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(84*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 +
 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4)*arc
tan((sqrt(b^8*sqrt(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2
*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15) + (49*B^2*b^2*c^2 - 154
*A*B*b*c^3 + 121*A^2*c^4)*x)*b^11*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4
 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(3/4) +
 (7*B*b^12*c - 11*A*b^11*c^2)*sqrt(x)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3
*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(3/
4))/(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268
*A^3*B*b*c^6 + 14641*A^4*c^7)) + 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c
^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 1464
1*A^4*c^7)/b^15)^(1/4)*log(b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 +
35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7
*B*b*c - 11*A*c^2)*sqrt(x)) - 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3
- 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A
^4*c^7)/b^15)^(1/4)*log(-b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35
574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B
*b*c - 11*A*c^2)*sqrt(x)) - 4*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7
*B*b^2 - 11*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

giac [A] time = 0.25, size = 292, normalized size = 0.94

$$-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}-\frac{\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\right.}{}\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/8*sqrt(2)*(7*(b*c^
3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(
b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1/
16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x
)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/2*(B*b*c*sqrt(x) - A*c^2*sqrt(x))/((
c*x^2 + b)*b^3) - 2/21*(7*B*b*x^2 - 14*A*c*x^2 + 3*A*b)/(b^3*x^(7/2))
```

maple [A] time = 0.06, size = 348, normalized size = 1.12

$$\frac{A\,c^2\sqrt{x}}{2\left(c\,x^2+b\right)b^3}-\frac{Bc\sqrt{x}}{2\left(c\,x^2+b\right)b^2}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^4}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^4}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] 1/2/b^3*c^2*x^(1/2)/(c*x^2+b)*A-1/2/b^2*c*x^(1/2)/(c*x^2+b)*B+11/8/b^4*c^2*
(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+11/8/b^4*c^2*(b
/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+11/16/b^4*c^2*(b/
c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^
(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*B*arctan(
2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)+1)-7/16/b^3*c*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4
)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))
-2/7*A/b^2/x^(7/2)+4/3/b^3/x^(3/2)*A*c-2/3/b^2/x^(3/2)*B
```

maxima [A] time = 3.12, size = 286, normalized size = 0.92

$$-\frac{7\left(7\,Bbc-11\,Ac^{2}\right)x^{4}+12\,Ab^{2}+4\left(7\,Bb^{2}-11\,Abc\right)x^{2}}{42\left(b^{3}cx^{\frac{11}{2}}+b^{4}x^{\frac{7}{2}}\right)}-\frac{\frac{2\sqrt{2}\left(7\,Bbc-11\,Ac^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\left(7\,Bbc-11\,A\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/42*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7*B*b^2 - 11*A*b*c)*x^2)/
(b^3*c*x^(11/2) + b^4*x^(7/2)) - 1/16*(2*sqrt(2)*(7*B*b*c - 11*A*c^2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 11*A*c^2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 11*A*c^2)*
log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)
) - sqrt(2)*(7*B*b*c - 11*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

mupad [B] time = 0.40, size = 595, normalized size = 1.92

$$\frac{\frac{2\,x^{2}\,(11\,A\,c-7\,B\,b)}{21\,b^{2}}-\frac{2\,A}{7\,b}+\frac{c\,x^{4}\,(11\,A\,c-7\,B\,b)}{6\,b^{3}}}{b\,x^{7/2}+c\,x^{11/2}}+\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(3872\,A^{2}\,b^{9}\,c^{7}-4928\,A\,B\,b^{10}\,c^{6}+1568\,B^{2}\,b^{11}\,c^{5}\right)-\frac{(-c)^{3/4}\,(11\,A\,c-7\cdots}{\cdots}\right)}{8\,b^{15/4}}\cdots}{\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(3872\,A^{2}\,b^{9}\,c^{7}-4928\,A\,B\,b^{10}\,c^{6}+1568\,B^{2}\,b^{11}\,c^{5}\right)-\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\cdots}{\cdots}\right)}{8\,b^{15/4}}\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*x^2*(11*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (c*x^4*(11*A*c - 7*B*b))
/(6*b^3))/(b*x^(7/2) + c*x^(11/2)) + ((-c)^(3/4)*atan((((-c)^(3/4)*(11*A*c
- 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6
) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8
*b^(15/4))))/(8*b^(15/4)) + ((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2
*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6) + ((-c)^(3/4)*(11*A*c - 7
*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^(15/4))))/(8*b^(15/4)))/
(((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^
5 - 4928*A*B*b^10*c^6) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 17
92*B*b^14*c^4)*1i)/(8*b^(15/4)))*1i)/(8*b^(15/4)) - ((-c)^(3/4)*(11*A*c - 7
*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6) +
```

```
((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^
(15/4)))*1i)/(8*b^(15/4))))*(11*A*c - 7*B*b))/(4*b^(15/4)) - ((-c)^(3/4)*at
an((A^3*c^8*x^(1/2)*1331i - B^3*b^3*c^5*x^(1/2)*343i - A^2*B*b*c^7*x^(1/2)*
2541i + A*B^2*b^2*c^6*x^(1/2)*1617i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(1331*A^3*c
 - 2541*A^2*B*b) + 1617*A*B^2*b^2) - 343*B^3*b^3)))*(11*A*c - 7*B*b)*1i)/(4
*b^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.206 $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=332

$$\frac{c^{5/4}(9bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9}{}$$

```
[Out] 1/18*(-13*A*c+9*B*b)/b^2/c/x^(9/2)+1/10*(13*A*c-9*B*b)/b^3/x^(5/2)+1/2*(A*c
-B*b)/b/c/x^(9/2)/(c*x^2+b)-1/8*c^(5/4)*(-13*A*c+9*B*b)*arctan(1-c^(1/4)*2^
(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/8*c^(5/4)*(-13*A*c+9*B*b)*arctan(
1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/16*c^(5/4)*(-13*A*c+9
*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2
)-1/16*c^(5/4)*(-13*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)
*x^(1/2))/b^(17/4)*2^(1/2)+1/2*c*(-13*A*c+9*B*b)/b^4/x^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(9bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}-\frac{c^{5/4}(9}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (9*b*B - 13*A*c)/(18*b^2*c*x^(9/2)) - (9*b*B - 13*A*c)/(10*b^3*x^(5/2)) + (
c*(9*b*B - 13*A*c))/(2*b^4*Sqrt[x]) - (b*B - A*c)/(2*b*c*x^(9/2)*(b + c*x^2
)) - (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/
4)) - (c^(5/4)*(9*b*B - 13*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(-\frac{9bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(9bB-13Ac)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{(c(9bB-13Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(c^2(9bB-13Ac)\right)\int}{4b^4}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(c^2(9bB-13Ac)\right)\text{S}}{}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{\left(c^{3/2}(9bB-13Ac)\right)}{}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(c(9bB-13Ac))\,\text{Su}}{}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{c^{5/4}(9bB-13Ac)\,\text{lo}}{}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{c^{5/4}(9bB-13Ac)\,\text{ta}}{4\sqrt{2}}$$

<u>Mathematica</u> [C] time = 0.59, size = 176, normalized size = 0.53

$$\frac{2c^2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^5} + \frac{2c(2bB-3Ac)}{b^4\sqrt{x}} - \frac{2(bB-2Ac)}{5b^3x^{5/2}} - \frac{2A}{9b^2x^{9/2}} + \frac{c^{5/4}(2bB-3Ac)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{17/4}} + \frac{c^{5/4}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-2*A)/(9*b^2*x^(9/2)) - (2*(b*B - 2*A*c))/(5*b^3*x^(5/2)) + (2*c*(2*b*B -
3*A*c))/(b^4*Sqrt[x]) + (c^(5/4)*(2*b*B - 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(
-b)^(1/4)])/(-b)^(17/4) + (c^(5/4)*(-2*b*B + 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x
])/(-b)^(1/4)])/(-b)^(17/4) + (2*c^2*(b*B - A*c)*x^(3/2)*Hypergeometric2F1[
3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^5)
```

fricas [B] time = 1.06, size = 1024, normalized size = 3.08

$$180\left(b^{4}cx^{7}+b^{5}x^{5}\right)\left(-\frac{6561\,B^{4}b^{4}c^{5}-37908\,AB^{3}b^{3}c^{6}+82134\,A^{2}B^{2}b^{2}c^{7}-79092\,A^{3}Bbc^{8}+28561\,A^{4}c^{9}}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{(531441\,B^{6}b^{6}c^{8}-460582\cdots}}{\cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6
+ 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*ar
ctan((sqrt((531441*B^6*b^6*c^8 - 4605822*A*B^5*b^5*c^9 + 16632135*A^2*B^4*b
^4*c^10 - 32032260*A^3*B^3*b^3*c^11 + 34701615*A^4*B^2*b^2*c^12 - 20049822*
A^5*B*b*c^13 + 4826809*A^6*c^14)*x - (6561*B^4*b^13*c^5 - 37908*A*B^3*b^12*
c^6 + 82134*A^2*B^2*b^11*c^7 - 79092*A^3*B*b^10*c^8 + 28561*A^4*b^9*c^9)*sq
rt(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092
*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17))*b^4*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*
b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^
(1/4) + (729*B^3*b^7*c^4 - 3159*A*B^2*b^6*c^5 + 4563*A^2*B*b^5*c^6 - 2197*A
^3*b^4*c^7)*sqrt(x)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B
^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4))/(6561*B^4*b^4*
c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 285
61*A^4*c^9)) - 45*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b
^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(
1/4)*log(b^13*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2
*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 -
3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 45*(b^4*c*
x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^
2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*log(-b^13*(-(6561*B^
4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8
 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 456
3*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 4*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 +
 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*A*b^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)
*sqrt(x))/(b^4*c*x^7 + b^5*x^5)
```

giac [A] time = 0.21, size = 328, normalized size = 0.99

$$\frac{\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{3}{4}}Bb-13\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{5}c}+\frac{\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{3}{4}}Bb-13\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\cdots\right.}{\cdots}\right)}{8\,b^{5}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) + 1/8*sqrt(2)*(9*(b
*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b
 - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b^5*c) + 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) + 1/2*(B*b*c^2*x^(3/2) -
A*c^3*x^(3/2))/((c*x^2 + b)*b^4) + 2/45*(90*B*b*c*x^4 - 135*A*c^2*x^4 - 9*B
*b^2*x^2 + 18*A*b*c*x^2 - 5*A*b^2)/(b^4*x^(9/2))
```

maple [A] time = 0.07, size = 372, normalized size = 1.12

$$-\frac{A\,c^3 x^{\frac{3}{2}}}{2\left(c\,x^2+b\right)b^4}+\frac{B\,c^2 x^{\frac{3}{2}}}{2\left(c\,x^2+b\right)b^3}-\frac{13\sqrt{2}\,A\,c^2 \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}} b^4}-\frac{13\sqrt{2}\,A\,c^2 \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}} b^4}-\frac{13\sqrt{2}\,A\,c^2 \ln}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x^(3/2)/(c*x^2+b)*A+1/2/b^3*c^2*x^(3/2)/(c*x^2+b)*B-13/16/b^4*
c^2/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)
*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+9/16/b^3*c/(b/c)^(1/4)*2^(1/2)*B*ln(
(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+
(b/c)^(1/2)))+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^
(1/2)+1)+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
-1)-2/9*A/b^2/x^(9/2)+4/5/b^3/x^(5/2)*A*c-2/5/b^2/x^(5/2)*B-6*c^2/b^4/x^(1/
2)*A+4*c/b^3/x^(1/2)*B
```

maxima [A] time = 3.01, size = 276, normalized size = 0.83

$$\frac{45\left(9\,Bbc^2-13\,Ac^3\right)x^6+36\left(9\,Bb^2c-13\,Abc^2\right)x^4-20\,Ab^3-4\left(9\,Bb^3-13\,Ab^2c\right)x^2}{90\left(b^4cx^{\frac{13}{2}}+b^5x^{\frac{9}{2}}\right)}+\frac{\left(9\,Bbc^2-13\,Ac^3\right)\left(\frac{2\sqrt{2}\,\mathrm{arcta}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/90*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 + 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*
A*b^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c*x^(13/2) + b^5*x^(9/2)) + 1/16
*(9*B*b*c^2 - 13*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1
/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt
(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*lo
g(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
+ sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1
/4)*c^(3/4)))/b^4
```

mupad [B] time = 0.20, size = 142, normalized size = 0.43

$$\frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)(13\,A\,c-9\,B\,b)}{4\,b^{17/4}}-\frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(13\,A\,c-9\,B\,b)}{45\,b^2}+\frac{c^2\,x^6\,(13\,A\,c-9\,B\,b)}{2\,b^4}+\frac{2\,c\,x^4\,(13\,A\,c-9\,B\,b)}{5\,b^3}}{b\,x^{9/2}+c\,x^{13/2}}-\frac{(-c)^{5/4}\,\mathrm{atanh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))/(4*b^(17/4
)) - ((2*A)/(9*b) - (2*x^2*(13*A*c - 9*B*b))/(45*b^2) + (c^2*x^6*(13*A*c -
```

```
9*B*b))/(2*b^4) + (2*c*x^4*(13*A*c - 9*B*b))/(5*b^3))/(b*x^(9/2) + c*x^(13/
2)) - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))/(4*
b^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.207 $\int \frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{9\sqrt[4]{b}\,(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,c^{17/4}}+\frac{9\sqrt[4]{b}\,(13bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,c^{17/4}}-9$$

```
[Out] 9/80*(-5*A*c+13*B*b)*x^(5/2)/b/c^3-1/4*(-A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)^2-
1/16*(-5*A*c+13*B*b)*x^(9/2)/b/c^2/(c*x^2+b)-9/64*b^(1/4)*(-5*A*c+13*B*b)*a
rctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+9/64*b^(1/4)*(-5*
A*c+13*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-9/12
8*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1
/2))/c^(17/4)*2^(1/2)+9/128*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)+b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)*2^(1/2)-9/16*(-5*A*c+13*B*b)*x^(1/2
)/c^4
```

Rubi [A] time = 0.28, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{9/2}(13bB-5Ac)}{16bc^2\left(b+cx^2\right)}+\frac{9x^{5/2}(13bB-5Ac)}{80bc^3}-\frac{9\sqrt{x}\,(13bB-5Ac)}{16c^4}-\frac{9\sqrt[4]{b}\,(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\right.}{64\sqrt{2}\,c^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-9*(13*b*B - 5*A*c)*Sqrt[x])/(16*c^4) + (9*(13*b*B - 5*A*c)*x^(5/2))/(80*b
*c^3) - ((b*B - A*c)*x^(13/2))/(4*b*c*(b + c*x^2)^2) - ((13*b*B - 5*A*c)*x^
(9/2))/(16*b*c^2*(b + c*x^2)) - (9*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B
- 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4
)) - (9*b^(1/4)*(13*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B - 5*A*c)*Log[S
qrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{23/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{13bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{11/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}+\frac{(9(13bB-5Ac))\int \frac{x^{7/2}}{b+cx^2}\,dx}{32bc^2}$$

$$=\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}-\frac{(9(13bB-5Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{32c^3}$$

$$=-\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}+\frac{(9b(1}{}$$

$$=-\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}+\frac{(9b(1}{}$$

$$=-\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}+\frac{\left(9\sqrt{b}\right.}{}$$

$$=-\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}+\frac{\left(9\sqrt{b}\right.}{}$$

$$=-\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}-\frac{9\sqrt[4]{b}\,(}{}$$

$$=-\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}-\frac{9\sqrt[4]{b}\,(}{}$$

Mathematica [A] time = 0.63, size = 435, normalized size = 1.27

$$-\frac{160Ab^2c^{5/4}\sqrt{x}}{\left(b+cx^2\right)^2}-90\sqrt{2}\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+90\sqrt{2}\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)+\frac{680}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3840*b*B*c^(1/4)*Sqrt[x] + 1280*A*c^(5/4)*Sqrt[x] + 256*B*c^(5/4)*x^(5/2)
 + (160*b^3*B*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (160*A*b^2*c^(5/4)*Sqrt[x])/
(b + c*x^2)^2 - (1000*b^2*B*c^(1/4)*Sqrt[x])/(b + c*x^2) + (680*A*b*c^(5/4)
```

```
*Sqrt[x])/(b + c*x^2) - 90*Sqrt[2]*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 90*Sqrt[2]*b^(1/4)*(13*b*B - 5*A*c)*ArcTan
[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 585*Sqrt[2]*b^(5/4)*B*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 225*Sqrt[2]*A*b^(1/4)*c*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 585*Sqrt[2]*b^(
5/4)*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 225*Sqr
t[2]*A*b^(1/4)*c*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(640*c^(17/4))
```

fricas [B] time = 1.01, size = 817, normalized size = 2.38

$$-\frac{180\left(c^{6}x^{4}+2\,bc^{5}x^{2}+b^{2}c^{4}\right)\left(-\frac{28561\,B^{4}b^{5}-43940\,AB^{3}b^{4}c+25350\,A^{2}B^{2}b^{3}c^{2}-6500\,A^{3}Bb^{2}c^{3}+625\,A^{4}bc^{4}}{c^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^{8}\sqrt{-\frac{28561\,B^{4}\cdots}{}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43940*A*B^
3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)
^(1/4)*arctan((sqrt(c^8*sqrt(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^
2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17) + (169*B^2*b^2 -
130*A*B*b*c + 25*A^2*c^2)*x)*c^13*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25
350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(3/4) + (13
*B*b*c^13 - 5*A*c^14)*sqrt(x)*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*
A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(3/4))/(28561*B
^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 6
25*A^4*b*c^4)) + 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43
940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^
4)/c^17)^(1/4)*log(9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B
^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b -
5*A*c)*sqrt(x)) - 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 4
3940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c
^4)/c^17)^(1/4)*log(-9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2
*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b
- 5*A*c)*sqrt(x)) - 4*(32*B*c^3*x^6 - 32*(13*B*b*c^2 - 5*A*c^3)*x^4 - 585*B
*b^3 + 225*A*b^2*c - 81*(13*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(x))/(c^6*x^4 + 2
*b*c^5*x^2 + b^2*c^4)
```

giac [A] time = 0.23, size = 321, normalized size = 0.94

$$\frac{9\sqrt{2}\left(13\,\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\,\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^{5}}+\frac{9\sqrt{2}\left(13\,\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\,\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\cdots\right.}{64\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 9/64*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/64*sqrt(2)*(13*(b*
c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 -
```

```
9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqr
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 1/16*(25*B*b^2*c*x^(5/2) - 17*A*b*c
^2*x^(5/2) + 21*B*b^3*sqrt(x) - 13*A*b^2*c*sqrt(x))/((c*x^2 + b)^2*c^4) + 2
/5*(B*c^12*x^(5/2) - 15*B*b*c^11*sqrt(x) + 5*A*c^12*sqrt(x))/c^15
```

maple [A] time = 0.08, size = 381, normalized size = 1.11

$$\frac{17Abx^{\frac{5}{2}}}{16\left(cx^2+b\right)^2c^2}-\frac{25Bb^2x^{\frac{5}{2}}}{16\left(cx^2+b\right)^2c^3}+\frac{13Ab^2\sqrt{x}}{16\left(cx^2+b\right)^2c^3}-\frac{21Bb^3\sqrt{x}}{16\left(cx^2+b\right)^2c^4}+\frac{2Bx^{\frac{5}{2}}}{5c^3}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right.}{64c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/5/c^3*B*x^(5/2)+2/c^3*A*x^(1/2)-6/c^4*b*B*x^(1/2)+17/16*b/c^2/(c*x^2+b)^2
*x^(5/2)*A-25/16*b^2/c^3/(c*x^2+b)^2*x^(5/2)*B+13/16*b^2/c^3/(c*x^2+b)^2*A*
x^(1/2)-21/16*b^3/c^4/(c*x^2+b)^2*B*x^(1/2)-45/64/c^3*(b/c)^(1/4)*2^(1/2)*A
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)*A*arct
an(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b
/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)
^(1/2)))+117/64*b/c^4*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)+1)+117/64*b/c^4*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)+117/128*b/c^4*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+
(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))
```

maxima [A] time = 3.08, size = 306, normalized size = 0.89

$$-\frac{\left(25\,Bb^2c-17\,Abc^2\right)x^{\frac{5}{2}}+\left(21\,Bb^3-13\,Ab^2c\right)\sqrt{x}}{16\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}+\frac{9\left(\frac{2\,\sqrt{2}\,(13\,Bb-5\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,(13\,Bb-5\,Ac)\,\mathrm{arcta}}{\sqrt{b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((25*B*b^2*c - 17*A*b*c^2)*x^(5/2) + (21*B*b^3 - 13*A*b^2*c)*sqrt(x))
/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 9/128*(2*sqrt(2)*(13*B*b - 5*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b - 5*A*c)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b - 5*A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(13*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sq
rt(b))/(b^(3/4)*c^(1/4)))*b/c^4 + 2/5*(B*c*x^(5/2) - 5*(3*B*b - A*c)*sqrt(x
))/c^4
```

mupad [B] time = 0.25, size = 865, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(5/2)*((17*A*b*c^2)/16 - (25*B*b^2*c)/16) - x^(1/2)*((21*B*b^3)/16 - (13
*A*b^2*c)/16))/(b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x^(1/2)*((2*A)/c^3 - (6*
B*b)/c^4) + (2*B*x^(5/2))/(5*c^3) + ((-b)^(1/4)*atan((((-b)^(1/4)*((81*x^(1
/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - (81*(-b)^(1/
4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b)
*9i)/(64*c^(17/4)) + ((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2
 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5
*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b)*9i)/(64*c^(17/4)))/((9*(-b)^(1/4
)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - (
81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*
c - 13*B*b))/(64*c^(17/4)) - (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A
^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13
*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(17/4))))*(5*A*
c - 13*B*b)*9i)/(32*c^(17/4)) + (9*(-b)^(1/4)*atan(((9*(-b)^(1/4)*((81*x^(1
/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)^(1/4)*
(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b
))/(64*c^(17/4)) + (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2
 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*
b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(17/4)))/(((-b)^(1/4)*((
81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)
^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c -
 13*B*b)*9i)/(64*c^(17/4)) - ((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2
*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^
3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b)*9i)/(64*c^(17/4))))*(5*
A*c - 13*B*b))/(32*c^(17/4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.208 $\int \frac{x^{21/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{7(11bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}+\frac{7(11bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}+\frac{7(11bB -$$

```
[Out] 7/48*(-3*A*c+11*B*b)*x^(3/2)/b/c^3-1/4*(-A*c+B*b)*x^(11/2)/b/c/(c*x^2+b)^2-
1/16*(-3*A*c+11*B*b)*x^(7/2)/b/c^2/(c*x^2+b)+7/64*(-3*A*c+11*B*b)*arctan(1-
c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-7/64*(-3*A*c+11*B
*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-7/12
8*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(
1/4)/c^(15/4)*2^(1/2)+7/128*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^
(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(15/4)*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{7/2}(11bB-3Ac)}{16bc^2\left(b+cx^2\right)}+\frac{7x^{3/2}(11bB-3Ac)}{48bc^3}-\frac{7(11bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}+\frac{7(11bB-3Ac)\log(}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (7*(11*b*B - 3*A*c)*x^(3/2))/(48*b*c^3) - ((b*B - A*c)*x^(11/2))/(4*b*c*(b
+ c*x^2)^2) - ((11*b*B - 3*A*c)*x^(7/2))/(16*b*c^2*(b + c*x^2)) + (7*(11*b*
B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/
4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(32*Sqrt[2]*b^(1/4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*Log[Sqrt[b] - Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4)) +
 (7*(11*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{21/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$
$$=-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{11bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{9/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$
$$=-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}+\frac{(7(11bB-3Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{32bc^2}$$
$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{(7(11bB-3Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32c^3}$$
$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{(7(11bB-3Ac))\,\text{Subst}\left(\int \frac{x}{b+\ldots}\right.}{16c^3}$$
$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}+\frac{(7(11bB-3Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}\ldots}{\ldots}\right.}{32c^{7/2}}$$
$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{(7(11bB-3Ac))\,\text{Subst}\left(\int \frac{\ldots}{\frac{\sqrt{b}}{\sqrt{c}}\ldots}\right.}{64c^4}$$
$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}-\frac{7(11bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\ldots\right.}{64\sqrt{2}\sqrt[4]{b}\,c^{1\ldots}}$$
$$=\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}+\frac{7(11bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\ldots}\right.}{32\sqrt{2}\sqrt[4]{b}\,c^{15/4}}$$

Mathematica [C] time = 0.47, size = 176, normalized size = 0.55

$$\frac{\frac{2c^{3/4}x^{3/2}(3bB-2Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{b}+\frac{2c^{3/4}x^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{b}+\frac{(3Ac-9bB)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{\sqrt[4]{-b}}+\frac{(9bB-3Ac)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{\sqrt[4]{-b}}+2Bc^{3/\ldots}}{3c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^(3/4)*x^(3/2) + ((-9*b*B + 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4
)])/(-b)^(1/4) + ((9*b*B - 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-
b)^(1/4) + (2*c^(3/4)*(3*b*B - 2*A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4
, -((c*x^2)/b)])/b + (2*c^(3/4)*(-(b*B) + A*c)*x^(3/2)*Hypergeometric2F1[3/
4, 3, 7/4, -((c*x^2)/b)])/b)/(3*c^(15/4))
```

fricas [B] time = 0.78, size = 993, normalized size = 3.08

$$-\frac{84\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{14641\,B^4b^4-15972\,AB^3b^3c+6534\,A^2B^2b^2c^2-1188\,A^3Bbc^3+81\,A^4c^4}{bc^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(1771561\,B^6b^6-289\cdots\right.}}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1
/4)*arctan((sqrt((1771561*B^6*b^6 - 2898918*A*B^5*b^5*c + 1976535*A^2*B^4*b
^4*c^2 - 718740*A^3*B^3*b^3*c^3 + 147015*A^4*B^2*b^2*c^4 - 16038*A^5*B*b*c^
5 + 729*A^6*c^6)*x - (14641*B^4*b^5*c^7 - 15972*A*B^3*b^4*c^8 + 6534*A^2*B^
2*b^3*c^9 - 1188*A^3*B*b^2*c^10 + 81*A^4*b*c^11)*sqrt(-(14641*B^4*b^4 - 159
72*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c
^15)))*c^4*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 11
88*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4) + (1331*B^3*b^3*c^4 - 1089*A*B
^2*b^2*c^5 + 297*A^2*B*b*c^6 - 27*A^3*c^7)*sqrt(x)*(-(14641*B^4*b^4 - 15972
*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
5))^(1/4))/(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188
*A^3*B*b*c^3 + 81*A^4*c^4)) - 21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641
*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81
*A^4*c^4)/(b*c^15))^(1/4)*log(343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3
*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4)
- 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt
(x)) + 21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*
b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/
4)*log(-343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*
c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 -
1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 4*(32*B*c^2*x^5
 + 11*(11*B*b*c - 3*A*c^2)*x^3 + 7*(11*B*b^2 - 3*A*b*c)*x)*sqrt(x))/(c^5*x^
4 + 2*b*c^4*x^2 + b^2*c^3)
```

giac [A] time = 0.23, size = 304, normalized size = 0.94

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c^3}+\frac{19\,Bbcx^{\frac{7}{2}}-11\,Ac^2x^{\frac{7}{2}}+15\,Bb^2x^{\frac{3}{2}}-7\,Abcx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2c^3}-\frac{7\sqrt{2}\left(11\left(bc^3\right)^{\frac{3}{4}}Bb-3\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\cdots\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^3 + 1/16*(19*B*b*c*x^(7/2) - 11*A*c^2*x^(7/2) + 15*B*b^2*x^
(3/2) - 7*A*b*c*x^(3/2))/((c*x^2 + b)^2*c^3) - 7/64*sqrt(2)*(11*(b*c^3)^(3/
4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/(b*c^6) - 7/64*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b*c^6) + 7/128*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^6) - 7/128*sqrt(2)*(1
1*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4)
 + x + sqrt(b/c))/(b*c^6)
```

maple [A] time = 0.07, size = 357, normalized size = 1.11

$$-\frac{11 A x^{\frac{7}{2}}}{16 \left(c x^{2}+b\right)^{2} c}+\frac{19 B b x^{\frac{7}{2}}}{16 \left(c x^{2}+b\right)^{2} c^{2}}-\frac{7 A b x^{\frac{3}{2}}}{16 \left(c x^{2}+b\right)^{2} c^{2}}+\frac{15 B b^{2} x^{\frac{3}{2}}}{16 \left(c x^{2}+b\right)^{2} c^{3}}+\frac{2 B x^{\frac{3}{2}}}{3 c^{3}}+\frac{21 \sqrt{2}\, A \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64 \left(\frac{b}{c}\right)^{\frac{1}{4}} c^{3}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/3*B*x^(3/2)/c^3-11/16/c/(c*x^2+b)^2*A*x^(7/2)+19/16/c^2/(c*x^2+b)^2*B*x^(
7/2)*b-7/16/c^2/(c*x^2+b)^2*x^(3/2)*A*b+15/16/c^3/(c*x^2+b)^2*x^(3/2)*B*b^2
+21/128/c^3/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(
1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^
(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)
*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-77/128/c^4/(b/c)^(1/4)*2^(1/2)*b*B
*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1
/2)+(b/c)^(1/2)))-77/64/c^4/(b/c)^(1/4)*2^(1/2)*b*B*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)+1)-77/64/c^4/(b/c)^(1/4)*2^(1/2)*b*B*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1)
```

maxima [A] time = 3.10, size = 256, normalized size = 0.80

$$\frac{\left(19\, Bbc - 11\, Ac^{2}\right) x^{\frac{7}{2}} + \left(15\, Bb^{2} - 7\, Abc\right) x^{\frac{3}{2}}}{16 \left(c^{5} x^{4} + 2\, bc^{4} x^{2} + b^{2} c^{3}\right)}+\frac{2\, Bx^{\frac{3}{2}}}{3\, c^{3}}-\frac{7 \left(11\, Bb - 3\, Ac\right) \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} + 2 \sqrt{c} \sqrt{x}\right)}{2 \sqrt{\sqrt{b} \sqrt{c}}}\right)}{\sqrt{\sqrt{b} \sqrt{c}} \sqrt{c}} + \frac{2 \sqrt{2} \arctan\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((19*B*b*c - 11*A*c^2)*x^(7/2) + (15*B*b^2 - 7*A*b*c)*x^(3/2))/(c^5*x^
4 + 2*b*c^4*x^2 + b^2*c^3) + 2/3*B*x^(3/2)/c^3 - 7/128*(11*B*b - 3*A*c)*(2*
sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c
)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*s
qrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3
```

mupad [B] time = 0.26, size = 138, normalized size = 0.43

$$\frac{x^{3/2} \left(\frac{15\, B\, b^2}{16} - \frac{7\, A\, b\, c}{16}\right) - x^{7/2} \left(\frac{11\, A\, c^2}{16} - \frac{19\, B\, b\, c}{16}\right)}{b^2\, c^3 + 2\, b\, c^4\, x^2 + c^5\, x^4}+\frac{2\, B\, x^{3/2}}{3\, c^3}+\frac{7 \operatorname{atan}\left(\frac{c^{1/4} \sqrt{x}}{(-b)^{1/4}}\right) (3\, A\, c - 11\, B\, b)}{32\, (-b)^{1/4}\, c^{15/4}}+\frac{\operatorname{atan}\left(\frac{c^{1/4} \sqrt{x}\, 1\mathrm{i}}{(-b)^{1/4}}\right) (3\, A\, c - \cdots}{32\, (-b)^{1/4}\, c^{15/4}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(3/2)*((15*B*b^2)/16 - (7*A*b*c)/16) - x^(7/2)*((11*A*c^2)/16 - (19*B*b*
c)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(3/2))/(3*c^3) + (7*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 11*B*b))/(32*(-b)^(1/4)*c^(15/4)) +
```

```
 (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 11*B*b)*7i)/(32*(-b)^(1/4)
*c^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.209 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=322

$$\frac{5(9bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}+\frac{5(9bB-Ac)\,\mathrm{ta}}{32\sqrt{}}$$

```
[Out] -1/4*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)^2-1/16*(-A*c+9*B*b)*x^(5/2)/b/c^2/(c*
x^2+b)+5/64*(-A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/
c^(13/4)*2^(1/2)-5/64*(-A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(3/4)/c^(13/4)*2^(1/2)+5/128*(-A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1/2)-5/128*(-A*c+9*B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1/2)+5/
16*(-A*c+9*B*b)*x^(1/2)/b/c^3
```

Rubi [A] time = 0.25, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5(9bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}+\frac{5(9bB-Ac)\,\mathrm{ta}}{32\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*(9*b*B - A*c)*Sqrt[x])/(16*b*c^3) - ((b*B - A*c)*x^(9/2))/(4*b*c*(b + c*
x^2)^2) - ((9*b*B - A*c)*x^(5/2))/(16*b*c^2*(b + c*x^2)) + (5*(9*b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(13/4
)) - (5*(9*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sq
rt[2]*b^(3/4)*c^(13/4)) + (5*(9*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(13/4)) - (5*(9*b*B - A*c
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(3/4)*c^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$
$$=-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{9bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$
$$=-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}+\frac{(5(9bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{32bc^2}$$
$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^3}$$
$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,\right.}{16c^3}$$
$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\right.}{32\sqrt{b}\,c^3}$$
$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}-\frac{(5(9bB-Ac))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{64\sqrt{b}\,c^{7/2}}$$
$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}+\frac{5(9bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{64\sqrt{2}\,b^{3/4}c^{13/4}}$$
$$=\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)}+\frac{5(9bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{13/4}}$$

Mathematica [A] time = 0.63, size = 403, normalized size = 1.25

$$\frac{10\sqrt{2}(9bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}-\frac{10\sqrt{2}(9bB-Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{5\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+\frac{5\sqrt{2}Ac\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (256*B*c^(1/4)*Sqrt[x] - (32*b^2*B*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (32*A*b
*c^(5/4)*Sqrt[x])/(b + c*x^2)^2 + (136*b*B*c^(1/4)*Sqrt[x])/(b + c*x^2) - (
72*A*c^(5/4)*Sqrt[x])/(b + c*x^2) + (10*Sqrt[2]*(9*b*B - A*c)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) - (10*Sqrt[2]*(9*b*B - A*c)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + 45*Sqrt[2]*b^(1/4)*B*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - (5*Sqrt[2]*A*c*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) - 45*Sqr
```

t[2]*b^(1/4)*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + (5*Sqrt[2]*A*c*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(128*c^(13/4))

fricas [B] time = 0.84, size = 793, normalized size = 2.46

$$\frac{20\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2c^6\sqrt{-\frac{6561\,B^4b^4-2916\,AB^3b^3c+}{}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3
*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*arct
an((sqrt(b^2*c^6*sqrt(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c
^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13)) + (81*B^2*b^2 - 18*A*B*b*c + A^2
*c^2)*x)*b^2*c^10*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2
- 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(3/4) + (9*B*b^3*c^10 - A*b^2*c^11)
*sqrt(x)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*
B*b*c^3 + A^4*c^4)/(b^3*c^13))^(3/4))/(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 48
6*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)) + 5*(c^5*x^4 + 2*b*c^4*x^2 +
 b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3
*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(5*b*c^3*(-(6561*B^4*b^4 - 2916*A*
B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/
4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561
*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^
4)/(b^3*c^13))^(1/4)*log(-5*b*c^3*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*
A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b -
A*c)*sqrt(x)) + 4*(32*B*c^2*x^4 + 45*B*b^2 - 5*A*b*c + 9*(9*B*b*c - A*c^2)*
x^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

giac [A] time = 0.20, size = 304, normalized size = 0.94

$$\frac{2\,B\sqrt{x}}{c^3}-\frac{5\,\sqrt{2}\left(9\,\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^4}-\frac{5\,\sqrt{2}\left(9\,\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\right.}{64\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^3 - 5/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5
/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5/128*sqrt(2)*(9*(b
*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
sqrt(b/c))/(b*c^4) + 5/128*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4) + 1/16*(17*B*b*
c*x^(5/2) - 9*A*c^2*x^(5/2) + 13*B*b^2*sqrt(x) - 5*A*b*c*sqrt(x))/((c*x^2 +
 b)^2*c^3)
```

maple [A] time = 0.07, size = 363, normalized size = 1.13

$$-\frac{9A\,x^{\frac{5}{2}}}{16\left(c\,x^{2}+b\right)^{2}c}+\frac{17Bb\,x^{\frac{5}{2}}}{16\left(c\,x^{2}+b\right)^{2}c^{2}}-\frac{5Ab\sqrt{x}}{16\left(c\,x^{2}+b\right)^{2}c^{2}}+\frac{13B\,b^{2}\sqrt{x}}{16\left(c\,x^{2}+b\right)^{2}c^{3}}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b\,c^{2}}+\frac{5\left(\frac{b}{c}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*B/c^3*x^(1/2)-9/16/c/(c*x^2+b)^2*x^(5/2)*A+17/16/c^2/(c*x^2+b)^2*x^(5/2)*
b*B-5/16/c^2/(c*x^2+b)^2*A*x^(1/2)*b+13/16/c^3/(c*x^2+b)^2*B*x^(1/2)*b^2+5/
64/c^2*(b/c)^(1/4)/b*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+5/128/
c^2*(b/c)^(1/4)/b*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/
(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)
*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*B*ln((x+
(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/
c)^(1/2)))-45/64/c^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/
2)+1)
```

maxima [A] time = 3.06, size = 283, normalized size = 0.88

$$\frac{\left(17\,Bbc-9\,Ac^{2}\right)x^{\frac{5}{2}}+\left(13\,Bb^{2}-5\,Abc\right)\sqrt{x}}{16\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)}+\frac{2\,B\sqrt{x}}{c^{3}}-\frac{5\left(\frac{2\,\sqrt{2}\,(9\,Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,(9\,Bb-Ac)\arctan\left(\cdots\right)}{\sqrt{b}\,\sqrt{\cdots}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((17*B*b*c - 9*A*c^2)*x^(5/2) + (13*B*b^2 - 5*A*b*c)*sqrt(x))/(c^5*x^4
 + 2*b*c^4*x^2 + b^2*c^3) + 2*B*sqrt(x)/c^3 - 5/128*(2*sqrt(2)*(9*B*b - A*c
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b - A*c)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(9*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(3/4)*c^(1/4)))/c^3
```

mupad [B] time = 0.23, size = 760, normalized size = 2.36

$$\frac{\sqrt{x}\,\left(\frac{13\,B\,b^{2}}{16}-\frac{5\,A\,b\,c}{16}\right)-x^{5/2}\,\left(\frac{9\,A\,c^{2}}{16}-\frac{17\,B\,b\,c}{16}\right)}{b^{2}\,c^{3}+2\,b\,c^{4}\,x^{2}+c^{5}\,x^{4}}+\frac{2\,B\,\sqrt{x}}{c^{3}}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^{2}\,c^{2}-18\,A\,B\,b\,c+81\,B^{2}\,b^{2}\right)}{64\,c^{3}}-\frac{5\left(45\,B\,b^{2}-5\,A\,b\,c\right)(A\,c-9\,B\,b)}{64\,(-b)^{3/4}\,c^{13/4}}\right)5\cdots}{64\,(-b)^{3/4}\,c^{13/4}}\cdots}{\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^{2}\,c^{2}-18\,A\,B\,b\,c+81\,B^{2}\,b^{2}\right)}{64\,c^{3}}-\frac{5\left(45\,B\,b^{2}-5\,A\,b\,c\right)(A\,c-9\,B\,b)}{64\,(-b)^{3/4}\,c^{13/4}}\right)}{64\,(-b)^{3/4}\,c^{13/4}}\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(1/2)*((13*B*b^2)/16 - (5*A*b*c)/16) - x^(5/2)*((9*A*c^2)/16 - (17*B*b*c
)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(1/2))/c^3 - (atan((((A*c
 - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (5*(
45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(
3/4)*c^(13/4)) + ((A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B
*b*c))/(64*c^3) + (5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(
13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)))/((5*(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c
^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*
B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/4)) - (5*(A*c - 9*B*b
)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) + (5*(45*B*b^2
 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/
4))))*(A*c - 9*B*b)*5i)/(32*(-b)^(3/4)*c^(13/4)) - (5*atan(((5*(A*c - 9*B*b
)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - ((45*B*b^2 -
 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13
/4)) + (5*(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(
64*c^3) + ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4)))
)/(64*(-b)^(3/4)*c^(13/4)))/(((A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*
b^2 - 18*A*B*b*c))/(64*c^3) - ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(
-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)) - ((A*c - 9*B*b)*((25*x^(
1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) + ((45*B*b^2 - 5*A*b*c)*
(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4))))*
(A*c - 9*B*b))/(32*(-b)^(3/4)*c^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.210 $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=293

$$\frac{3(Ac+7bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\ \text{ta}}{32\sqrt{}}$$

```
[Out] -1/4*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)^2-1/16*(A*c+7*B*b)*x^(3/2)/b/c^2/(c*x
^2+b)-3/64*(A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^
(11/4)*2^(1/2)+3/64*(A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(5/4)/c^(11/4)*2^(1/2)+3/128*(A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1
/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)-3/128*(A*c+7*B*b)*ln(b^(1/2)+
x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3(Ac+7bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\ \text{ta}}{32\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*x^(7/2))/(4*b*c*(b + c*x^2)^2) - ((7*b*B + A*c)*x^(3/2))/(16*
b*c^2*(b + c*x^2)) - (3*(7*b*B + A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B + A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B +
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(5/4)*c^(11/4)) - (3*(7*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{7bB}{2}+\frac{Ac}{2}\right)\int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}+\frac{(3(7bB+Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc^2}$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}+\frac{(3(7bB+Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16bc^2}$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}-\frac{(3(7bB+Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32bc^{5/2}}+\frac{(3(7\cdots}{\cdots}$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}+\frac{(3(7bB+Ac))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64bc^3}+\cdots$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}+\frac{3(7bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}-\frac{3\cdots}{\cdots}$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}-\frac{3(7bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}}+\frac{3(7bB+Ac)\,\text{ta}\cdots}{32\sqrt{\cdots}}$$

Mathematica [**C**] time = 0.32, size = 137, normalized size = 0.47

$$\frac{2c^{3/4}x^{3/2}(Ac-2bB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)+2c^{3/4}x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)+3(-b)^{7/4}B\left(\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+\tanh^{\cdots}\right.}{3b^2c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*(-b)^(7/4)*B*(ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)] + ArcTanh[(b*c^(1/4)*
Sqrt[x])/(-b)^(5/4)]) + 2*c^(3/4)*(-2*b*B + A*c)*x^(3/2)*Hypergeometric2F1[
3/4, 2, 7/4, -((c*x^2)/b)] + 2*c^(3/4)*(b*B - A*c)*x^(3/2)*Hypergeometric2F
1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^2*c^(11/4))
```

fricas [**B**] time = 0.94, size = 990, normalized size = 3.38

$$-\frac{12\left(bc^4x^4+2\,b^2c^3x^2+b^3c^2\right)\left(-\frac{2401\,B^4b^4+1372\,AB^3b^3c+294\,A^2B^2b^2c^2+28\,A^3Bbc^3+A^4c^4}{b^5c^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(117649\,B^6b^6+100842\,AB^5b^5\cdots\right.}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^
3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)
*arctan((sqrt((117649*B^6*b^6 + 100842*A*B^5*b^5*c + 36015*A^2*B^4*b^4*c^2
+ 6860*A^3*B^3*b^3*c^3 + 735*A^4*B^2*b^2*c^4 + 42*A^5*B*b*c^5 + A^6*c^6)*x
- (2401*B^4*b^7*c^5 + 1372*A*B^3*b^6*c^6 + 294*A^2*B^2*b^5*c^7 + 28*A^3*B*b
^4*c^8 + A^4*b^3*c^9)*sqrt(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*
b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11)))*b*c^3*(-(2401*B^4*b^4 + 13
72*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11)
)^(1/4) - (343*B^3*b^4*c^3 + 147*A*B^2*b^3*c^4 + 21*A^2*B*b^2*c^5 + A^3*b*c
^6)*sqrt(x)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A
^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4))/(2401*B^4*b^4 + 1372*A*B^3*b^3*c +
 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)) - 3*(b*c^4*x^4 + 2*b^2*c^
3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 +
 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*log(27*b^4*c^8*(-(2401*B^4*b^4
 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*
c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3
)*sqrt(x)) + 3*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372
*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^
(1/4)*log(-27*b^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*
c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A
*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) + 4*((11*B*b*c - 3*A*c^2)*x
^3 + (7*B*b^2 + A*b*c)*x)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

giac [A] time = 0.23, size = 293, normalized size = 1.00

$$-\frac{11\,Bbcx^{\frac{7}{2}} - 3\,Ac^{2}x^{\frac{7}{2}} + 7\,Bb^{2}x^{\frac{3}{2}} + Abcx^{\frac{3}{2}}}{16\,\left(cx^{2}+b\right)^{2}bc^{2}} + \frac{3\,\sqrt{2}\left(7\,\left(bc^{3}\right)^{\frac{3}{4}}Bb + \left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{2}c^{5}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*b*c*x^(7/2) - 3*A*c^2*x^(7/2) + 7*B*b^2*x^(3/2) + A*b*c*x^(3/2)
)/((c*x^2 + b)^2*b*c^2) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
2*c^5) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^5) - 3/128*s
qrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^
(1/4) + x + sqrt(b/c))/(b^2*c^5) + 3/128*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*
c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^5)
```

maple [A] time = 0.06, size = 325, normalized size = 1.11

$$\frac{3\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b\,c^{2}} + \frac{3\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b\,c^{2}} + \frac{3\sqrt{2}\;A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b\,c^{2}} + \frac{21\sqrt{2}\;B\arctan\left(\cdots\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(3*A*c-11*B*b)/b/c*x^(7/2)-1/32*(A*c+7*B*b)/c^2*x^(3/2))/(c*x^2+b)^
2+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/
64/c^2/b/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/128/
```

```
c^2/b/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/
(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)*B*arc
tan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+21/128/c^3/(b/c)^(1/4)*2^(1/2)*B*ln((x-(
b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c
)^(1/2)))
```

maxima [A] time = 3.11, size = 251, normalized size = 0.86

$$-\frac{\left(11\,Bbc-3\,Ac^2\right)x^{\frac{7}{2}}+\left(7\,Bb^2+Abc\right)x^{\frac{3}{2}}}{16\left(bc^4x^4+2\,b^2c^3x^2+b^3c^2\right)}+\frac{3\,(7\,Bb+Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\right.}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right.}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}\right.}{128\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((11*B*b*c - 3*A*c^2)*x^(7/2) + (7*B*b^2 + A*b*c)*x^(3/2))/(b*c^4*x^4
 + 2*b^2*c^3*x^2 + b^3*c^2) + 3/128*(7*B*b + A*c)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt
(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c^2)
```

mupad [B] time = 0.17, size = 122, normalized size = 0.42

$$\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+7\,B\,b)}{32\,(-b)^{5/4}\,c^{11/4}}-\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+7\,B\,b)}{32\,(-b)^{5/4}\,c^{11/4}}-\frac{\frac{x^{3/2}\,(A\,c+7\,B\,b)}{16\,c^2}-\frac{x^{7/2}\,(3\,A\,c-11\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)*c^(11/
4)) - (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)*c
^(11/4)) - ((x^(3/2)*(A*c + 7*B*b))/(16*c^2) - (x^(7/2)*(3*A*c - 11*B*b))/(
16*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.211 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{(3Ac+5bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}-\frac{(3Ac+5b}{}$$

```
[Out] -1/4*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)^2-1/64*(3*A*c+5*B*b)*arctan(1-c^(1/4)
*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)+1/64*(3*A*c+5*B*b)*arctan
(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)-1/128*(3*A*c+5*
B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(9/4)*
2^(1/2)+1/128*(3*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(7/4)/c^(9/4)*2^(1/2)-1/16*(3*A*c+5*B*b)*x^(1/2)/b/c^2/(c*x^2+b)
```

Rubi [A] time = 0.24, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(3Ac+5bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}-\frac{(3Ac+5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*x^(5/2))/(4*b*c*(b + c*x^2)^2) - ((5*b*B + 3*A*c)*Sqrt[x])/(1
6*b*c^2*(b + c*x^2)) - ((5*b*B + 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) - ((5*b*B + 3*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} + \frac{\left(\frac{5bB}{2}+\frac{3Ac}{2}\right)\int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} + \frac{(5bB+3Ac)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32bc^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} + \frac{(5bB+3Ac)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} + \frac{(5bB+3Ac)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{3/2}c^2} + \frac{(5}{}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} + \frac{(5bB+3Ac)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{3/2}c^{5/2}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} - \frac{(5bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{9/4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} - \frac{(5bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{9/4}} + \frac{(5bB+3A}{}$$

<u>**Mathematica**</u> **[A]** time = 0.71, size = 389, normalized size = 1.31

$$-\frac{2\sqrt{2}(3Ac+5bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{7/4}} + \frac{2\sqrt{2}(3Ac+5bB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{7/4}} - \frac{3\sqrt{2}Ac\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{7/4}} + \frac{3\sqrt{2}Ac\log\left(\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b*B*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (32*A*c^(5/4)*Sqrt[x])/(b + c*x^2
)^2 - (72*B*c^(1/4)*Sqrt[x])/(b + c*x^2) + (8*A*c^(5/4)*Sqrt[x])/(b^2 + b*c
*x^2) - (2*Sqrt[2]*(5*b*B + 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/b^(7/4) + (2*Sqrt[2]*(5*b*B + 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/b^(7/4) - (5*Sqrt[2]*B*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) - (3*Sqrt[2]*A*c*Log[Sqrt[b] - Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4) + (5*Sqrt[2]*B*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) + (3*Sqrt[2]*A*c*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4))/(128*c^(9/4))
```

fricas [B] time = 0.88, size = 806, normalized size = 2.70

$$\frac{4\left(bc^4x^4+2\,b^2c^3x^2+b^3c^2\right)\left(-\frac{625\,B^4b^4+1500\,AB^3b^3c+1350\,A^2B^2b^2c^2+540\,A^3Bbc^3+81\,A^4c^4}{b^7c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4c^4\sqrt{-\frac{625\,B^4b^4+1500\,AB^3b^3c\cdots}{}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b
^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4
)*arctan((sqrt(b^4*c^4*sqrt(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2
*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9)) + (25*B^2*b^2 + 30*A*B*
b*c + 9*A^2*c^2)*x)*b^5*c^7*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^
2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(3/4) - (5*B*b^6*c^7 +
 3*A*b^5*c^8)*sqrt(x)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*
c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(3/4))/(625*B^4*b^4 + 1500*A
*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)) + (b*c^4
*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^
2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log(b^2*c^2*
(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3
+ 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - (b*c^4*x^4 + 2*
b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2
*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log(-b^2*c^2*(-(625*B
^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4
*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - 4*(5*B*b^2 + 3*A*b*c +
(9*B*b*c - A*c^2)*x^2)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

giac [A] time = 0.19, size = 298, normalized size = 1.00

$$\frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{1}{4}}Bb+3\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3}+\frac{\sqrt{2}\left(5\left(bc^3\right)^{\frac{1}{4}}Bb+3\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)\cdots\right.}{2\left(\frac{b}{c}\right)\cdots}\right.}{64\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/64*sqrt(2)*(5
*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/128*sqrt(2)*(5*(b*c^3)^(1/4
)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^3) - 1/128*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3) - 1/16*(9*B*b*c*x
^(5/2) - A*c^2*x^(5/2) + 5*B*b^2*sqrt(x) + 3*A*b*c*sqrt(x))/((c*x^2 + b)^2*
b*c^2)
```

maple [A] time = 0.06, size = 334, normalized size = 1.12

$$\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^2c}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^2c}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^2c}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(A*c-9*B*b)/b/c*x^(5/2)-1/32*(3*A*c+5*B*b)/c^2*x^(1/2))/(c*x^2+b)^2
+3/64/c/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/6
4/c/b^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/128/c
/b^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(
x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+5/64/c^2/b*(b/c)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+5/64/c^2/b*(b/c)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+5/128/c^2/b*(b/c)^(1/4)*2^(1/2)*B*ln((x
+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b
/c)^(1/2)))
```

maxima [A] time = 2.97, size = 280, normalized size = 0.94

$$-\frac{\left(9\,Bbc-Ac^{2}\right)x^{\frac{5}{2}}+\left(5\,Bb^{2}+3\,Abc\right)\sqrt{x}}{16\left(bc^{4}x^{4}+2\,b^{2}c^{3}x^{2}+b^{3}c^{2}\right)}+\frac{\frac{2\sqrt{2}\,(5\,Bb+3\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\,(5\,Bb+3\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}\cdots\right)}{2\sqrt{\cdots}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((9*B*b*c - A*c^2)*x^(5/2) + (5*B*b^2 + 3*A*b*c)*sqrt(x))/(b*c^4*x^4
+ 2*b^2*c^3*x^2 + b^3*c^2) + 1/128*(2*sqrt(2)*(5*B*b + 3*A*c)*arctan(1/2*sq
rt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/
(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(5*B*b + 3*A*c)*arctan(-1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(5*B*b + 3*A*c)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(5*B*b
 + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3
/4)*c^(1/4)))/(b*c^2)
```

mupad [B] time = 0.38, size = 799, normalized size = 2.68

$$-\frac{\frac{\sqrt{x}\,(3\,A\,c+5\,B\,b)}{16\,c^{2}}-\frac{x^{5/2}\,(A\,c-9\,B\,b)}{16\,b\,c}}{b^{2}+2\,b\,c\,x^{2}+c^{2}\,x^{4}}+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^{2}\,c^{2}+30\,A\,B\,b\,c+25\,B^{2}\,b^{2})}{64\,b^{2}\,c}-\frac{(3\,A\,c^{2}+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}+\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^{2}\,c^{2}+\cdots}{\cdots}\cdots\right)\cdots}{\cdots}}{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^{2}\,c^{2}+30\,A\,B\,b\,c+25\,B^{2}\,b^{2})}{64\,b^{2}\,c}-\frac{(3\,A\,c^{2}+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}-\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^{2}\,c^{2}+\cdots}{\cdots}\cdots\right)}{\cdots}}\right)\cdots}{32\,(-b)^{7/4}\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(6
4*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4)))*1
i)/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2
*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*
(-b)^(7/4)*c^(9/4)))*1i)/(64*(-b)^(7/4)*c^(9/4)))/(((3*A*c + 5*B*b)*((x^(1/
2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*B*b*c)
*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)) - ((3*A
*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + (
(3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/
4)*c^(9/4))))*(3*A*c + 5*B*b)*1i)/(32*(-b)^(7/4)*c^(9/4)) - ((x^(1/2)*(3*A*
c + 5*B*b))/(16*c^2) - (x^(5/2)*(A*c - 9*B*b))/(16*b*c))/(b^2 + c^2*x^4 + 2
```

```
*b*c*x^2) + (atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*
A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7
/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*
c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c +
5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)))/((((3*A*c + 5*
B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^
2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4)))*1i)/(64*(-b)^(7/4
)*c^(9/4)) - ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*
c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^
(9/4)))*1i)/(64*(-b)^(7/4)*c^(9/4))))*(3*A*c + 5*B*b))/(32*(-b)^(7/4)*c^(9/
4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.212 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=298

$$\frac{(5Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB}{3}$$

```
[Out] -1/4*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)^2+1/16*(5*A*c+3*B*b)*x^(3/2)/b^2/c/(c
*x^2+b)-1/64*(5*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4
)/c^(7/4)*2^(1/2)+1/64*(5*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/b^(9/4)/c^(7/4)*2^(1/2)+1/128*(5*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4
)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(1/2)-1/128*(5*A*c+3*B*b)*ln(b
^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(5Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}-\frac{(5Ac+3bB}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*x^(3/2))/(4*b*c*(b + c*x^2)^2) + ((3*b*B + 5*A*c)*x^(3/2))/(1
6*b^2*c*(b + c*x^2)) - ((3*b*B + 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(7/4)) - ((3*b*B + 5*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{\left(\frac{3bB}{2}+\frac{5Ac}{2}\right)\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} - \frac{(3bB+5Ac)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2c^{3/2}} + \frac{(3}{}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^2c^2}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} + \frac{(3bB+5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)} - \frac{(3bB+5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}} + \frac{(3bB+5A}{}$$

Mathematica [C] time = 0.09, size = 62, normalized size = 0.21

$$\frac{2x^{3/2}\left((Ac-bB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)+bB\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{3b^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(b*B*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)] + (-(b*B) + A*
c)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)]))/(3*b^3*c)
```

fricas [B] time = 0.84, size = 1005, normalized size = 3.37

$$-\frac{4\left(b^2c^3x^4+2\,b^3c^2x^2+b^4c\right)\left(-\frac{81\,B^4b^4+540\,AB^3b^3c+1350\,A^2B^2b^2c^2+1500\,A^3Bbc^3+625\,A^4c^4}{b^9c^7}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(729\,B^6b^6+7290\,AB^5b^5\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^
3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/
```

```
4)*arctan((sqrt((729*B^6*b^6 + 7290*A*B^5*b^5*c + 30375*A^2*B^4*b^4*c^2 + 6
7500*A^3*B^3*b^3*c^3 + 84375*A^4*B^2*b^2*c^4 + 56250*A^5*B*b*c^5 + 15625*A^
6*c^6)*x - (81*B^4*b^9*c^3 + 540*A*B^3*b^8*c^4 + 1350*A^2*B^2*b^7*c^5 + 150
0*A^3*B*b^6*c^6 + 625*A^4*b^5*c^7)*sqrt(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 13
50*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7)))*b^2*c^2*(-
(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 6
25*A^4*c^4)/(b^9*c^7))^(1/4) - (27*B^3*b^5*c^2 + 135*A*B^2*b^4*c^3 + 225*A^
2*B*b^3*c^4 + 125*A^3*b^2*c^5)*sqrt(x)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 13
50*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4))/(81*
B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A
^4*c^4)) - (b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*
b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(
1/4)*log(b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1
500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b
^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x)) + (b^2*c^3*x^4 + 2*b^3*c^2*x
^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A
^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*log(-b^7*c^5*(-(81*B^4*b^4 + 540
*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*
c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3
)*sqrt(x)) - 4*((3*B*b*c + 5*A*c^2)*x^3 - (B*b^2 - 9*A*b*c)*x)*sqrt(x))/(b^
2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

giac [A] time = 0.23, size = 298, normalized size = 1.00

$$\frac{3\,Bbcx^{\frac{7}{2}}+5\,Ac^2x^{\frac{7}{2}}-Bb^2x^{\frac{3}{2}}+9\,Abcx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^2c}+\frac{\sqrt{2}\left(3\left(bc^3\right)^{\frac{3}{4}}Bb+5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^4}+\frac{\sqrt{2}\left(3\left(bc^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*B*b*c*x^(7/2) + 5*A*c^2*x^(7/2) - B*b^2*x^(3/2) + 9*A*b*c*x^(3/2))/
((c*x^2 + b)^2*b^2*c) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
3*c^4) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^4) - 1/128
*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^4) + 1/128*sqrt(2)*(3*(b*c^3)^(3/4)*B*b +
 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^4)
```

maple [A] time = 0.07, size = 335, normalized size = 1.12

$$\frac{5\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2c}+\frac{5\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2c}+\frac{5\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2c}+\frac{3\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(5*A*c+3*B*b)/b^2*x^(7/2)+1/32*(9*A*c-B*b)/b/c*x^(3/2))/(c*x^2+b)^2
+5/128/b^2/c/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^
(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+5/64/b^2/c/(b/c)^(1/4)*
```

```
2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+5/64/b^2/c/(b/c)^(1/4)*2^(1
/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/128/b/c^2/(b/c)^(1/4)*2^(1/2)
*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2)))+3/64/b/c^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)-1)+3/64/b/c^2/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)+1)
```

maxima [A] time = 3.01, size = 253, normalized size = 0.85

$$\frac{\left(3\,Bbc+5\,Ac^{2}\right)x^{\frac{7}{2}}-\left(Bb^{2}-9\,Abc\right)x^{\frac{3}{2}}}{16\left(b^{2}c^{3}x^{4}+2\,b^{3}c^{2}x^{2}+b^{4}c\right)}+\frac{(3\,Bb+5\,Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\ldots\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}\ldots\right)}{128\,b^{2}c\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*B*b*c + 5*A*c^2)*x^(7/2) - (B*b^2 - 9*A*b*c)*x^(3/2))/(b^2*c^3*x^4
 + 2*b^3*c^2*x^2 + b^4*c) + 1/128*(3*B*b + 5*A*c)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt
(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b^2*c)
```

mupad [B] time = 0.16, size = 124, normalized size = 0.42

$$\frac{\frac{x^{7/2}\,(5\,A\,c+3\,B\,b)}{16\,b^2}+\frac{x^{3/2}\,(9\,A\,c-B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,(5\,A\,c+3\,B\,b)}{32\,{(-b)}^{9/4}\,c^{7/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,(5\,A\,c+3\,B\,b)}{32\,{(-b)}^{9/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(7/2)*(5*A*c + 3*B*b))/(16*b^2) + (x^(3/2)*(9*A*c - B*b))/(16*b*c))/(b^
2 + c^2*x^4 + 2*b*c*x^2) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c + 3*B
*b))/(32*(-b)^(9/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c
+ 3*B*b))/(32*(-b)^(9/4)*c^(7/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.213 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=293

$$-\frac{3(7Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(7Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}-\frac{3(7Ac+bB)}{32\cdots}$$

```
[Out] -3/64*(7*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(5/4
)*2^(1/2)+3/64*(7*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/
4)/c^(5/4)*2^(1/2)-3/128*(7*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)+3/128*(7*A*c+B*b)*ln(b^(1/2)+x*c^(
1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)-1/4*(-A*c+B*
b)*x^(1/2)/b/c/(c*x^2+b)^2+1/16*(7*A*c+B*b)*x^(1/2)/b^2/c/(c*x^2+b)
```

Rubi [A] time = 0.23, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1584, 457, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(7Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(7Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}-\frac{3(7Ac+bB)}{32\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*B - A*c)*Sqrt[x])/(4*b*c*(b + c*x^2)^2) + ((b*B + 7*A*c)*Sqrt[x])/(16*
b^2*c*(b + c*x^2)) - (3*(b*B + 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) - (3*(b*B + 7*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{A+Bx^2}{\sqrt{x}\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{bB}{2}+\frac{7Ac}{2}\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16b^2c}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{5/2}c}+\frac{(3(b\cdots}{\cdots}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{5/2}c^{3/2}}+\cdots$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}-\frac{3(bB+7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}c^{5/4}}+\frac{3\cdots}{\cdots}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}-\frac{3(bB+7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}c^{5/4}}+\frac{3(bB+7Ac)\,\mathrm{t}\cdots}{32\sqrt{\cdots}}$$

Mathematica [A] time = 0.38, size = 230, normalized size = 0.78

$$\frac{\frac{(7Ac+bB)\left(7\left(b+cx^2\right)\left(8b^{3/4}\sqrt[4]{c}\sqrt{x}-3\sqrt{2}\left(b+cx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\tan^{-1}\left(\cdots\right.\right.\right.\right.}{b^{11/4}\sqrt[4]{c}}}{896c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-256*B*Sqrt[x] + ((b*B + 7*A*c)*(32*b^(7/4)*c^(1/4)*Sqrt[x] + 7*(b + c*x^2
)*(8*b^(3/4)*c^(1/4)*Sqrt[x] - 3*Sqrt[2]*(b + c*x^2)*(2*ArcTan[1 - (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
] + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b
] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))))/(b^(11/4)*c^(1/4)))/(8
96*c*(b + c*x^2)^2)
```

fricas [B] time = 0.90, size = 793, normalized size = 2.71

$$12\left(b^2c^3x^4+2\,b^3c^2x^2+b^4c\right)\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6c^2\sqrt{-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B\cdots}{\cdots}}}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c
+ 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*
arctan((sqrt(b^6*c^2*sqrt(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2
+ 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5)) + (B^2*b^2 + 14*A*B*b*c + 49
*A^2*c^2)*x)*b^8*c^4*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 13
72*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(3/4) - (B*b^9*c^4 + 7*A*b^8*c^5
)*sqrt(x)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*
c^3 + 2401*A^4*c^4)/(b^11*c^5))^(3/4))/(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*
B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)) + 3*(b^2*c^3*x^4 + 2*b^3*c^
2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3
*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*log(3*b^3*c*(-(B^4*b^4 + 28*A*B^
3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5)
)^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c
)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 24
01*A^4*c^4)/(b^11*c^5))^(1/4)*log(-3*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 29
4*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*
(B*b + 7*A*c)*sqrt(x)) - 4*(3*B*b^2 - 11*A*b*c - (B*b*c + 7*A*c^2)*x^2)*sqr
t(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

giac [A] time = 0.20, size = 293, normalized size = 1.00

$$\frac{3\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb+7\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^2}+\frac{3\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb+7\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\right.\right.}{2\left(\right.}\right.}{64\,b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/64*sqrt(2)*((b*
c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b
+ 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^2) - 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(
2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) + 1/16*(B*b*c*x^(5/2) + 7
*A*c^2*x^(5/2) - 3*B*b^2*sqrt(x) + 11*A*b*c*sqrt(x))/((c*x^2 + b)^2*b^2*c)
```

maple [A] time = 0.06, size = 325, normalized size = 1.11

$$\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^3}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^3}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32*(7*A*c+B*b)/b^2*x^(5/2)+1/32*(11*A*c-3*B*b)/b/c*x^(1/2))/(c*x^2+b)^
2+21/64/b^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+21/
128/b^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+21/64/b^3*(b/c)^(1/4)*2^(1/2
```

```
)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/64/b^2/c*(b/c)^(1/4)*2^(1/2)*B*
arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+3/128/b^2/c*(b/c)^(1/4)*2^(1/2)*B*ln(
(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+
(b/c)^(1/2)))+3/64/b^2/c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)
```

maxima [A] time = 3.05, size = 276, normalized size = 0.94

$$\frac{\left(Bbc+7\,Ac^{2}\right)x^{\frac{5}{2}}-\left(3\,Bb^{2}-11\,Abc\right)\sqrt{x}}{16\left(b^{2}c^{3}x^{4}+2\,b^{3}c^{2}x^{2}+b^{4}c\right)}+\frac{3\left(\frac{2\sqrt{2}\,(Bb+7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bb+7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-\cdots\right.}{2\,\sqrt{\sqrt{b}\,\cdots}}\right.}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((B*b*c + 7*A*c^2)*x^(5/2) - (3*B*b^2 - 11*A*b*c)*sqrt(x))/(b^2*c^3*x^
4 + 2*b^3*c^2*x^2 + b^4*c) + 3/128*(2*sqrt(2)*(B*b + 7*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 7*A*c)*arctan(-1/2*sqrt(2)
*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt
(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 7*A*c)*log(sqrt(2)*b^(1/4)*c^(1
/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b + 7*A*c
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1
/4)))/(b^2*c)
```

mupad [B] time = 0.39, size = 780, normalized size = 2.66

$$\frac{\frac{x^{5/2}\,(7\,A\,c+B\,b)}{16\,b^{2}}+\frac{\sqrt{x}\,(11\,A\,c-3\,B\,b)}{16\,b\,c}}{b^{2}+2\,b\,c\,x^{2}+c^{2}\,x^{4}}-\frac{\mathrm{atan}\left(\frac{\frac{(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^{2}\,c^{3}+14\,A\,B\,b\,c^{2}+B^{2}\,b^{2}\,c\right)}{64\,b^{4}}-\frac{9\,(7\,A\,c+B\,b)\left(7\,A\,c^{3}+B\,b\,c^{2}\right)}{64\,(-b)^{15/4}\,c^{5/4}}\right)3\mathrm{i}}{64\,(-b)^{11/4}\,c^{5/4}}+\frac{(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^{2}\,c^{3}+14\,A\cdots\right.}{64\,b^{4}}\cdots\right.}{\cdots}}{\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^{2}\,c^{3}+14\,A\,B\,b\,c^{2}+B^{2}\,b^{2}\,c\right)}{64\,b^{4}}-\frac{9\,(7\,A\,c+B\,b)\left(7\,A\,c^{3}+B\,b\,c^{2}\right)}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}-\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\,\sqrt{x}\,\left(49\,A^{2}\,c^{3}+14\cdots\right.}{64\,b\cdots}\cdots\right.}{\cdots}}\right)\cdots}{32\,(-b)^{11/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(5/2)*(7*A*c + B*b))/(16*b^2) + (x^(1/2)*(11*A*c - 3*B*b))/(16*b*c))/(b
^2 + c^2*x^4 + 2*b*c*x^2) - (atan((((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 +
 B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2)
)/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4)) + ((7*A*c + B*b)*(
(9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A*c +
B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(
5/4)))/((3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2
))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)*c^(5/4)
)))/(64*(-b)^(11/4)*c^(5/4)) - (3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B
^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2))/
(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4))))*(7*A*c + B*b)*3i)/(32
*(-b)^(11/4)*c^(5/4)) - (3*atan(((3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 +
 B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9
i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4)) + (3*(7*A*c + B*b)*(
(9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + ((7*A*c + B*
b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/
4)))/(((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(
```

```
64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4)))*
3i)/(64*(-b)^(11/4)*c^(5/4)) - ((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2
*b^2*c + 14*A*B*b*c^2))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(
64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4))))*(7*A*c + B*b))/(32*
(-b)^(11/4)*c^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.214 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=316

$$\frac{5(bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\,\mathrm{ta}}{32\sqrt{}}$$

```
[Out] -5/64*(-9*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)/c^(3/
4)*2^(1/2)+5/64*(-9*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1
3/4)/c^(3/4)*2^(1/2)+5/128*(-9*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4
)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)-5/128*(-9*A*c+B*b)*ln(b^(1/2)+x
*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)+5/16*(-9
*A*c+B*b)/b^3/c/x^(1/2)+1/4*(A*c-B*b)/b/c/(c*x^2+b)^2/x^(1/2)+1/16*(9*A*c-B
*b)/b^2/c/(c*x^2+b)/x^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5(bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\,\mathrm{ta}}{32\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*(b*B - 9*A*c))/(16*b^3*c*Sqrt[x]) - (b*B - A*c)/(4*b*c*Sqrt[x]*(b + c*x^
2)^2) - (b*B - 9*A*c)/(16*b^2*c*Sqrt[x]*(b + c*x^2)) - (5*(b*B - 9*A*c)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)*c^(3/4)) +
 (5*(b*B - 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2
]*b^(13/4)*c^(3/4)) + (5*(b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)*c^(3/4)) - (5*(b*B - 9*A*c)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13
/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}+\frac{\left(-\frac{bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)}-\frac{(5(bB-9Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$=\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)}+\frac{(5(bB-9Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$=\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)}+\frac{(5(bB-9Ac))\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,d\right.}{16b^3}$$

$$=\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)}-\frac{(5(bB-9Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}}{b+cx^4}\right.}{32b^3\sqrt{c}}$$

$$=\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)}+\frac{(5(bB-9Ac))\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}-\sqrt{2}}\right.}{64b^3c}$$

$$=\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)}+\frac{5(bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{64\sqrt{2}\,b^{13/4}c^{3/4}}$$

$$=\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)}-\frac{5(bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{13/4}c^{3/4}}$$

Mathematica [C] time = 0.29, size = 147, normalized size = 0.47

$$\frac{2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^4}-\frac{2Acx^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^4}-\frac{2A}{b^3\sqrt{x}}+\frac{A\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{(-b)^{13/4}}+\frac{Ab\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{}}{\sqrt[4]{-b}}\right.}{(-b)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]

[Out] (-2*A)/(b^3*Sqrt[x]) + (A*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b
)^(13/4) + (A*b*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4)
- (2*A*c*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^4) + (2
*(b*B - A*c)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^4)
```

fricas [B] time = 0.99, size = 988, normalized size = 3.13

$$20\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)\left(-\frac{B^4b^4-36\,AB^3b^3c+486\,A^2B^2b^2c^2-2916\,A^3Bbc^3+6561\,A^4c^4}{b^{13}c^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^6-54\,AB^5b^5c+1215\,A^2B\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c +
486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*ar
ctan((sqrt((B^6*b^6 - 54*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 - 14580*A^3*B^3
*b^3*c^3 + 98415*A^4*B^2*b^2*c^4 - 354294*A^5*B*b*c^5 + 531441*A^6*c^6)*x -
 (B^4*b^11*c - 36*A*B^3*b^10*c^2 + 486*A^2*B^2*b^9*c^3 - 2916*A^3*B*b^8*c^4
 + 6561*A^4*b^7*c^5)*sqrt(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2
- 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3)))*b^3*c*(-(B^4*b^4 - 36*A*B^3
*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))
^(1/4) + (B^3*b^6*c - 27*A*B^2*b^5*c^2 + 243*A^2*B*b^4*c^3 - 729*A^3*b^3*c^
4)*sqrt(x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b
*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4))/(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2
*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)) - 5*(b^3*c^2*x^5 + 2*b^4*c
*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*
B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*log(125*b^10*c^2*(-(B^4*b^4 - 36*
A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*
c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3
)*sqrt(x)) + 5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^
3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1
/4)*log(-125*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2
916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2
*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) + 4*(5*(B*b*c - 9*A*c^2)*x
^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^
3 + b^5*x)
```

giac [A] time = 0.21, size = 300, normalized size = 0.95

$$-\frac{2\,A}{b^3\sqrt{x}}+\frac{5\,Bbcx^{\frac{7}{2}}-13\,Ac^2x^{\frac{7}{2}}+9\,Bb^2x^{\frac{3}{2}}-17\,Abcx^{\frac{3}{2}}}{16\,\left(cx^2+b\right)^2b^3}+\frac{5\,\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-9\,\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right.}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{64\,b^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2*A/(b^3*sqrt(x)) + 1/16*(5*B*b*c*x^(7/2) - 13*A*c^2*x^(7/2) + 9*B*b^2*x^(
3/2) - 17*A*b*c*x^(3/2))/((c*x^2 + b)^2*b^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*
B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^4*c^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(
3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^4*c^3) - 5/128*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^3) + 5/128*sqrt(2)*((b*c
^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b^4*c^3)
```

maple [A] time = 0.07, size = 363, normalized size = 1.15

$$-\frac{13A\,c^2x^{\frac{7}{2}}}{16\left(c\,x^2+b\right)^2b^3}+\frac{5Bc\,x^{\frac{7}{2}}}{16\left(c\,x^2+b\right)^2b^2}-\frac{17Ac\,x^{\frac{3}{2}}}{16\left(c\,x^2+b\right)^2b^2}+\frac{9B\,x^{\frac{3}{2}}}{16\left(c\,x^2+b\right)^2b}-\frac{45\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3}-\frac{45\sqrt{2}\,A\,\mathrm{a}\cdots}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -13/16/b^3/(c*x^2+b)^2*x^(7/2)*A*c^2+5/16/b^2/(c*x^2+b)^2*x^(7/2)*B*c-17/16
/b^2/(c*x^2+b)^2*A*x^(3/2)*c+9/16/b/(c*x^2+b)^2*B*x^(3/2)-45/128/b^3/(b/c)^
(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/
4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-45/64/b^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/b^3/(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)-1)+5/128/b^2/c/(b/c)^(1/4)*2^(1/2)*B*ln((x-(b/c)^(1/4)*
2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+5
/64/b^2/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+5/64/
b^2/c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2*A/b^3/x
^(1/2)
```

maxima [A] time = 3.07, size = 255, normalized size = 0.81

$$\frac{5\left(Bbc-9\,Ac^2\right)x^4-32\,Ab^2+9\left(Bb^2-9\,Abc\right)x^2}{16\left(b^3c^2x^{\frac{9}{2}}+2\,b^4cx^{\frac{5}{2}}+b^5\sqrt{x}\right)}+\frac{5\left(Bb-9\,Ac\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\cdots\right.}{2\cdots}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\cdots}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*(B*b*c - 9*A*c^2)*x^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)/(b^3*c^
2*x^(9/2) + 2*b^4*c*x^(5/2) + b^5*sqrt(x)) + 5/128*(B*b - 9*A*c)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sq
rt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(
1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

mupad [B] time = 0.26, size = 133, normalized size = 0.42

$$\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(9\,A\,c-B\,b)}{32\,(-b)^{13/4}\,c^{3/4}}-\frac{\frac{2\,A}{b}+\frac{9\,x^2\,(9\,A\,c-B\,b)}{16\,b^2}+\frac{5\,c\,x^4\,(9\,A\,c-B\,b)}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(9\,A\,c-B\,b)}{32\,(-b)^{13/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/4
)) - ((2*A)/b + (9*x^2*(9*A*c - B*b))/(16*b^2) + (5*c*x^4*(9*A*c - B*b))/(1
```

```
6*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*x^(5/2)) - (5*atanh((c^(1/4)*x^(
1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.215 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{7(3bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}+\frac{7(3bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}-\frac{7(3bB-}{}$$

```
[Out] 7/48*(-11*A*c+3*B*b)/b^3/c/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)^2+1/
16*(11*A*c-3*B*b)/b^2/c/x^(3/2)/(c*x^2+b)-7/64*(-11*A*c+3*B*b)*arctan(1-c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)+7/64*(-11*A*c+3*B*b)
*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)-7/128*(
-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/
4)/c^(1/4)*2^(1/2)+7/128*(-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/
4)*2^(1/2)*x^(1/2))/b^(15/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)}+\frac{7(3bB-11Ac)}{48b^3cx^{3/2}}-\frac{7(3bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}+\frac{7(3bB-11Ac)\log\left(\sqrt{}\right.}{64\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (7*(3*b*B - 11*A*c))/(48*b^3*c*x^(3/2)) - (b*B - A*c)/(4*b*c*x^(3/2)*(b + c
*x^2)^2) - (3*b*B - 11*A*c)/(16*b^2*c*x^(3/2)*(b + c*x^2)) - (7*(3*b*B - 11
*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)*c
^(1/4)) + (7*(3*b*B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(15/4)*c^(1/4)) - (7*(3*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)*c^(1/4)) + (7*(
3*b*B - 11*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(15/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{3bB}{2}+\frac{11Ac}{2}\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{(7(3bB-11Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}}{32b^3}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\,\mathrm{Subst}\left(\int \frac{}{b+}\right.}{16b^3}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{}}{}\right.}{32b^{7/2}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{}}{\sqrt{}}}\right.}{64b^{7/2}\sqrt{}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{7(3bB-11Ac)\log\left(\sqrt{b}-\sqrt{}\right.}{64\sqrt{2}\,b^{15/4}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{7(3bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{}}{}\right.}{32\sqrt{2}\,b^{15/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.50, size = 400, normalized size = 1.24

$$-\frac{96Ab^{7/4}c\sqrt{x}}{\left(b+cx^2\right)^2} - \frac{360Ab^{3/4}c\sqrt{x}}{b+cx^2} - \frac{256Ab^{3/4}}{x^{3/2}} + \frac{42\sqrt{2}(11Ac-3bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2}(3bB-11Ac)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}} + 231\sqrt{2}\,A$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-256*A*b^(3/4))/x^(3/2) + (96*b^(11/4)*B*Sqrt[x])/(b + c*x^2)^2 - (96*A*b
^(7/4)*c*Sqrt[x])/(b + c*x^2)^2 + (168*b^(7/4)*B*Sqrt[x])/(b + c*x^2) - (36
0*A*b^(3/4)*c*Sqrt[x])/(b + c*x^2) + (42*Sqrt[2]*(-3*b*B + 11*A*c)*ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (42*Sqrt[2]*(3*b*B - 11*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (63*Sqrt[2]*b*B
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + 231*
```

```
Sqrt[2]*A*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
] + (63*Sqrt[2]*b*B*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/c^(1/4) - 231*Sqrt[2]*A*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x] + Sqrt[c]*x])/(384*b^(15/4))
```

fricas [B] time = 0.85, size = 809, normalized size = 2.51

$$-\frac{84\left(b^3c^2x^6+2\,b^4cx^4+b^5x^2\right)\left(-\frac{81\,B^4b^4-1188\,AB^3b^3c+6534\,A^2B^2b^2c^2-15972\,A^3Bbc^3+14641\,A^4c^4}{b^{15}c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^8\sqrt{-\frac{81\,B^4b^4-11\cdots}{\cdots}}\cdots}}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c)
)^(1/4)*arctan((sqrt(b^8*sqrt(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^
2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c)) + (9*B^2*b^2 - 66*
A*B*b*c + 121*A^2*c^2)*x)*b^11*c*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^
2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(3/4) + (3*B*b
^12*c - 11*A*b^11*c^2)*sqrt(x)*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*
B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(3/4))/(81*B^4*b
^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^
4*c^4)) + 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B
^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*
c))^(1/4)*log(7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2
 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c)*
sqrt(x)) - 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*
B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15
*c))^(1/4)*log(-7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c
^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c
)*sqrt(x)) - 4*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*
b*c)*x^2)*sqrt(x))/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
```

giac [A] time = 0.20, size = 304, normalized size = 0.94

$$\frac{7\sqrt{2}\left(3\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c}+\frac{7\sqrt{2}\left(3\left(bc^3\right)^{\frac{1}{4}}Bb-11\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\cdots\right)}{64\,b^4c}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/64*sqrt(2)*(3*
(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*
B*b - 11*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c)
)/(b^4*c) - 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(
-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) - 2/3*A/(b^3*x^(3/2))
 + 1/16*(7*B*b*c*x^(5/2) - 15*A*c^2*x^(5/2) + 11*B*b^2*sqrt(x) - 19*A*b*c*s
qrt(x))/((c*x^2 + b)^2*b^3)
```

maple [A] time = 0.07, size = 357, normalized size = 1.11

$$-\frac{15A\,c^2x^{\frac{5}{2}}}{16\left(c\,x^2+b\right)^2b^3}+\frac{7Bc\,x^{\frac{5}{2}}}{16\left(c\,x^2+b\right)^2b^2}-\frac{19Ac\sqrt{x}}{16\left(c\,x^2+b\right)^2b^2}+\frac{11B\sqrt{x}}{16\left(c\,x^2+b\right)^2b}-\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^4}-\frac{77\ \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/16/b^3/(c*x^2+b)^2*x^(5/2)*A*c^2+7/16/b^2/(c*x^2+b)^2*x^(5/2)*B*c-19/16
/b^2/(c*x^2+b)^2*A*x^(1/2)*c+11/16/b/(c*x^2+b)^2*B*x^(1/2)-77/64/b^4*(b/c)^
(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)*c-77/128/b^4*(b/c)^(1
/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)
*2^(1/2)*x^(1/2)+(b/c)^(1/2)))*c-77/64/b^4*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)*c+21/64/b^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2
)/(b/c)^(1/4)*x^(1/2)-1)+21/128/b^3*(b/c)^(1/4)*2^(1/2)*B*ln((x+(b/c)^(1/4)
*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+
21/64/b^3*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-2/3*A
/b^3/x^(3/2)
```

maxima [A] time = 3.09, size = 285, normalized size = 0.89

$$\frac{7\left(3\,Bbc-11\,Ac^2\right)x^4-32\,Ab^2+11\left(3\,Bb^2-11\,Abc\right)x^2}{48\left(b^3c^2x^{\frac{11}{2}}+2\,b^4cx^{\frac{7}{2}}+b^5x^{\frac{3}{2}}\right)}+\frac{7\left(\frac{2\sqrt{2}\,(3\,Bb-11\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(3\,Bb-11\,A\cdots}{\cdots}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*b*c)*x^2)/
(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7/2) + b^5*x^(3/2)) + 7/128*(2*sqrt(2)*(3*B*
b - 11*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(3*B*b
 - 11*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(3*B*b -
 11*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4
)*c^(1/4)) - sqrt(2)*(3*B*b - 11*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

mupad [B] time = 0.45, size = 888, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((2*A)/(3*b) + (11*x^2*(11*A*c - 3*B*b))/(48*b^2) + (7*c*x^4*(11*A*c - 3*
B*b))/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11/2) + 2*b*c*x^(7/2)) - (atan((((11*
A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 529858
56*A*B*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^
```

```
14*c^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)) + ((11*A*c
- 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A
*B*b^10*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c
^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)))/((7*(11*A*c -
3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B
*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3
))/(64*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4)) - (7*(11*A*c - 3*B*b
)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10
*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3))/(6
4*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4))))*(11*A*c - 3*B*b)*7i)/(3
2*(-b)^(15/4)*c^(1/4)) - (7*atan(((7*(11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^
2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) - ((11*A*c - 3*B*
b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4))
))/(64*(-b)^(15/4)*c^(1/4)) + (7*(11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^
9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) + ((11*A*c - 3*B*b)*(
80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4))))/(
64*(-b)^(15/4)*c^(1/4)))/(((11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5
+ 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) - ((11*A*c - 3*B*b)*(807403
52*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*
(-b)^(15/4)*c^(1/4)) - ((11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7
225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) + ((11*A*c - 3*B*b)*(80740352*
A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b
)^(15/4)*c^(1/4))))*(11*A*c - 3*B*b))/(32*(-b)^(15/4)*c^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.216 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{9\sqrt[4]{c}\,(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}+\frac{9\sqrt[4]{c}\,(5bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}+\cdots$$

```
[Out] 9/80*(-13*A*c+5*B*b)/b^3/c/x^(5/2)+1/4*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)^2+1/
16*(13*A*c-5*B*b)/b^2/c/x^(5/2)/(c*x^2+b)+9/64*c^(1/4)*(-13*A*c+5*B*b)*arct
an(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/64*c^(1/4)*(-13*A*
c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/128*c
^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/b^(17/4)*2^(1/2)+9/128*c^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/
4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)-9/16*(-13*A*c+5*B*b)/b^4/x^(1/
2)
```

Rubi [A] time = 0.28, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}+\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{9\sqrt[4]{c}\,(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (9*(5*b*B - 13*A*c))/(80*b^3*c*x^(5/2)) - (9*(5*b*B - 13*A*c))/(16*b^4*Sqrt
[x]) - (b*B - A*c)/(4*b*c*x^(5/2)*(b + c*x^2)^2) - (5*b*B - 13*A*c)/(16*b^2
*c*x^(5/2)*(b + c*x^2)) + (9*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (9*c^(1/4)*(5*b*B - 13*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (
9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4)) + (9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{5bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \frac{(9(5bB-13Ac))\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + \frac{(9(5bB-13Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - (9c(5bB-$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - (9c(5bB-$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + \left(9\sqrt{c}(5bB\right.$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - (9(5bB-1$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - 9\sqrt[4]{c}(5bB$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + 9\sqrt[4]{c}(5bB$$

Mathematica [C] time = 0.52, size = 189, normalized size = 0.55

$$-\frac{2cx^{3/2}(bB-2Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^5}+\frac{2cx^{3/2}(Ac-bB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^5}+\frac{6Ac-2bB}{b^4\sqrt{x}}-\frac{2A}{5b^3x^{5/2}}+\frac{\sqrt[4]{c}(3Ac-bB)\,t}{(-b)^1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*A)/(5*b^3*x^(5/2)) + (-2*b*B + 6*A*c)/(b^4*Sqrt[x]) + (c^(1/4)*(-(b*B)
+ 3*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) + (c^(1/4)*(b*B
- 3*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(-b)^(17/4) - (2*c*(b*B - 2
*A*c)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^5) + (2*c*
```

```
(-(b*B) + A*c)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^5
)
```

fricas [B] time = 0.93, size = 1043, normalized size = 3.04

$$-\frac{180\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)\left(-\frac{625\,B^4b^4c-6500\,AB^3b^3c^2+25350\,A^2B^2b^2c^3-43940\,A^3Bbc^4+28561\,A^4c^5}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(15625\,B^6b\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A
*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b
^17)^(1/4)*arctan((sqrt((15625*B^6*b^6*c^2 - 243750*A*B^5*b^5*c^3 + 1584375
*A^2*B^4*b^4*c^4 - 5492500*A^3*B^3*b^3*c^5 + 10710375*A^4*B^2*b^2*c^6 - 111
38790*A^5*B*b*c^7 + 4826809*A^6*c^8)*x - (625*B^4*b^13*c - 6500*A*B^3*b^12*
c^2 + 25350*A^2*B^2*b^11*c^3 - 43940*A^3*B*b^10*c^4 + 28561*A^4*b^9*c^5)*sq
rt(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3
*B*b*c^4 + 28561*A^4*c^5)/b^17))*b^4*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2
+ 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4) +
(125*B^3*b^7*c - 975*A*B^2*b^6*c^2 + 2535*A^2*B*b^5*c^3 - 2197*A^3*b^4*c^4)
*sqrt(x)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43
940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4))/(625*B^4*b^4*c - 6500*A*B^3*b
^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)) - 45*(
b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2
+ 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*lo
g(729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 -
43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A
*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 45*(b^4*c^2*x^7
+ 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*
B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*log(-729*b^13*
(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B
*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A*B^2*b^2*c^
2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 4*(45*(5*B*b*c^2 - 13*A*c^3
)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32*A*b^3 + 32*(5*B*b^3 - 13*A*b^2
*c)*x^2)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

giac [A] time = 0.22, size = 326, normalized size = 0.95

$$-\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c^2}-\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\right.}{64\,b^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -9/64*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) - 9/64*sqrt(2)*
(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) + 9/128*sqrt(2)*(5*(b*c^3)^(
3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt
(b/c))/(b^5*c^2) - 9/128*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*
```

```
c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c^2) - 1/16*(13*B
*b*c^2*x^(7/2) - 21*A*c^3*x^(7/2) + 17*B*b^2*c*x^(3/2) - 25*A*b*c^2*x^(3/2)
)/((c*x^2 + b)^2*b^4) - 2/5*(5*B*b*x^2 - 15*A*c*x^2 + A*b)/(b^4*x^(5/2))
```

maple [A] time = 0.07, size = 381, normalized size = 1.11

$$\frac{21Ac^3x^{\frac{7}{2}}}{16\left(cx^2+b\right)^2b^4}-\frac{13Bc^2x^{\frac{7}{2}}}{16\left(cx^2+b\right)^2b^3}+\frac{25Ac^2x^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^3}-\frac{17Bcx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^2}+\frac{117\sqrt{2}\,Ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^4}+\frac{117\sqrt{2}\,\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 21/16/b^4*c^3/(c*x^2+b)^2*x^(7/2)*A-13/16/b^3*c^2/(c*x^2+b)^2*x^(7/2)*B+25/
16/b^3*c^2/(c*x^2+b)^2*A*x^(3/2)-17/16/b^2*c/(c*x^2+b)^2*B*x^(3/2)+117/128/
b^4*c/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/
(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+117/64/b^4*c/(b/c)^(1/4)*2^(1/
2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+117/64/b^4*c/(b/c)^(1/4)*2^(1/2)
*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-45/128/b^3/(b/c)^(1/4)*2^(1/2)*B*l
n((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2
)+(b/c)^(1/2)))-45/64/b^3/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)+1)-45/64/b^3/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/
2)-1)-2/5*A/b^3/x^(5/2)+6/b^4/x^(1/2)*A*c-2/b^3/x^(1/2)*B
```

maxima [A] time = 3.04, size = 285, normalized size = 0.83

$$-\frac{45\left(5\,Bbc^2-13\,Ac^3\right)x^6+81\left(5\,Bb^2c-13\,Abc^2\right)x^4+32\,Ab^3+32\left(5\,Bb^3-13\,Ab^2c\right)x^2}{80\left(b^4c^2x^{\frac{13}{2}}+2\,b^5cx^{\frac{9}{2}}+b^6x^{\frac{5}{2}}\right)}-\frac{9\left(5\,Bbc-13\,Ac^2\right)\left(\frac{2\sqrt{2}\,\ldots}{\ldots}\ldots\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/80*(45*(5*B*b*c^2 - 13*A*c^3)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32
*A*b^3 + 32*(5*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c^2*x^(13/2) + 2*b^5*c*x^(9/2)
 + b^6*x^(5/2)) - 9/128*(5*B*b*c - 13*A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*
(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(
sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(
c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

mupad [B] time = 0.19, size = 152, normalized size = 0.44

$$\frac{\frac{2\,x^2\,\left(13\,A\,c-5\,B\,b\right)}{5\,b^2}-\frac{2\,A}{5\,b}+\frac{9\,c^2\,x^6\,\left(13\,A\,c-5\,B\,b\right)}{16\,b^4}+\frac{81\,c\,x^4\,\left(13\,A\,c-5\,B\,b\right)}{80\,b^3}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}}+\frac{9\,{\left(-c\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,\left(13\,A\,c-5\,B\,b\right)}{32\,b^{17/4}}-\frac{9\,{\left(-c\right)}^{1/4}\,\mathrm{at}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((2*x^2*(13*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (9*c^2*x^6*(13*A*c - 5*B*
b))/(16*b^4) + (81*c*x^4*(13*A*c - 5*B*b))/(80*b^3))/(b^2*x^(5/2) + c^2*x^(
13/2) + 2*b*c*x^(9/2)) + (9*(-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(
13*A*c - 5*B*b))/(32*b^(17/4)) - (9*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b
^(1/4))*(13*A*c - 5*B*b))/(32*b^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.217 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=343

$$\frac{11c^{3/4}(7bB-15Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+$$

```
[Out] 11/112*(-15*A*c+7*B*b)/b^3/c/x^(7/2)-11/48*(-15*A*c+7*B*b)/b^4/x^(3/2)+1/4*
(A*c-B*b)/b/c/x^(7/2)/(c*x^2+b)^2+1/16*(15*A*c-7*B*b)/b^2/c/x^(7/2)/(c*x^2+
b)+11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(19/4)*2^(1/2)-11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(
1/2)/b^(1/4))/b^(19/4)*2^(1/2)+11/128*c^(3/4)*(-15*A*c+7*B*b)*ln(b^(1/2)+x*
c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)-11/128*c^(3/4)*(-
15*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4
)*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11c^{3/4}(7bB-15Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (11*(7*b*B - 15*A*c))/(112*b^3*c*x^(7/2)) - (11*(7*b*B - 15*A*c))/(48*b^4*x
^(3/2)) - (b*B - A*c)/(4*b*c*x^(7/2)*(b + c*x^2)^2) - (7*b*B - 15*A*c)/(16*
b^2*c*x^(7/2)*(b + c*x^2)) + (11*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B -
15*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)
) + (11*c^(3/4)*(7*b*B - 15*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B - 15*A*c)*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4
))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}+\frac{\left(-\frac{7bB}{2}+\frac{15Ac}{2}\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$=-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11(7bB-15Ac))\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}+\frac{(11(7bB-15Ac))\int \frac{1}{x^{5/2}\left(b+cx\right.}}{32b^3}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11c(7bE}{}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11c(7bE}{}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11c(7bE}{}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{\left(11\sqrt{c}\,(7\right.}{}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}+\frac{11c^{3/4}(7l}{}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}+\frac{11c^{3/4}(7l}{}$$

Mathematica [A] time = 0.57, size = 433, normalized size = 1.26

$$\frac{\frac{672Ab^{7/4}c^2\sqrt{x}}{\left(b+cx^2\right)^2}+\frac{3864Ab^{3/4}c^2\sqrt{x}}{b+cx^2}+\frac{5376Ab^{3/4}c}{x^{3/2}}-\frac{768Ab^{7/4}}{x^{7/2}}+462\sqrt{2}\,c^{3/4}(7bB-15Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)+462\sqrt{2}\,c^{3/4}(1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-768*A*b^(7/4))/x^(7/2) - (1792*b^(7/4)*B)/x^(3/2) + (5376*A*b^(3/4)*c)/x
^(3/2) - (672*b^(11/4)*B*c*Sqrt[x])/(b + c*x^2)^2 + (672*A*b^(7/4)*c^2*Sqrt
[x])/(b + c*x^2)^2 - (2520*b^(7/4)*B*c*Sqrt[x])/(b + c*x^2) + (3864*A*b^(3/
```

```
4)*c^2*Sqrt[x])/(b + c*x^2) + 462*Sqrt[2]*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 462*Sqrt[2]*c^(3/4)*(-7*b*B + 15*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 1617*Sqrt[2]*b*B*c^(3/4)
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 3465*Sqrt[2]*
A*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 1617
*Sqrt[2]*b*B*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x] + 3465*Sqrt[2]*A*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(2688*b^(19/4))
```

fricas [B] time = 1.03, size = 864, normalized size = 2.52

$$\frac{924\left(b^4c^2x^8+2\,b^5cx^6+b^6x^4\right)\left(-\frac{2401\,B^4b^4c^3-20580\,AB^3b^3c^4+66150\,A^2B^2b^2c^5-94500\,A^3Bbc^6+50625\,A^4c^7}{b^{19}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{10}\sqrt{-\frac{24\cdots}{}}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/1344*(924*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 205
80*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^
7)/b^19)^(1/4)*arctan((sqrt(b^10*sqrt(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*
c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19) + (4
9*B^2*b^2*c^2 - 210*A*B*b*c^3 + 225*A^2*c^4)*x)*b^14*(-(2401*B^4*b^4*c^3 -
20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4
*c^7)/b^19)^(3/4) + (7*B*b^15*c - 15*A*b^14*c^2)*sqrt(x)*(-(2401*B^4*b^4*c^
3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625
*A^4*c^7)/b^19)^(3/4))/(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*
B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)) + 231*(b^4*c^2*x^8 + 2*b^
5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^
2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4)*log(11*b^5*(-(24
01*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*
b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*A*c^2)*sqrt(x)) - 231
*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^
3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1
/4)*log(-11*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b
^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*
A*c^2)*sqrt(x)) - 4*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B*b^2*c - 15*A*
b*c^2)*x^4 + 96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)*sqrt(x))/(b^4*c^2*x^
8 + 2*b^5*c*x^6 + b^6*x^4)
```

giac [A] time = 0.22, size = 315, normalized size = 0.92

$$-\frac{11\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-15\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}-\frac{11\sqrt{2}\left(7\left(bc^3\right)^{\frac{1}{4}}Bb-15\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\cdots\right.}{64\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -11/64*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/64*sqrt(2)*(7*(
b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b
```

```
- 15*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^
5 + 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*log(-sqrt(2
)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 1/16*(15*B*b*c^2*x^(5/2) - 23*
A*c^3*x^(5/2) + 19*B*b^2*c*sqrt(x) - 27*A*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4
) - 2/21*(7*B*b*x^2 - 21*A*c*x^2 + 3*A*b)/(b^4*x^(7/2))
```

maple [A] time = 0.07, size = 390, normalized size = 1.14

$$\frac{23 A c^3 x^{\frac{5}{2}}}{16\left(c x^2+b\right)^2 b^4}-\frac{15 B c^2 x^{\frac{5}{2}}}{16\left(c x^2+b\right)^2 b^3}+\frac{27 A c^2 \sqrt{x}}{16\left(c x^2+b\right)^2 b^3}-\frac{19 B c \sqrt{x}}{16\left(c x^2+b\right)^2 b^2}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, A c^2 \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64 b^5}+ \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 23/16/b^4*c^3/(c*x^2+b)^2*x^(5/2)*A-15/16/b^3*c^2/(c*x^2+b)^2*x^(5/2)*B+27/
16/b^3*c^2/(c*x^2+b)^2*A*x^(1/2)-19/16/b^2*c/(c*x^2+b)^2*B*x^(1/2)+165/64/b
^5*c^2*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+165/128/
b^5*c^2*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+165/64/b^5*c^2*(b/c)^(1/4)*2
^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-77/64/b^4*c*(b/c)^(1/4)*2^(1
/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-77/128/b^4*c*(b/c)^(1/4)*2^(1/2
)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x
^(1/2)+(b/c)^(1/2)))-77/64/b^4*c*(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)+1)-2/7*A/b^3/x^(7/2)+2/b^4/x^(3/2)*A*c-2/3/b^3/x^(3/2)*B
```

maxima [A] time = 3.15, size = 321, normalized size = 0.94

$$-\frac{77\left(7 Bbc^2-15 Ac^3\right)x^6+121\left(7 Bb^2c-15 Abc^2\right)x^4+96 Ab^3+32\left(7 Bb^3-15 Ab^2c\right)x^2}{336\left(b^4c^2x^{\frac{15}{2}}+2 b^5cx^{\frac{11}{2}}+b^6x^{\frac{7}{2}}\right)}-\frac{11\left(\frac{2\sqrt{2}\left(7 Bbc-15 Ac^2\right)\operatorname{arcta}\cdots}{\sqrt{b}\sqrt{\cdots}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/336*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B*b^2*c - 15*A*b*c^2)*x^4 +
96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)/(b^4*c^2*x^(15/2) + 2*b^5*c*x^(11
/2) + b^6*x^(7/2)) - 11/128*(2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 15*A*c^2)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2
)*(7*B*b*c - 15*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(3/4)*c^(1/4)))/b^4
```

mupad [B] time = 0.45, size = 626, normalized size = 1.83

$$\frac{\frac{2\,x^2\,(15\,A\,c-7\,B\,b)}{21\,b^2}-\frac{2\,A}{7\,b}+\frac{11\,c^2\,x^6\,(15\,A\,c-7\,B\,b)}{48\,b^4}+\frac{121\,c\,x^4\,(15\,A\,c-7\,B\,b)}{336\,b^3}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}}+\frac{11\,(-c)^{3/4}\,\mathrm{atan}\left(\frac{11\,(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^{\cdots}\right.\right.}{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^7-4\cdots\right.\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((2*x^2*(15*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (11*c^2*x^6*(15*A*c - 7*
B*b))/(48*b^4) + (121*c*x^4*(15*A*c - 7*B*b))/(336*b^3))/(b^2*x^(7/2) + c^2
*x^(15/2) + 2*b*c*x^(11/2)) + (11*(-c)^(3/4)*atan(((11*(-c)^(3/4)*(15*A*c -
 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631744
0*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8074
0352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)) + (11*(-c)^(3/4)*(15*A*
c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631
7440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8
0740352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)))/(((-c)^(3/4)*(15*A*
c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631
7440*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8
0740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4)) - ((-c)^(3/4)*(1
5*A*c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 4
16317440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5
 - 80740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4))))*(15*A*c -
7*B*b))/(32*b^(19/4)) - ((-c)^(3/4)*atan((A^3*c^8*x^(1/2)*3375i - B^3*b^3*c
^5*x^(1/2)*343i - A^2*B*b*c^7*x^(1/2)*4725i + A*B^2*b^2*c^6*x^(1/2)*2205i)/
(b^(1/4)*(-c)^(19/4)*(c*(c*(3375*A^3*c - 4725*A^2*B*b) + 2205*A*B^2*b^2) -
343*B^3*b^3)))*(15*A*c - 7*B*b)*11i)/(32*b^(19/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.218 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=365

$$\frac{13c^{5/4}(9bB-17Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}-\frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}$$

```
[Out] 13/144*(-17*A*c+9*B*b)/b^3/c/x^(9/2)-13/80*(-17*A*c+9*B*b)/b^4/x^(5/2)+1/4*
(A*c-B*b)/b/c/x^(9/2)/(c*x^2+b)^2+1/16*(17*A*c-9*B*b)/b^2/c/x^(9/2)/(c*x^2+
b)-13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(21/4)*2^(1/2)+13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(
1/2)/b^(1/4))/b^(21/4)*2^(1/2)+13/128*c^(5/4)*(-17*A*c+9*B*b)*ln(b^(1/2)+x*
c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)-13/128*c^(5/4)*(-
17*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4
)*2^(1/2)+13/16*c*(-17*A*c+9*B*b)/b^5/x^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13c^{5/4}(9bB-17Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}-\frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (13*(9*b*B - 17*A*c))/(144*b^3*c*x^(9/2)) - (13*(9*b*B - 17*A*c))/(80*b^4*x
^(5/2)) + (13*c*(9*b*B - 17*A*c))/(16*b^5*Sqrt[x]) - (b*B - A*c)/(4*b*c*x^(
9/2)*(b + c*x^2)^2) - (9*b*B - 17*A*c)/(16*b^2*c*x^(9/2)*(b + c*x^2)) - (13
*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^
(21/4)) - (13*c^(5/4)*(9*b*B - 17*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{9bB}{2}+\frac{17Ac}{2}\right)\int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)} - \frac{(13(9bB-17Ac))\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)} + \frac{(13(9bB-17Ac))\int \frac{1}{x^{7/2}(b+c}}{32b^3}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)} - \frac{(13c(9bB}{}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-}{16b^2cx^{9/2}(}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-}{16b^2cx^{9/2}(}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-}{16b^2cx^{9/2}(}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-}{16b^2cx^{9/2}(}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-}{16b^2cx^{9/2}(}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2} - \frac{9bB-}{16b^2cx^{9/2}(}$$

Mathematica [C] time = 0.55, size = 216, normalized size = 0.59

$$\frac{2c^2x^{3/2}(2bB-3Ac)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^6} + \frac{2c^2x^{3/2}(bB-Ac)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^6} + \frac{6c(bB-2Ac)}{b^5\sqrt{x}} - \frac{2(bB-3Ac)}{5b^4x^{5/2}} - \frac{2A}{9b^3x^{9/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*A)/(9*b^3*x^(9/2)) - (2*(b*B - 3*A*c))/(5*b^4*x^(5/2)) + (6*c*(b*B - 2*
A*c))/(b^5*Sqrt[x]) - (3*c^(5/4)*(b*B - 2*A*c)*ArcTan[(c^(1/4)*Sqrt[x])/(-b
)^(1/4)])/(-b)^(21/4) + (3*c^(5/4)*(b*B - 2*A*c)*ArcTanh[(c^(1/4)*Sqrt[x])/
(-b)^(1/4)])/(-b)^(21/4) + (2*c^2*(2*b*B - 3*A*c)*x^(3/2)*Hypergeometric2F1
[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^6) + (2*c^2*(b*B - A*c)*x^(3/2)*Hypergeom
etric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^6)
```

fricas [B] time = 1.25, size = 1093, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2880*(2340*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49
572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4
*c^9)/b^21)^(1/4)*arctan((sqrt((531441*B^6*b^6*c^8 - 6022998*A*B^5*b^5*c^9
+ 28441935*A^2*B^4*b^4*c^10 - 71631540*A^3*B^3*b^3*c^11 + 101478015*A^4*B^2
*b^2*c^12 - 76672278*A^5*B*b*c^13 + 24137569*A^6*c^14)*x - (6561*B^4*b^15*c
^5 - 49572*A*B^3*b^14*c^6 + 140454*A^2*B^2*b^13*c^7 - 176868*A^3*B*b^12*c^8
 + 83521*A^4*b^11*c^9)*sqrt(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 1404
54*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21))*b^5*(-(6561
*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*
b*c^8 + 83521*A^4*c^9)/b^21)^(1/4) + (729*B^3*b^8*c^4 - 4131*A*B^2*b^7*c^5
+ 7803*A^2*B*b^6*c^6 - 4913*A^3*b^5*c^7)*sqrt(x)*(-(6561*B^4*b^4*c^5 - 4957
2*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c
^9)/b^21)^(1/4))/(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b
^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)) - 585*(b^5*c^2*x^9 + 2*b^6*c*
x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b
^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(2197*b^16*(-(6
561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3
*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*
b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) + 585*(b^5*c^2*x^9 + 2*
b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2
*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(-2197*b^
16*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176
868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131
*A*B^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) + 4*(585*(9*B*b*
c^3 - 17*A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^6 - 160*A*b^4 + 416
*(9*B*b^3*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^3*c)*x^2)*sqrt(x))/(
b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)
```

giac [A] time = 0.26, size = 351, normalized size = 0.96

$$\frac{13\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{3}{4}}Bb-17\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}c}+\frac{13\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{3}{4}}Bb-17\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\cdots\right)}{64\,b^{6}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 13/64*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) + 13/64*sqrt(2)*(
9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) - 13/128*sqrt(2)*(9*(b*c^3)^(3/
4)*B*b - 17*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
```

```
/c))/(b^6*c) + 13/128*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*
log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^6*c) + 1/16*(21*B*b*c^
3*x^(7/2) - 29*A*c^4*x^(7/2) + 25*B*b^2*c^2*x^(3/2) - 33*A*b*c^3*x^(3/2))/(
(c*x^2 + b)^2*b^5) + 2/45*(135*B*b*c*x^4 - 270*A*c^2*x^4 - 9*B*b^2*x^2 + 27
*A*b*c*x^2 - 5*A*b^2)/(b^5*x^(9/2))
```

maple [A] time = 0.07, size = 414, normalized size = 1.13

$$-\frac{29 A c^4 x^{\frac{7}{2}}}{16\left(c x^2+b\right)^2 b^5}+\frac{21 B c^3 x^{\frac{7}{2}}}{16\left(c x^2+b\right)^2 b^4}-\frac{33 A c^3 x^{\frac{3}{2}}}{16\left(c x^2+b\right)^2 b^4}+\frac{25 B c^2 x^{\frac{3}{2}}}{16\left(c x^2+b\right)^2 b^3}-\frac{221\sqrt{2}\, A c^2 \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^5}-\frac{221\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -29/16/b^5*c^4/(c*x^2+b)^2*A*x^(7/2)+21/16/b^4*c^3/(c*x^2+b)^2*B*x^(7/2)-33
/16/b^4*c^3/(c*x^2+b)^2*x^(3/2)*A+25/16/b^3*c^2/(c*x^2+b)^2*x^(3/2)*B-221/1
28/b^5*c^2/(b/c)^(1/4)*2^(1/2)*A*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1
/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-221/64/b^5*c^2/(b/c)^(1/4
)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-221/64/b^5*c^2/(b/c)^(1/4
)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+117/128/b^4*c/(b/c)^(1/4)
*2^(1/2)*B*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^
(1/2)*x^(1/2)+(b/c)^(1/2)))+117/64/b^4*c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)+1)+117/64/b^4*c/(b/c)^(1/4)*2^(1/2)*B*arctan(2^(1/2)
/(b/c)^(1/4)*x^(1/2)-1)-2/9*A/b^3/x^(9/2)+6/5/b^4/x^(5/2)*A*c-2/5/b^3/x^(5/
2)*B-12*c^2/b^5/x^(1/2)*A+6*c/b^4/x^(1/2)*B
```

maxima [A] time = 3.08, size = 311, normalized size = 0.85

$$\frac{585\left(9 Bbc^3-17 Ac^4\right)x^8+1053\left(9 Bb^2c^2-17 Abc^3\right)x^6-160 Ab^4+416\left(9 Bb^3c-17 Ab^2c^2\right)x^4-32\left(9 Bb^4-17 A\right.}{720\left(b^5c^2x^{\frac{17}{2}}+2 b^6cx^{\frac{13}{2}}+b^7x^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/720*(585*(9*B*b*c^3 - 17*A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^6
 - 160*A*b^4 + 416*(9*B*b^3*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^3*
c)*x^2)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) + 13/128*(9*B*b
*c^2 - 17*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(
2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(
2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(
3/4)))/b^5
```

mupad [B] time = 0.29, size = 173, normalized size = 0.47

$$\frac{13\,(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(17\,A\,c-9\,B\,b)}{32\,b^{21/4}}-\frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(17\,A\,c-9\,B\,b)}{45\,b^2}+\frac{117\,c^2\,x^6\,(17\,A\,c-9\,B\,b)}{80\,b^4}+\frac{13\,c^3\,x^8\,(17\,A\,c-9\,B\,b)}{16\,b^5}+\frac{26\,c\,x^4\,(}{}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (13*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b))/(32*b^(
21/4)) - ((2*A)/(9*b) - (2*x^2*(17*A*c - 9*B*b))/(45*b^2) + (117*c^2*x^6*(1
7*A*c - 9*B*b))/(80*b^4) + (13*c^3*x^8*(17*A*c - 9*B*b))/(16*b^5) + (26*c*x
^4*(17*A*c - 9*B*b))/(45*b^3))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(13/2)
) - (13*(-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b))/(3
2*b^(21/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.219 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=365

$$-\frac{15c^{7/4}(11bB-19Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}$$

```
[Out] 15/176*(-19*A*c+11*B*b)/b^3/c/x^(11/2)-15/112*(-19*A*c+11*B*b)/b^4/x^(7/2)+
5/16*c*(-19*A*c+11*B*b)/b^5/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(11/2)/(c*x^2+b)^2+
1/16*(19*A*c-11*B*b)/b^2/c/x^(11/2)/(c*x^2+b)-15/64*c^(7/4)*(-19*A*c+11*B*b
)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)+15/64*c^(7/4)*
(-19*A*c+11*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)
-15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/
2)*x^(1/2))/b^(23/4)*2^(1/2)+15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1584, 457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{15c^{7/4}(11bB-19Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] (15*(11*b*B - 19*A*c))/(176*b^3*c*x^(11/2)) - (15*(11*b*B - 19*A*c))/(112*b
^4*x^(7/2)) + (5*c*(11*b*B - 19*A*c))/(16*b^5*x^(3/2)) - (b*B - A*c)/(4*b*c
*x^(11/2)*(b + c*x^2)^2) - (11*b*B - 19*A*c)/(16*b^2*c*x^(11/2)*(b + c*x^2)
) - (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) - (15*c^(7/4)*(11*b*B
 - 19*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{11bB}{2}+\frac{19Ac}{2}\right)\int \frac{1}{x^{13/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} - \frac{(15(11bB-19Ac))\int \frac{1}{x^{13/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} + \frac{(15(11bB-19Ac))\int \cdots}{32b^3}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} - \cdots$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \cdots$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \cdots$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \cdots$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \cdots$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \cdots$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \cdots$$

Mathematica [A] time = 0.63, size = 467, normalized size = 1.28

$$\frac{-\frac{19096Ab^{3/4}c^3\sqrt{x}}{b+cx^2} - \frac{2464Ab^{7/4}c^3\sqrt{x}}{\left(b+cx^2\right)^2} - \frac{39424Ab^{3/4}c^2}{x^{3/2}} + \frac{8448Ab^{7/4}c}{x^{7/2}} - \frac{1792Ab^{11/4}}{x^{11/2}} + 2310\sqrt{2}\,c^{7/4}(19Ac-11bB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\sqrt[4]{b}\cdots}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-1792*A*b^(11/4))/x^(11/2) - (2816*b^(11/4)*B)/x^(7/2) + (8448*A*b^(7/4)*
c)/x^(7/2) + (19712*b^(7/4)*B*c)/x^(3/2) - (39424*A*b^(3/4)*c^2)/x^(3/2) +
(2464*b^(11/4)*B*c^2*Sqrt[x])/(b + c*x^2)^2 - (2464*A*b^(7/4)*c^3*Sqrt[x])/
(b + c*x^2)^2 + (14168*b^(7/4)*B*c^2*Sqrt[x])/(b + c*x^2) - (19096*A*b^(3/4
)*c^3*Sqrt[x])/(b + c*x^2) + 2310*Sqrt[2]*c^(7/4)*(-11*b*B + 19*A*c)*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 2310*Sqrt[2]*c^(7/4)*(11*b*B - 19
*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 12705*Sqrt[2]*b*B*c^(
7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 21945*Sqr
t[2]*A*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
+ 12705*Sqrt[2]*b*B*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x] - 21945*Sqrt[2]*A*c^(11/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(9856*b^(23/4))
```

fricas [B] time = 0.83, size = 894, normalized size = 2.45

$$-\frac{4620\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)\left(-\frac{14641\,B^4b^4c^7-101156\,AB^3b^3c^8+262086\,A^2B^2b^2c^9-301796\,A^3Bbc^{10}+130321\,A^4c^{11}}{b^{23}}\right)^{\frac{1}{4}}\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(4620*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 -
 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 1303
21*A^4*c^11)/b^23)^(1/4)*arctan((sqrt(b^12*sqrt(-(14641*B^4*b^4*c^7 - 10115
6*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4
*c^11)/b^23) + (121*B^2*b^2*c^4 - 418*A*B*b*c^5 + 361*A^2*c^6)*x)*b^17*(-(1
4641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A
^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(3/4) + (11*B*b^18*c^2 - 19*A*b^17*c^3
)*sqrt(x)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*
c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(3/4))/(14641*B^4*b^4*c^
7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 1
30321*A^4*c^11)) + 1155*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4
*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c
^10 + 130321*A^4*c^11)/b^23)^(1/4)*log(15*b^6*(-(14641*B^4*b^4*c^7 - 101156
*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*
c^11)/b^23)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqrt(x)) - 1155*(b^5*c^2*x^1
0 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 26
2086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4)*l
og(-15*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2
*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4) - 15*(11*B*b*c^2
- 19*A*c^3)*sqrt(x)) - 4*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c
^2 - 19*A*b*c^3)*x^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32
*(11*B*b^4 - 19*A*b^3*c)*x^2)*sqrt(x))/(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^
6)
```

giac [A] time = 0.28, size = 351, normalized size = 0.96

$$\frac{15\sqrt{2}\left(11\left(bc^3\right)^{\frac{1}{4}}Bbc-19\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}+\frac{15\sqrt{2}\left(11\left(bc^3\right)^{\frac{1}{4}}Bbc-19\left(bc^3\right)^{\frac{1}{4}}Ac^2\right)\mathrm{arc}}{64\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 15/64*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/64*sqrt(2)*
(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/128*sqrt(2)*(11*(b*c^3)^
(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/b^6 - 15/128*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)
*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 + 1/16*(23*B*
b*c^3*x^(5/2) - 31*A*c^4*x^(5/2) + 27*B*b^2*c^2*sqrt(x) - 35*A*b*c^3*sqrt(x
))/((c*x^2 + b)^2*b^5) + 2/77*(77*B*b*c*x^4 - 154*A*c^2*x^4 - 11*B*b^2*x^2
+ 33*A*b*c*x^2 - 7*A*b^2)/(b^5*x^(11/2))
```

maple [A] time = 0.07, size = 420, normalized size = 1.15

$$-\frac{31 A c^{4} x^{\frac{5}{2}}}{16 \left(c x^{2}+b\right)^{2} b^{5}}+\frac{23 B c^{3} x^{\frac{5}{2}}}{16 \left(c x^{2}+b\right)^{2} b^{4}}-\frac{35 A c^{3} \sqrt{x}}{16 \left(c x^{2}+b\right)^{2} b^{4}}+\frac{27 B c^{2} \sqrt{x}}{16 \left(c x^{2}+b\right)^{2} b^{3}}-\frac{285 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, A c^{3} \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64 b^{6}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] -31/16/b^5*c^4/(c*x^2+b)^2*x^(5/2)*A+23/16/b^4*c^3/(c*x^2+b)^2*x^(5/2)*B-35
/16/b^4*c^3/(c*x^2+b)^2*A*x^(1/2)+27/16/b^3*c^2/(c*x^2+b)^2*B*x^(1/2)-285/6
4/b^6*c^3*(b/c)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-285/1
28/b^6*c^3*(b/c)^(1/4)*2^(1/2)*A*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1
/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-285/64/b^6*c^3*(b/c)^(1/4
)*2^(1/2)*A*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+165/64/b^5*c^2*(b/c)^(1/4
)*2^(1/2)*B*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)+165/128/b^5*c^2*(b/c)^(1/
4)*2^(1/2)*B*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*
2^(1/2)*x^(1/2)+(b/c)^(1/2)))+165/64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*B*arctan(2
^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-2/11*A/b^3/x^(11/2)+6/7/b^4/x^(7/2)*A*c-2/7/b
^3/x^(7/2)*B-4*c^2/b^5/x^(3/2)*A+2*c/b^4/x^(3/2)*B
```

maxima [A] time = 3.10, size = 353, normalized size = 0.97

$$\frac{385 \left(11 B b c^{3}-19 A c^{4}\right) x^{8}+605 \left(11 B b^{2} c^{2}-19 A b c^{3}\right) x^{6}-224 A b^{4}+160 \left(11 B b^{3} c-19 A b^{2} c^{2}\right) x^{4}-32 \left(11 B b^{4}-1\right.}{1232 \left(b^{5} c^{2} x^{\frac{19}{2}}+2 b^{6} c x^{\frac{15}{2}}+b^{7} x^{\frac{11}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/1232*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c^2 - 19*A*b*c^3)*x
^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32*(11*B*b^4 - 19*A*
b^3*c)*x^2)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) + 15/128*(
2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c
))) + 2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4
)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)
*sqrt(c))) + sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(11*B*b*c^2 - 19*A
```

```
*c^3)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*
c^(1/4)))/b^5
```

mupad [B] time = 0.51, size = 639, normalized size = 1.75

$$\frac{15\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{6859\,A^3\,c^{10}\,\sqrt{x}-1331\,B^3\,b^3\,c^7\,\sqrt{x}-11913\,A^2\,B\,b\,c^9\,\sqrt{x}+6897\,A\,B^2\,b^2\,c^8\,\sqrt{x}}{b^{1/4}\,{\left(-c\right)}^{27/4}\,\left(c\,\left(c\,\left(6859\,A^3\,c-11913\,A^2\,B\,b\right)+6897\,A\,B^2\,b^2\right)-1331\,B^3\,b^3\right)}\right)\,\left(19\,A\,c-11\,B\,b\right)}{32\,b^{23/4}}-\frac{\frac{2\,A}{11\,b}-\frac{2\,x^2\,\left(19\,A\,c-\right.}{77\,b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (15*(-c)^(7/4)*atan((6859*A^3*c^10*x^(1/2) - 1331*B^3*b^3*c^7*x^(1/2) - 119
13*A^2*B*b*c^9*x^(1/2) + 6897*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(
c*(c*(6859*A^3*c - 11913*A^2*B*b) + 6897*A*B^2*b^2) - 1331*B^3*b^3)))*(19*A
*c - 11*B*b))/(32*b^(23/4)) - ((2*A)/(11*b) - (2*x^2*(19*A*c - 11*B*b))/(77
*b^2) + (55*c^2*x^6*(19*A*c - 11*B*b))/(112*b^4) + (5*c^3*x^8*(19*A*c - 11*
B*b))/(16*b^5) + (10*c*x^4*(19*A*c - 11*B*b))/(77*b^3))/(b^2*x^(11/2) + c^2
*x^(19/2) + 2*b*c*x^(15/2)) - ((-c)^(7/4)*atan((((-c)^(7/4)*(19*A*c - 11*B*
b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 1540915200*
A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21*c^6 - 17
3015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)) + ((-c)^(7/4)*(19*A*
c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 15
40915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21
*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)))/((15*(-c)^
(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b
^17*c^7 - 1540915200*A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(2988
44160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))/(64*b^(23/4)) - (
15*(-c)^(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 4460544
00*B^2*b^17*c^7 - 1540915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*
b)*(298844160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))/(64*b^(23
/4))))*(19*A*c - 11*B*b)*15i)/(32*b^(23/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.220 $\int x^{5/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=243

$$-\frac{2b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}+\frac{4b^2\sqrt{bx^2+cx^4}(3bB-5Ac)}{231c^3\sqrt{x}}-\frac{4bx^{3/2}\sqrt{bx^2}}{}$$

```
[Out] 2/15*B*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c-4/385*b*(-5*A*c+3*B*b)*x^(3/2)*(c*x^4+
b*x^2)^(1/2)/c^2-2/55*(-5*A*c+3*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+4/231*b^
2*(-5*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-2/231*b^(11/4)*(-5*A*c+3*B
*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(
1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/
(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$\frac{4b^2\sqrt{bx^2+cx^4}(3bB-5Ac)}{231c^3\sqrt{x}}-\frac{2b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}-\frac{4bx^{3/2}\sqrt{bx^2+}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b^2*(3*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (4*b*(3*b*B
 - 5*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^2) - (2*(3*b*B - 5*A*c)*x^(7/
2)*Sqrt[b*x^2 + c*x^4])/(55*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/(15*c)
 - (2*b^(11/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sq
rt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/
(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{5/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}-\frac{\left(2\left(\frac{9bB}{2}-\frac{15Ac}{2}\right)\right)\int x^{5/2}\sqrt{bx^2+cx^4}\,dx}{15c}$$

$$=-\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c}+\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}-\frac{(2b(3bB-5Ac))}{55}$$

$$=-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}-\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c}+\frac{2Bx^{3/2}}{}$$

$$=\frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}-\frac{2(3bB-5}{}$$

$$=\frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}-\frac{2(3bB-5}{}$$

$$=\frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}-\frac{2(3bB-5}{}$$

$$=\frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}-\frac{2(3bB-5}{}$$

Mathematica [C] time = 0.18, size = 136, normalized size = 0.56

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\left(-3bc\left(25A+21Bx^2\right)+7c^2x^2\left(15A+11Bx^2\right)+45b^2B\right)+15b^2(5Ac-3b\right.}{1155c^3\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 7*c^2
*x^2*(15*A + 11*B*x^2) - 3*b*c*(25*A + 21*B*x^2)) + 15*b^2*(-3*b*B + 5*A*c)
*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(1155*c^3*Sqrt[x]*Sqrt[1
 + (c*x^2)/b])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bx^4 + Ax^2\right)\sqrt{cx^4 + bx^2}\,\sqrt{x}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

maple [A] time = 0.20, size = 307, normalized size = 1.26

$$2\sqrt{c\,x^4 + b\,x^2}\,\left(77B\,c^5x^9 + 105A\,c^5x^7 + 91Bb\,c^4x^7 + 135Ab\,c^4x^5 - 4B\,b^2c^3x^5 - 20A\,b^2c^3x^3 + 12B\,b^3c^2x^3 - 50A\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(77*B*x^9*c^5+105*A*x^7*c^5+91
*B*x^7*b*c^4+25*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c+135*A*x^5*
b*c^4-15*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4-4*B*x^5*b^2*c^3-20*
A*x^3*b^2*c^3+12*B*x^3*b^3*c^2-50*A*x*b^3*c^2+30*B*x*b^4*c)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{5/2}\,\left(B\,x^2 + A\right)\,\sqrt{c\,x^4 + b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.221 $\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=369

$$\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-13}{195c^{11/4}\sqrt{bx^2+cx}}$$

```
[Out] 2/13*B*(c*x^4+b*x^2)^(3/2)*x^(1/2)/c+4/195*b^2*(-13*A*c+7*B*b)*x^(3/2)*(c*x
^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+7*B*b)
*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-4/585*b*(-13*A*c+7*B*b)*x^(1/2)*(c*x^4+b*x^2
)^(1/2)/c^2-4/195*b^(9/4)*(-13*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+2/195*b^(9/4)*(-1
3*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/
c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{4b^2x^{3/2}\left(b+cx^2\right)(7bB-13Ac)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-4b^{9/4}x\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b^2*(7*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(7*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^
4])/(585*c^2) - (2*(7*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117*c) +
(2*B*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(13*c) - (4*b^(9/4)*(7*b*B - 13*A*c)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4]
) + (2*b^(9/4)*(7*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{3/2}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{13Ac}{2}\right)\right)\int x^{3/2}\sqrt{bx^2+cx^4}\,dx}{13c}$$

$$= -\frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{13c} - \frac{(2b(7bB-13Ac))}{117c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\,(}{}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\,(}{}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\,(}{}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\,(}{}$$

$$= \frac{4b^2(7bB-13Ac)x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB}{}$$

Mathematica [C] time = 0.14, size = 111, normalized size = 0.30

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(b(7bB-13Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\left(-13Ac+7bB-9Bcx^2\right)\right)}{117c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(7*b*B
- 13*A*c - 9*B*c*x^2)) + b*(7*b*B - 13*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/
4, -((c*x^2)/b)]))/(117*c^2*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{cx^4+bx^2}\left(Bx^3+Ax\right)\sqrt{x}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^3 + A*x)*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2}\left(Bx^2+A\right)x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

maple [A] time = 0.10, size = 446, normalized size = 1.21

$$-\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(-45B\,c^4x^8-65A\,c^4x^6-55Bb\,c^3x^6-91Ab\,c^3x^4+4B\,b^2c^2x^4-26A\,b^2c^2x^2+14B\,b^3c\,x^2+78\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(-45*B*x^8*c^4-65*A*x^6*c^
4-55*B*x^6*b*c^3+78*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ellip
ticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-39*A*b^3*c*((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))-42*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*
EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+21*B*b^4*((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))-91*A*x^4*b*c^3+4*B*b^2*c^2*x^4-26*A*x^2*b^2*c^2+
14*B*x^2*b^3*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+bx^2}\left(Bx^2+A\right)x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.222 $\int \sqrt{x}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=204

$$\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{bx^2+cx^4}(5bB-11Ac)}{231c^2\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2+\cdots}}{}$$

```
[Out] 2/11*B*(c*x^4+b*x^2)^(3/2)/c/x^(1/2)-2/77*(-11*A*c+5*B*b)*x^(3/2)*(c*x^4+b*
x^2)^(1/2)/c-4/231*b*(-11*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+2/231*
b^(7/4)*(-11*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{bx^2+cx^4}(5bB-11Ac)}{231c^2\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2+\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-4*b*(5*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (2*(5*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) + (2*B*(b*x^2 + c*x^4)^(3/2))/
(11*c*Sqrt[x]) + (2*b^(7/4)*(5*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
```

```
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{11Ac}{2}\right)\right)\int\sqrt{x}\sqrt{bx^2+cx^4}\,dx}{11c}$$

$$= -\frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{11c\sqrt{x}} - \frac{(2b(5bB-11Ac))\int}{77c}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2\right.}{11c}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2\right.}{11c}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2\right.}{11c}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B\left(bx^2\right.}{11c}$$

Mathematica [C] time = 0.14, size = 111, normalized size = 0.54

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(b(5bB-11Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\left(-11Ac+5bB-7Bcx^2\right)\right)}{77c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(5*b*B - 11*A*c
 - 7*B*c*x^2)) + b*(5*b*B - 11*A*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^2)/b)]))/(77*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

maple [A] time = 0.10, size = 283, normalized size = 1.39

$$-\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(-21B\,c^4x^7-33A\,c^4x^5-27Bb\,c^3x^5-55Ab\,c^3x^3+4B\,b^2c^2x^3-22A\,b^2c^2x+10B\,b^3cx+11\sqrt{-bc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(-21*B*x^7*c^4+11*A*(-b*c)^(1/
2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*b^2*c-33*A*x^5*c^4-5*B*(-b*c)^(1/2)*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/
(-b*c)^(1/2)*c*x)^(1/2)*b^3-27*B*x^5*b*c^3-55*A*x^3*b*c^3+4*B*x^3*b^2*c^2-2
2*A*x*b^2*c^2+10*B*x*b^3*c)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{x}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \sqrt{x^2 \left(b + cx^2\right)} \left(A + Bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

3.223 $\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal. Leaf size=326

$$-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/9*B*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)-4/15*b*(-3*A*c+B*b)*x^(3/2)*(c*x^2+b)/c
^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-3*A*c+B*b)*x^(1/2)*(c
*x^4+b*x^2)^(1/2)/c+4/15*b^(5/4)*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-2/15*b^(5/4)*(
-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c
^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2039, 2021, 2032, 329, 305, 220, 1196}

$$-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

```
[Out] (-4*b*(b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[b*x^2 + c*x^4]) - (2*(b*B - 3*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(15*c)
 + (2*B*(b*x^2 + c*x^4)^(3/2))/(9*c*x^(3/2)) + (4*b^(5/4)*(b*B - 3*A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (2*b^(5/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{9c}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{(2b(bB-3Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,d}{15c}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(2b(bB-3Ac)x\sqrt{b+cx^2}\right)\int}{15c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(4b(bB-3Ac)x\sqrt{b+cx^2}\right)\mathrm{S}}{15c\sqrt{bx^2}}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(4b^{3/2}(bB-3Ac)x\sqrt{b+cx^2}\right.}{15c^{3/2}\sqrt{b}}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4}{9cx^{3/2}}$$

Mathematica [C] time = 0.09, size = 94, normalized size = 0.29

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left((3Ac-bB)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+B\sqrt{\frac{cx^2}{b}+1}\,(b+cx^2)\right)}{9c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*
B) + 3*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 +
 (c*x^2)/b])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,(Bx^2+A)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\,(Bx^2+A)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)

maple [A] time = 0.09, size = 422, normalized size = 1.29

$$\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\,\left(5B\,c^{3}x^{6}+9A\,c^{3}x^{4}+7Bb\,c^{2}x^{4}+9Ab\,c^{2}x^{2}+2B\,b^{2}c\,x^{2}+18\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x)

[Out] 2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(5*B*c^3*x^6+18*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c-9*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3+3*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3+9*A*x^4*c^3+7*B*b*c^2*x^4+9*A*x^2*b*c^2+2*B*x^2*b^2*c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\,\left(Bx^2+A\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2),x)

[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(1/2),x)

[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/sqrt(x), x)

3.224 $\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(bB-7Ac)}{21c\sqrt{x}}+\frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}}$$

```
[Out] 2/7*B*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)-2/21*(-7*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c
/x^(1/2)-2/21*b^(3/4)*(-7*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2
)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2021, 2032, 329, 220}

$$-\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(bB-7Ac)}{21c\sqrt{x}}+\frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

```
[Out] (-2*(b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4
)^(3/2))/(7*c*x^(5/2)) - (2*b^(3/4)*(b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{7c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}} - \frac{(2b(bB-7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}} - \frac{\left(2b(bB-7Ac)x\sqrt{b+cx^2}\right)\int}{21c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}} - \frac{\left(4b(bB-7Ac)x\sqrt{b+cx^2}\right)\mathrm{Su}}{21c\sqrt{bx^2}}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B\left(bx^2+cx^4\right)^{3/2}}{7cx^{5/2}} - \frac{2b^{3/4}(bB-7Ac)x\left(\sqrt{b}+\sqrt{c}\right.}{21c^{5}}$$

Mathematica [C] time = 0.06, size = 94, normalized size = 0.57

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left((7Ac-bB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)+B\sqrt{\frac{cx^2}{b}+1}\left(b+cx^2\right)\right)}{7c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*B) + 7*A
*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 +
 (c*x^2)/b])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

maple [A] time = 0.09, size = 257, normalized size = 1.56

$$\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(3B\,c^3x^5+7A\,c^3x^3+5Bb\,c^2x^3+7Ab\,c^2x+2B\,b^2cx+7\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}}{21\left(c\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(7*A*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1
/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(-b*c)^(1/2)*b*c-B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2+3*B*c^
3*x^5+7*A*x^3*c^3+5*B*x^3*b*c^2+7*A*x*b*c^2+2*B*x*b^2*c)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(3/2), x)
```

3.225 $\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal. Leaf size=323

$$\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5Ac+b}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2*A*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)+4/5*(5*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/2
)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/5*(5*A*c+B*b)*x^(1/2)*(c*x^4+b*
x^2)^(1/2)/b-4/5*b^(1/4)*(5*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*b^(1/4)*(5*A*c+B*b)
*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2038, 2021, 2032, 329, 305, 220, 1196}

$$\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5Ac+b}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

```
[Out] (4*(b*B + 5*A*c)*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[b*x^2 + c*x^4]) + (2*(b*B + 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*A*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (4*b^(1/4)*(b*B + 5*A*c)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(1
/4)*(b*B + 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sq
rt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{b}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{1}{5}(2(bB+5Ac))\int \frac{x^{3/}}{\sqrt{bx^2+}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{\left(2(bB+5Ac)x\sqrt{b+cx^2}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{\left(4(bB+5Ac)x\sqrt{b+cx^2}\right)}{5\sqrt{bx^2}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} + \frac{\left(4\sqrt{b}(bB+5Ac)x\sqrt{b+cx}\right.}{5\sqrt{c}\,\sqrt{}}$$

$$= \frac{4(bB+5Ac)x^{3/2}\left(b+cx^2\right)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A\left(bx^2+cx\right.}{bx^{7/2}}$$

Mathematica [C] time = 0.05, size = 97, normalized size = 0.30

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(x^2(5Ac+bB)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-3A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{3bx^{3/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (b*B + 5*A
*c)*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*x^(3/2)*Sqrt
[1 + (c*x^2)/b])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)

maple [A] time = 0.11, size = 399, normalized size = 1.24

$$2\sqrt{c\,x^4+b\,x^2}\,\left(B\,c^2x^4-5A\,c^2x^2+Bbc\,x^2+10\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x)

[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(10*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-5*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c+2*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+B*c^2*x^4-5*A*x^2*c^2+B*x^2*b*c-5*A*b*c)/c

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2),x)

[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(5/2),x)

[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(5/2), x)

3.226 $\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal. Leaf size=163

$$\frac{2\sqrt{bx^2+cx^4}\,(Ac+bB)}{3b\sqrt{x}}+\frac{2x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}}$$

```
[Out] -2/3*A*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)+2/3*(A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b/x^
(1/2)+2/3*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(1/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2021, 2032, 329, 220}

$$\frac{2\sqrt{bx^2+cx^4}\,(Ac+bB)}{3b\sqrt{x}}+\frac{2x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*(b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(3*b*Sqrt[x]) - (2*A*(b*x^2 + c*x^4)^(3
/2))/(3*b*x^(9/2)) + (2*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(3*b^(1/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + -\frac{\left(2\left(-\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{3b}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{1}{3}(2(bB+Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+c}}}{3\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{\left(4(bB+Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int\right.}{3\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{3bx^{9/2}} + \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c}{\left(\sqrt{b}+\right.}}}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2}}$$

Mathematica [C] time = 0.05, size = 97, normalized size = 0.60

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(3x^2(Ac+bB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{3bx^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b]) + 3*(b*B + A
*c)*x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*Sqrt
[1 + (c*x^2)/b])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

maple [A] time = 0.08, size = 239, normalized size = 1.47

$$\frac{2\sqrt{c\,x^4 + b\,x^2}\,\left(B\,c^2x^4 - A\,c^2x^2 + Bbc\,x^2 + \sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Acx\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{3\left(c\,x^2 + b\right)c\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x)
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1
/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*x*c+B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*E
llipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x*b+B*c^2*x^4
-A*c^2*x^2+B*b*c*x^2-A*b*c)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2 + A\right)\sqrt{c\,x^4 + b\,x^2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(7/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(7/2), x)
```

3.227 $\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal. Leaf size=328

$$\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+5bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+5bB)E}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/5*A*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)+4/5*(A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^(
1/2)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/5*(A*c+5*B*b)*(c*x^4+b*x^2
)^(1/2)/b/x^(3/2)-4/5*c^(1/4)*(A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*c^(1/4)*(A*c+5
*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2
^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)
/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2038, 2020, 2032, 329, 305, 220, 1196}

$$\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+5bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+5bB)E}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

```
[Out] (4*Sqrt[c]*(5*b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*(5*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(5*b*x^(3/2)) -
(2*A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^(11/2)) - (4*c^(1/4)*(5*b*B + A*c)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2
*c^(1/4)*(5*b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{\left(2\left(-\frac{5bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx}{5b}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{(2c(5bB+Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(2c(5bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{}{\sqrt{}}}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(4c(5bB+Ac)x\sqrt{b+cx^2}\right)\text{Sub}}{5b\sqrt{bx^2+c}}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(4\sqrt{c}\,(5bB+Ac)x\sqrt{b+cx^2}\right)\text{Su}}{5\sqrt{b}\,\sqrt{bx^2}}$$

$$= \frac{4\sqrt{c}\,(5bB+Ac)x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - $$

Mathematica [C] time = 0.05, size = 96, normalized size = 0.29

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(x^2(Ac+5bB)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)+A(b+cx^2)\sqrt{\frac{cx^2}{b}+1}\right)}{5bx^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (5*b*B + A*c
)*x^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*Sqrt[
1 + (c*x^2)/b])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,(Bx^2+A)}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\,(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)

maple [A] time = 0.10, size = 422, normalized size = 1.29

$$\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(-2A\,c^2x^4-5Bbc\,x^4+2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x)

[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c+10*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2-5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2-2*A*c^2*x^4-5*B*b*c*x^4-3*A*b*c*x^2-5*B*b^2*x^2-b^2*A)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2),x)

[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(9/2),x)

[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(9/2), x)

3.228 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal. Leaf size=167

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(7bB-Ac)}{21bx^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}}$$

```
[Out] -2/7*A*(c*x^4+b*x^2)^(3/2)/b/x^(13/2)-2/21*(-A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)
/b/x^(5/2)+2/21*c^(3/4)*(-A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2020, 2032, 329, 220}

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(7bB-Ac)}{21bx^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

```
[Out] (-2*(7*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4
)^(3/2))/(7*b*x^(13/2)) + (2*c^(3/4)*(7*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx}{7b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{(2c(7bB-Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,d}{21b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{\left(2c(7bB-Ac)x\sqrt{b+cx^2}\right)\int}{21b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{\left(4c(7bB-Ac)x\sqrt{b+cx^2}\right)\mathrm{S}}{21b\sqrt{bx^2}}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{7bx^{13/2}} + \frac{2c^{3/4}(7bB-Ac)x\left(\sqrt{b}+\sqrt{c}\right.}{21b^5}$$

Mathematica [C] time = 0.06, size = 98, normalized size = 0.59

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(x^2(7bB-Ac)\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)+3A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{21bx^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (7*b*B - A
*c)*x^2*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)]))/(21*b*x^(9/2)*Sq
rt[1 + (c*x^2)/b])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

maple [A] time = 0.09, size = 255, normalized size = 1.53

$$-\frac{2\sqrt{c\,x^4 + b\,x^2}\left(2A\,c^2x^4 + 7Bbc\,x^4 + \sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ac\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right.\right.}{21\left(c\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))*x^3*c-7*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^3*b+2
*A*c^2*x^4+7*B*b*c*x^4+5*A*b*c*x^2+7*B*b^2*x^2+3*A*b^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2 + A\right)\,\sqrt{c\,x^4 + b\,x^2}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}\left(A + Bx^2\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(11/2), x)
```

3.229 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal. Leaf size=369

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-A\dots}{15b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/9*A*(c*x^4+b*x^2)^(3/2)/b/x^(15/2)+4/15*c^(3/2)*(-A*c+3*B*b)*x^(3/2)*(c*
x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-A*c+3*B*b)*(c*x^4
+b*x^2)^(1/2)/b/x^(7/2)-4/15*c*(-A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)
-4/15*c^(5/4)*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1
/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+2/15*c^(5/4)*(-A*c+3*B*b)*x*(cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1
/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2
)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4c^{3/2}x^{3/2}\left(b+cx^2\right)(3bB-Ac)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}\dots\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (4*c^(3/2)*(3*b*B - A*c)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(7/2)
) - (4*c*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) - (2*A*(b*x^2
+ c*x^4)^(3/2))/(9*b*x^(15/2)) - (4*c^(5/4)*(3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*(3
*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*
x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}dx}{9b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} + \frac{(2c(3bB-Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}}{15b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} +$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} +$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} +$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{3/2}}{9bx^{15/2}} +$$

$$= \frac{4c^{3/2}(3bB-Ac)x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx}}{15b^2x^{3/2}}$$

Mathematica [C] time = 0.06, size = 99, normalized size = 0.27

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(3x^2(3bB-Ac)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)+5A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{45bx^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + 3*(3*b*B -
 A*c)*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(11/2
)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

maple [A] time = 0.10, size = 452, normalized size = 1.22

$$-\frac{2\sqrt{c\,x^4+b\,x^2}\left(-6A\,c^3x^6+18Bb\,c^2x^6+6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ab\,c^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{\cdots}}{2}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^
(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*x^4*b*c^2-3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-18*B*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*x^4*b^2*c+9*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*
EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c-6*
A*c^3*x^6+18*B*b*c^2*x^6-4*A*b*c^2*x^4+27*B*b^2*c*x^4+7*A*b^2*c*x^2+9*B*b^3
*x^2+5*A*b^3)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

3.230 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}(11bB-5Ac)}{231b^2x^{5/2}}-\frac{2\sqrt{bx^2+}}{}$$

```
[Out] -2/11*A*(c*x^4+b*x^2)^(3/2)/b/x^(17/2)-2/77*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^(
1/2)/b/x^(9/2)-4/231*c*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-2/23
1*c^(7/4)*(-5*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/
2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2038, 2020, 2025, 2032, 329, 220}

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}(11bB-5Ac)}{231b^2x^{5/2}}-\frac{2\sqrt{bx^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

```
[Out] (-2*(11*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(77*b*x^(9/2)) - (4*c*(11*b*B - 5
*A*c)*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(3/2))/
(11*b*x^(17/2)) - (2*c^(7/4)*(11*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+Bx^2\right)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx &= -\frac{2A\left(bx^2+cx^4\right)^{3/2}}{11bx^{17/2}}-\frac{\left(2\left(-\frac{11bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx}{11b}\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{11bx^{17/2}}+\frac{(2c(11bB-5Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+c}}\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{11bx^{17/2}}-\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{11bx^{17/2}}-\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{11bx^{17/2}}-\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2A\left(bx^2+cx^4\right)^{3/2}}{11bx^{17/2}}-
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 98, normalized size = 0.48

$$
-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(x^2(11bB-5Ac)\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)+7A\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}\right)}{77bx^{13/2}\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(7*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (11*b*B -
5*A*c)*x^2*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(13/
2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

maple [A] time = 0.10, size = 283, normalized size = 1.39

$$2\sqrt{c\,x^4+b\,x^2}\,\left(10A\,c^3x^6-22Bb\,c^2x^6+5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,A\,c^2x^5\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{\cdot}}{\sqrt{-b}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x)
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^
(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(-b*c)^(1/2)*x^5*c^2-11*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*El
lipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x
^5*b*c+10*A*c^3*x^6-22*B*b*c^2*x^6+4*A*b*c^2*x^4-55*B*b^2*c*x^4-27*A*b^2*c*
x^2-33*B*b^3*x^2-21*A*b^3)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(15/2),x)
[Out] Timed out
```

3.231 $\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=486

$$\frac{44b^{21/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}-\frac{88b^{21/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3b\ldots}{16575c^{19/4}\sqrt{bx\ldots}}$$

```
[Out] -2/105*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(3/2)/c+2/25*B*x^(5/2)*(c*x^4+b
*x^2)^(5/2)/c+88/16575*b^5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(9/2)/(b^(1/2
)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+88/69615*b^3*(-5*A*c+3*B*b)*x^(5/2)*(c*x^4
+b*x^2)^(1/2)/c^3-8/7735*b^2*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1/2)/c^2
-4/595*b*(-5*A*c+3*B*b)*x^(13/2)*(c*x^4+b*x^2)^(1/2)/c-88/49725*b^4*(-5*A*c
+3*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^4-88/16575*b^(21/4)*(-5*A*c+3*B*b)*x*
(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*
(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/4)/(c*x^4
+b*x^2)^(1/2)+44/16575*b^(21/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8b^2x^{9/2}\sqrt{bx^2+cx^4}\,(3bB-5Ac)}{7735c^2}+\frac{88b^3x^{5/2}\sqrt{bx^2+cx^4}\,(3bB-5Ac)}{69615c^3}+\frac{88b^5x^{3/2}\left(b+cx^2\right)(3bB-5Ac)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{88b\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (88*b^5*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(16575*c^(9/2)*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[b*x^2 + c*x^4]) - (88*b^4*(3*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 +
c*x^4])/(49725*c^4) + (88*b^3*(3*b*B - 5*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/
(69615*c^3) - (8*b^2*(3*b*B - 5*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(7735*c^2
) - (4*b*(3*b*B - 5*A*c)*x^(13/2)*Sqrt[b*x^2 + c*x^4])/(595*c) - (2*(3*b*B
- 5*A*c)*x^(9/2)*(b*x^2 + c*x^4)^(3/2))/(105*c) + (2*B*x^(5/2)*(b*x^2 + c*x
^4)^(5/2))/(25*c) - (88*b^(21/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt[b*x^2 + c*x^4]) + (44*b^(21/4)*(3*b*
B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{\left(2\left(\frac{15bB}{2}-\frac{25Ac}{2}\right)\right)\int x^{7/2}\left(bx^2+cx^4\right)^{3/2}\,dx}{25c}$$

$$=-\frac{2(3bB-5Ac)x^{9/2}\left(bx^2+cx^4\right)^{3/2}}{105c}+\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}-\frac{(2b(3bB-5$$

$$=-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}-\frac{2(3bB-5Ac)x^{9/2}\left(bx^2+cx^4\right)^{3/2}}{105c}+$$

$$=-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}-\frac{2}{}$$

$$=\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}-\frac{4}{}$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-$$

$$=\frac{88b^5(3bB-5Ac)x^{3/2}\left(b+cx^2\right)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}$$

Mathematica [C] time = 0.30, size = 160, normalized size = 0.33

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(385b^4(3bB-5Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-55b^2c\left(35A+39Bx^2\right)+6\right.\right.}{116025c^4\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(1155
*b^3*B - 221*c^3*x^4*(25*A + 21*B*x^2) - 55*b^2*c*(35*A + 39*B*x^2) + 65*b*
c^2*x^2*(55*A + 51*B*x^2))) + 385*b^4*(3*b*B - 5*A*c)*Hypergeometric2F1[-3/
2, 3/4, 7/4, -((c*x^2)/b)]))/(116025*c^4*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bcx^9+(Bb+Ac)x^7+Abx^5\right)\sqrt{cx^4+bx^2}\,\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^9 + (B*b + A*c)*x^7 + A*b*x^5)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right) x^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

maple [A] time = 0.11, size = 518, normalized size = 1.07

$$-\frac{2 \left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}} \left(-13923 B\,c^7 x^{14} - 16575 A\,c^7 x^{12} - 31824 B b\,c^6 x^{12} - 39000 A b\,c^6 x^{10} - 18369 B\,b^2 c^5 x^{10} - 23325 A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -2/348075*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^5*(-13923*B*x^14*c^7-16
575*A*x^12*c^7-31824*B*x^12*b*c^6-39000*A*x^10*b*c^6-18369*B*x^10*b^2*c^5-2
3325*A*x^8*b^2*c^5+72*B*x^8*b^3*c^4+200*A*x^6*b^3*c^4-120*B*x^6*b^4*c^3+462
0*A*b^6*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2310*A*b^6*c*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*
c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))-2772*B*b^7*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1386*B*b^7*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))-440*A*x^4*b^4*c^3+264*B*x^4*b^5*c^2-1540*A*x^2*b^5*c^2+924*B
*x^2*b^6*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right) x^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\, \left(B\,x^2 + A\right)\, \left(c\,x^4 + b\,x^2\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

<u>**sympy**</u> [**F(-1)**]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.232 $\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=321

$$\frac{12b^{19/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(13bB-23Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{33649c^{17/4}\sqrt{bx^2+cx^4}}-\frac{24b^4\sqrt{bx^2+cx^4}\,(13bB-23Ac)}{33649c^4\sqrt{x}}+\frac{72b^3x}{}$$

```
[Out] -2/437*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(3/2)/c+2/23*B*x^(3/2)*(c*x^4
+b*x^2)^(5/2)/c+72/168245*b^3*(-23*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/
c^3-8/24035*b^2*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2-4/2185*b*(
-23*A*c+13*B*b)*x^(11/2)*(c*x^4+b*x^2)^(1/2)/c-24/33649*b^4*(-23*A*c+13*B*b
)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+12/33649*b^(19/4)*(-23*A*c+13*B*b)*x*(cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(
1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(17/4)/(c*x^4+b*x
^2)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$-\frac{8b^2x^{7/2}\sqrt{bx^2+cx^4}\,(13bB-23Ac)}{24035c^2}+\frac{72b^3x^{3/2}\sqrt{bx^2+cx^4}\,(13bB-23Ac)}{168245c^3}-\frac{24b^4\sqrt{bx^2+cx^4}\,(13bB-23Ac)}{33649c^4\sqrt{x}}+\frac{12b^{19}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-24*b^4*(13*b*B - 23*A*c)*Sqrt[b*x^2 + c*x^4])/(33649*c^4*Sqrt[x]) + (72*b
^3*(13*b*B - 23*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(168245*c^3) - (8*b^2*(13
*b*B - 23*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(24035*c^2) - (4*b*(13*b*B - 23
*A*c)*x^(11/2)*Sqrt[b*x^2 + c*x^4])/(2185*c) - (2*(13*b*B - 23*A*c)*x^(7/2)
*(b*x^2 + c*x^4)^(3/2))/(437*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(5/2))/(23*c
) + (12*b^(19/4)*(13*b*B - 23*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(33649*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
```

```
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c} - \frac{\left(2\left(\frac{13bB}{2}-\frac{23Ac}{2}\right)\right)\int x^{5/2}\left(bx^2+cx^4\right)^{3/2}\,dx}{23c}$$

$$= -\frac{2(13bB-23Ac)x^{7/2}\left(bx^2+cx^4\right)^{3/2}}{437c} + \frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c} - \frac{(6b(13bB-$$

$$= -\frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}\left(bx^2+cx^4\right)^{3/2}}{437c} +$$

$$= -\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} -$$

$$= \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} - \frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} -$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} -$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} -$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} -$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} -$$

Mathematica [C] time = 0.23, size = 160, normalized size = 0.50

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(15b^4(13bB-23Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-3b^2c\left(115A+143Bx^2\right)+11bc^2\right.\right.}{24035c^4\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(195*b^3*B -
55*c^3*x^4*(23*A + 19*B*x^2) + 11*b*c^2*x^2*(69*A + 65*B*x^2) - 3*b^2*c*(11
5*A + 143*B*x^2))) + 15*b^4*(13*b*B - 23*A*c)*Hypergeometric2F1[-3/2, 1/4,
5/4, -((c*x^2)/b)]))/(24035*c^4*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bcx^8+(Bb+Ac)x^6+Abx^4\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^8 + (B*b + A*c)*x^6 + A*b*x^4)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

maple [A] time = 0.10, size = 355, normalized size = 1.11

$$-\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-7315B\,c^7x^{13}-8855A\,c^7x^{11}-16940Bb\,c^6x^{11}-21252Ab\,c^6x^9-9933B\,b^2c^5x^9-13041A\,b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -2/168245*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7315*B*x^13*c^7-8855*A*
x^11*c^7-16940*B*x^11*b*c^6-21252*A*x^9*b*c^6-9933*B*x^9*b^2*c^5-13041*A*x^
7*b^2*c^5+56*B*x^7*b^3*c^4+690*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2
)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b^5*c+184*A*x^5*b^3*c^4-390*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*
c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b
^6-104*B*x^5*b^4*c^3-552*A*x^3*b^4*c^3+312*B*x^3*b^5*c^2-1380*A*x*b^5*c^2+7
80*B*x*b^6*c)/c^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{5/2}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.233 $\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=447

$$-\frac{4b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-21Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}+\frac{8b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB}{3315c^{15/4}\sqrt{bx^2}}$$

```
[Out] -2/357*(-21*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(3/2)/c+2/21*B*(c*x^4+b*x^2)^
(5/2)*x^(1/2)/c-8/3315*b^4*(-21*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^(1
/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-8/13923*b^2*(-21*A*c+11*B*b)*x^(5/2)*(c*
x^4+b*x^2)^(1/2)/c^2-4/1547*b*(-21*A*c+11*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1/2)/
c+8/9945*b^3*(-21*A*c+11*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^3+8/3315*b^(17/
4)*(-21*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-4/3315*b^(17/4)*(-21*A*c+11*B*b)*x*(cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(
1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x
^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}(11bB-21Ac)}{13923c^2}-\frac{8b^4x^{3/2}\left(b+cx^2\right)(11bB-21Ac)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3\sqrt{x}\sqrt{bx^2+cx^4}(11bB-21Ac)}{9945c^3}-\frac{4b^{17/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^4*(11*b*B - 21*A*c)*x^(3/2)*(b + c*x^2))/(3315*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (8*b^3*(11*b*B - 21*A*c)*Sqrt[x]*Sqrt[b*x^2
+ c*x^4])/(9945*c^3) - (8*b^2*(11*b*B - 21*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4]
)/(13923*c^2) - (4*b*(11*b*B - 21*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(1547*c
) - (2*(11*b*B - 21*A*c)*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/(357*c) + (2*B*Sqrt
[x]*(b*x^2 + c*x^4)^(5/2))/(21*c) + (8*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3315*c^(15/4)*Sqrt[b*x^2 + c*x^4]) - (4
*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(33
15*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2B\sqrt{x}\,\left(bx^2+cx^4\right)^{5/2}}{21c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{21Ac}{2}\right)\right)\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx}{21c}$$

$$= -\frac{2(11bB-21Ac)x^{5/2}\left(bx^2+cx^4\right)^{3/2}}{357c} + \frac{2B\sqrt{x}\,\left(bx^2+cx^4\right)^{5/2}}{21c} - \frac{(2b(11bB-2}{}$$

$$= -\frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}\left(bx^2+cx^4\right)^{3/2}}{357c} + \frac{2}{}$$

$$= -\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2}{}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b}{}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b}{}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b}{}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b}{}$$

$$= -\frac{8b^4(11bB-21Ac)x^{3/2}\left(b+cx^2\right)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{8b^3(11bB-21Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{9945c^3} -$$

Mathematica [C] time = 0.19, size = 138, normalized size = 0.31

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(7b^3(21Ac-11bB)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-bc\left(147A+143Bx^2\right)+13c^2x\right.\right.}{4641c^3\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(77*b^2
*B + 13*c^2*x^2*(21*A + 17*B*x^2) - b*c*(147*A + 143*B*x^2)) + 7*b^3*(-11*b
*B + 21*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(4641*c^3*Sq
rt[1 + (c*x^2)/b])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bcx^7+(Bb+Ac)x^5+Abx^3\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^7 + (B*b + A*c)*x^5 + A*b*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right) x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

maple [A] time = 0.08, size = 494, normalized size = 1.11

$$\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}} \left(3315 B\,c^6 x^{12} + 4095 A\,c^6 x^{10} + 7800 B b\,c^5 x^{10} + 10080 A b\,c^5 x^8 + 4665 B\,b^2 c^4 x^8 + 6405 A\,b^2 c^4 x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x)
[Out] 2/69615*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^4*(3315*B*x^12*c^6+4095*A
*x^10*c^6+7800*B*x^10*b*c^5+10080*A*x^8*b*c^5+4665*B*x^8*b^2*c^4+6405*A*x^6
*b^2*c^4-40*B*x^6*b^3*c^3+1764*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*
EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^5*c-882*A*
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*b^5*c-924*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*
x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6
+462*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6-168*A*x^4*b^3*c^3+88*B*x^4*b^4*c^2
-588*A*x^2*b^4*c^2+308*B*x^2*b^5*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right) x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \left(B\,x^2 + A\right) \left(c\,x^4 + b\,x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

3.234 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=282

$$-\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-19Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4389c^{13/4}\sqrt{bx^2+cx^4}}+\frac{8b^3\sqrt{bx^2+cx^4}\,(9bB-19Ac)}{4389c^3\sqrt{x}}-\frac{8b^2x^{3/2}\sqrt{\ldots}}{\ldots}$$

```
[Out] -2/285*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c+2/19*B*(c*x^4+b*x^2)^(
5/2)/c/x^(1/2)-8/7315*b^2*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2-4
/1045*b*(-19*A*c+9*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+8/4389*b^3*(-19*A*c+9
*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-4/4389*b^(15/4)*(-19*A*c+9*B*b)*x*(co
s(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^
(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*
x^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$\frac{8b^3\sqrt{bx^2+cx^4}\,(9bB-19Ac)}{4389c^3\sqrt{x}}-\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}\,(9bB-19Ac)}{7315c^2}-\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-19Ac)\ldots}{4389c^{13/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^3*(9*b*B - 19*A*c)*Sqrt[b*x^2 + c*x^4])/(4389*c^3*Sqrt[x]) - (8*b^2*(9
*b*B - 19*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7315*c^2) - (4*b*(9*b*B - 19*A
*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(1045*c) - (2*(9*b*B - 19*A*c)*x^(3/2)*(b*
x^2 + c*x^4)^(3/2))/(285*c) + (2*B*(b*x^2 + c*x^4)^(5/2))/(19*c*Sqrt[x]) -
(4*b^(15/4)*(9*b*B - 19*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4
389*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{\left(2\left(\frac{9bB}{2}-\frac{19Ac}{2}\right)\right)\int\sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx}{19c}$$

$$=-\frac{2(9bB-19Ac)x^{3/2}\left(bx^2+cx^4\right)^{3/2}}{285c}+\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{(2b(9bB-19A$$

$$=-\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c}-\frac{2(9bB-19Ac)x^{3/2}\left(bx^2+cx^4\right)^{3/2}}{285c}+$$

$$=-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c}-$$

$$=\frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}}-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9$$

$$=\frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}}-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9$$

$$=\frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}}-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9$$

$$=\frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}}-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9$$

Mathematica [C] time = 0.18, size = 138, normalized size = 0.49

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(5b^3(19Ac-9bB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)+\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-bc\left(95A+99Bx^2\right)+11c^2x^2\left(19A\right.\right.\right.}{3135c^3\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 11*
c^2*x^2*(19*A + 15*B*x^2) - b*c*(95*A + 99*B*x^2)) + 5*b^3*(-9*b*B + 19*A*c
)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3135*c^3*Sqrt[x]*Sqrt[
1 + (c*x^2)/b])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bcx^6+(Bb+Ac)x^4+Abx^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^6 + (B*b + A*c)*x^4 + A*b*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

maple [A] time = 0.07, size = 331, normalized size = 1.17

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(1155B\,c^6x^{11}+1463A\,c^6x^9+2772Bb\,c^5x^9+3724Ab\,c^5x^7+1701B\,b^2c^4x^7+2489A\,b^2c^4x^5-24B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x)
```

```
[Out] 2/21945*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(1155*B*x^11*c^6+1463*A*x^9
*c^6+2772*B*x^9*b*c^5+3724*A*x^7*b*c^5+1701*B*x^7*b^2*c^4+190*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^4*c+2489*A*x^5*b^2*c^4-90*B*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^5-24*B*x^5*b^3*c^3-152*A*x^3*b^3*c^3+72*B*x
^3*b^4*c^2-380*A*x*b^4*c^2+180*B*x*b^5*c)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

3.235 $\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=408

$$\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-17Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}-\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-1}{1105c^{11/4}\sqrt{bx^2+}}$$

```
[Out] 2/17*B*(c*x^4+b*x^2)^(5/2)/c/x^(3/2)-2/221*(-17*A*c+7*B*b)*(c*x^4+b*x^2)^(3
/2)*x^(1/2)/c+8/1105*b^3*(-17*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/663*b*(-17*A*c+7*B*b)*x^(5/2)*(c*x^4+b*x^
2)^(1/2)/c-8/3315*b^2*(-17*A*c+7*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-8/110
5*b^(13/4)*(-17*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1
/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+4/1105*b^(13/4)*(-17*A*c+7*B*b)*
x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2)
)*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2039, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{8b^3x^{3/2}\left(b+cx^2\right)(7bB-17Ac)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}(7bB-17Ac)}{3315c^2}+\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-17}{1105c^{11/4}\sqrt{bx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

```
[Out] (8*b^3*(7*b*B - 17*A*c)*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (8*b^2*(7*b*B - 17*A*c)*Sqrt[x]*Sqrt[b*x^2 + c
*x^4])/(3315*c^2) - (4*b*(7*b*B - 17*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(663
*c) - (2*(7*b*B - 17*A*c)*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(221*c) + (2*B*(b*
x^2 + c*x^4)^(5/2))/(17*c*x^(3/2)) - (8*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (4
*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(110
5*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}} - \frac{\left(2\left(\frac{7bB}{2}-\frac{17Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx}{17c}$$

$$= -\frac{2(7bB-17Ac)\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{221c} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{17cx^{3/2}} - \frac{(6b(7bB-17Ac))\int x \cdots}{221c \cdots}$$

$$= -\frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}{221c} + \frac{2B\left(bx \cdots\right.}{17 \cdots}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB \cdots}{\cdots}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB \cdots}{\cdots}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB \cdots}{\cdots}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB \cdots}{\cdots}$$

$$= \frac{8b^3(7bB-17Ac)x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB \cdots}{\cdots}$$

Mathematica [C] time = 0.16, size = 115, normalized size = 0.28

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(b^2(7bB-17Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-17Ac+7bB-13Bcx^2\right)\right)}{221c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(7*b*
B - 17*A*c - 13*B*c*x^2)) + b^2*(7*b*B - 17*A*c)*Hypergeometric2F1[-3/2, 3/
4, 7/4, -((c*x^2)/b)]))/(221*c^2*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bcx^5+(Bb+Ac)x^3+Abx\right)\sqrt{cx^4+bx^2}\,\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^5 + (B*b + A*c)*x^3 + A*b*x)*sqrt(c*x^4 + b*x^2)*sqrt(x), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

maple [A] time = 0.07, size = 470, normalized size = 1.15

$$-\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-195B\,c^5x^{10}-255A\,c^5x^8-480Bb\,c^4x^8-680Ab\,c^4x^6-305B\,b^2c^3x^6-493A\,b^2c^3x^4+8B\,b^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(-195*B*x^10*c^5-255*A*
x^8*c^5-480*B*x^8*b*c^4-680*A*x^6*b*c^4-305*B*x^6*b^2*c^3+204*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*b^4*c-102*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*El
lipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c-84*B*((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*b^5+42*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/
2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^5-493*A
*x^4*b^2*c^3+8*B*x^4*b^3*c^2-68*A*x^2*b^3*c^2+28*B*x^2*b^4*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Timed out
```

3.236 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^2\sqrt{bx^2+cx^4}\,(bB-3Ac)}{231c^2\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{\ldots}}$$

```
[Out] 2/15*B*(c*x^4+b*x^2)^(5/2)/c/x^(5/2)-2/33*(-3*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/
c/x^(1/2)-4/77*b*(-3*A*c+B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-8/231*b^2*(-3*A
*c+B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+4/231*b^(11/4)*(-3*A*c+B*b)*x*(cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1
/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2
)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2021, 2024, 2032, 329, 220}

$$-\frac{8b^2\sqrt{bx^2+cx^4}\,(bB-3Ac)}{231c^2\sqrt{x}}+\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{4bx^{3/2}\sqrt{bx^2+\ldots}}{7\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

```
[Out] (-8*b^2*(b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (4*b*(b*B -
3*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) - (2*(b*B - 3*A*c)*(b*x^2 + c*x^
4)^(3/2))/(33*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4)^(5/2))/(15*c*x^(5/2)) + (4*
b^(11/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(
9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx = \frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{15Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx}{15c}$$

$$= -\frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}} - \frac{(2b(bB-3Ac))\int \sqrt{x}\,\sqrt{b}}{11c}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B\left(bx^2+c\right.}{15cx^{5/2}}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)}{33}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)}{33}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)}{33}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)}{33}$$

Mathematica [C] time = 0.15, size = 115, normalized size = 0.48

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(5b^2(bB-3Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\left(-15Ac+5bB-11Bcx^2\right)\right)}{165c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(5*b*B - 15*A
*c - 11*B*c*x^2)) + 5*b^2*(b*B - 3*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -
((c*x^2)/b)]))/(165*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bcx^4 + (Bb + Ac)x^2 + Ab\right)\sqrt{cx^4 + bx^2}\,\sqrt{x}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

maple [A] time = 0.07, size = 307, normalized size = 1.28

$$-\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(-77B\,c^5x^9 - 105A\,c^5x^7 - 196Bb\,c^4x^7 - 300Ab\,c^4x^5 - 131B\,b^2c^3x^5 - 255A\,b^2c^3x^3 + 8B\,b^3c^2x^3 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x)
```

```
[Out] -2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-77*B*c^5*x^9-105*A*c^5*x^
7-196*B*b*c^4*x^7+30*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/
2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c-300
*A*b*c^4*x^5-10*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*El
lipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4-131*B*b^2*
c^3*x^5-255*A*b^2*c^3*x^3+8*B*b^3*c^2*x^3-60*A*b^3*c^2*x+20*B*b^4*c*x)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(3/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(3/2), x)
```

3.237 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}} \, dx$

Optimal. Leaf size=369

$$-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-1}{195c^{7/4}\sqrt{bx^2+c}}$$

```
[Out] -2/117*(-13*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)+2/13*B*(c*x^4+b*x^2)^(
5/2)/c/x^(7/2)-8/195*b^2*(-13*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/195*b*(-13*A*c+3*B*b)*x^(1/2)*(c*x^4+b*x^
2)^(1/2)/c+8/195*b^(9/4)*(-13*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-4/195*b^(9/4)*(-13*
A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^
(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2039, 2021, 2032, 329, 305, 220, 1196}

$$-\frac{8b^2x^{3/2}\left(b+cx^2\right)(3bB-13Ac)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{9/4}x}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

```
[Out] (-8*b^2*(3*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(3/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(3*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x
^4])/(195*c) - (2*(3*b*B - 13*A*c)*(b*x^2 + c*x^4)^(3/2))/(117*c*x^(3/2)) +
 (2*B*(b*x^2 + c*x^4)^(5/2))/(13*c*x^(7/2)) + (8*b^(9/4)*(3*b*B - 13*A*c)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4]
) - (4*b^(9/4)*(3*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{13Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}dx}{13c}$$

$$= -\frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(2b(3bB-13Ac))\int \frac{\sqrt{bx^2}}{\sqrt{}}}{39c}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+}{13cx^7}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+}{13cx^7}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+}{13cx^7}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+}{13cx^7}$$

$$= -\frac{8b^2(3bB-13Ac)x^{3/2}(b+cx^2)}{195c^{3/2}(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB}{}$$

Mathematica [C] time = 0.11, size = 98, normalized size = 0.27

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(b(13Ac-3bB)\,_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+3B\sqrt{\frac{cx^2}{b}+1}\,(b+cx^2)^2\right)}{39c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(3*B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b
*(-3*b*B + 13*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(39*c*
Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

maple [A] time = 0.07, size = 446, normalized size = 1.21

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(45B\,c^4x^8+65A\,c^4x^6+120Bb\,c^3x^6+208Ab\,c^3x^4+87B\,b^2c^2x^4+143A\,b^2c^2x^2+12B\,b^3c\,x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x)
[Out] 2/585*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(45*B*c^4*x^8+65*A*c^4*x^
6+120*B*b*c^3*x^6+156*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ell
ipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-78*A*b^3*c*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))-36*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2
)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+18*B*b^4*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))+208*A*b*c^3*x^4+87*B*b^2*c^2*x^4+143*A*b^2*c^2
*x^2+12*B*b^3*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(5/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(5/2), x)
```

3.238 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{bx^2+cx^4}\,(bB-11Ac)}{77c\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{3}}{77c}$$

```
[Out] -2/77*(-11*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)+2/11*B*(c*x^4+b*x^2)^(5/2
)/c/x^(9/2)-4/77*b*(-11*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-4/77*b^(7/4)
*(-11*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2021, 2032, 329, 220}

$$-\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(bB-11Ac)}{77cx^{5/2}}-\frac{4b\sqrt{bx^2+cx}}{77}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

```
[Out] (-4*b*(b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) - (2*(b*B - 11*A*c
)*(b*x^2 + c*x^4)^(3/2))/(77*c*x^(5/2)) + (2*B*(b*x^2 + c*x^4)^(5/2))/(11*c
*x^(9/2)) - (4*b^(7/4)*(b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx = \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx}{11c}$$

$$= -\frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{11cx^{9/2}} - \frac{(6b(bB-11Ac))\int \frac{\sqrt{bx^2}}{x^{3}}}{77c}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right.}{11cx^{9/2}}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right.}{11cx^{9/2}}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right.}{11cx^{9/2}}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B\left(bx^2+cx^4\right.}{11cx^{9/2}}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 0.48

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(b(11Ac-bB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)+B\sqrt{\frac{cx^2}{b}+1}\left(b+cx^2\right)^2\right)}{11c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(-(b*B) +
 11*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sq
rt[1 + (c*x^2)/b])
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right)}{x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

maple [A] time = 0.07, size = 283, normalized size = 1.41

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(7B\,c^4x^7+11A\,c^4x^5+20Bb\,c^3x^5+44Ab\,c^3x^3+17B\,b^2c^2x^3+33A\,b^2c^2x+4B\,b^3cx+22\sqrt{-bc}\,\sqrt{\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x)
[Out] 2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(7*B*c^4*x^7+22*A*(-b*c)^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*b^2*c+11*A*c^4*x^5-2*B*(-b*c)^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-
b*c)^(1/2)*c*x)^(1/2)*b^3+20*B*b*c^3*x^5+44*A*b*c^3*x^3+17*B*b^2*c^2*x^3+33
*A*b^2*c^2*x+4*B*b^3*c*x)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(Bx^2 + A\right)}{x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2 + A\right)\,\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}} \left(A + Bx^2\right)}{x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(7/2), x)
```

3.239 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}} dx$

Optimal. Leaf size=356

$$\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)E}{15c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/9*(9*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(3/2)-2*A*(c*x^4+b*x^2)^(5/2)/b/x^(
11/2)+8/15*b*(9*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x
^4+b*x^2)^(1/2)+4/15*(9*A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)-8/15*b^(5/4)*(
9*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^
(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*b^(5/4)*(9*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF
(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2038, 2021, 2032, 329, 305, 220, 1196}

$$\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)E}{15c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

```
[Out] (8*b*(b*B + 9*A*c)*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (4*(b*B + 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2
*(b*B + 9*A*c)*(b*x^2 + c*x^4)^(3/2))/(9*b*x^(3/2)) - (2*A*(b*x^2 + c*x^4)^
(5/2))/(b*x^(11/2)) - (8*b^(5/4)*(b*B + 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(5/4)*(b*B + 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4
])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}dx}{b}$$

$$= \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}} + \frac{1}{3}(2(bB+9Ac))\int \frac{\sqrt{bx^2+c}}{\sqrt{x}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/}}{bx^{11/2}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/}}{bx^{11/2}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/}}{bx^{11/2}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/}}{bx^{11/2}}$$

$$= \frac{8b(bB+9Ac)x^{3/2}\left(b+cx^2\right)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac}{9}$$

Mathematica [C] time = 0.08, size = 85, normalized size = 0.24

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\frac{x^2(9Ac+bB)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{\sqrt{\frac{cx^2}{b}+1}} - \frac{3A\left(b+cx^2\right)^2}{b}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((-3*A*(b + c*x^2)^2)/b + ((b*B + 9*A*c)*x^2*Hyper
geometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)])/Sqrt[1 + (c*x^2)/b]))/(3*x^(3/2
))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

maple [A] time = 0.08, size = 429, normalized size = 1.21

$$\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(5B\,c^{3}x^{6}+9A\,c^{3}x^{4}+16Bb\,c^{2}x^{4}-36Ab\,c^{2}x^{2}+11B\,b^{2}c\,x^{2}+108\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(5*B*c^3*x^6+108*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*b^2*c-54*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ell
ipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c+12*B*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))*b^3-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3+9*A*c^3
*x^4+16*B*b*c^2*x^4-36*A*b*c^2*x^2+11*B*b^2*c*x^2-45*A*b^2*c)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(Bx^{2}+A\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^{2}+A\right)\,\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(9/2), x)
```

3.240 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}} \, dx$

Optimal. Leaf size=200

$$\frac{4b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7Ac+3bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{4\sqrt{bx^2+cx^4}(7Ac+3bB)}{21\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{21bx}$$

```
[Out] 2/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(5/2)-2/3*A*(c*x^4+b*x^2)^(5/2)/
b/x^(13/2)+4/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/21*b^(3/4)*(7*A
*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1
/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(
1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2038, 2021, 2032, 329, 220}

$$\frac{4b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7Ac+3bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\left(bx^2+cx^4\right)^{3/2}(7Ac+3bB)}{21bx^{5/2}}+\frac{4\sqrt{bx^2+cx^4}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

```
[Out] (4*(3*b*B + 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*Sqrt[x]) + (2*(3*b*B + 7*A*c)*(
b*x^2 + c*x^4)^(3/2))/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^(
13/2)) + (4*b^(3/4)*(3*b*B + 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(21*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{11/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{5/2}}{3bx^{13/2}} - \frac{\left(2\left(-\frac{3bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx}{3b}$$

$$= \frac{2(3bB+7Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{3bx^{13/2}} + \frac{1}{7}(2(3bB+7Ac))\int \frac{\sqrt{b}}{}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)}{3bx^{13/2}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)}{3bx^{13/2}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)}{3bx^{13/2}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{5/2}} - \frac{2A\left(bx^2+cx^4\right)}{3bx^{13/2}}$$

Mathematica [C] time = 0.09, size = 101, normalized size = 0.50

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(A\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}-bx^2(7Ac+3bB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3bx^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b*(3*b*B +
 7*A*c)*x^2*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*
Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

maple [A] time = 0.09, size = 260, normalized size = 1.30

$$\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(3B\,c^3x^6 + 7A\,c^3x^4 + 12Bb\,c^2x^4 + 9B\,b^2c\,x^2 + 14\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,Abcx}{21\left(c\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(14*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)
^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*(-b*c)^(1/2)*x*b*c+6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x*b
^2+3*B*c^3*x^6+7*A*c^3*x^4+12*B*b*c^2*x^4+9*B*b^2*c*x^2-7*A*b^2*c)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2 + A\right)\,\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}\left(A + Bx^2\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(11/2), x)
```

3.241 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=354

$$\frac{12c\sqrt{x}\sqrt{bx^2+cx^4}(Ac+bB)}{5b}+\frac{12\sqrt[4]{b}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24\sqrt[4]{b}\sqrt[4]{c}\,x}{}$$

```
[Out] -2*(A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)-2/5*A*(c*x^4+b*x^2)^(5/2)/b/x^(1
5/2)+24/5*(A*c+B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*
x^2)^(1/2)+12/5*c*(A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/b-24/5*b^(1/4)*c^(1
/4)*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/(c*x^4+b*x^2)^(1/2)+12/5*b^(1/4)*c^(1/4)*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*(
(c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2020, 2021, 2032, 329, 305, 220, 1196}

$$-\frac{2\left(bx^2+cx^4\right)^{3/2}(Ac+bB)}{bx^{7/2}}+\frac{12c\sqrt{x}\sqrt{bx^2+cx^4}(Ac+bB)}{5b}+\frac{24\sqrt{c}\,x^{3/2}\left(b+cx^2\right)(Ac+bB)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{12\sqrt[4]{b}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

```
[Out] (24*Sqrt[c]*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (12*c*(b*B + A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*(b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (2*A*(b*x^2 + c*x^4)^(5/2
))/(5*b*x^(15/2)) - (24*b^(1/4)*c^(1/4)*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(1/4)*(b*B + A
*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = -\frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + -\frac{\left(2\left(-\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}dx}{5b}$$

$$= -\frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + \frac{(6c(bB+Ac))\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}dx}{b}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}}$$

$$= \frac{24\sqrt{c}(bB+Ac)x^{3/2}(b+cx^2)}{5(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} + \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(b}{bx^{7/}}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.28

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(5bx^2(Ac+bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)+A(b+cx^2)^2\sqrt{\frac{cx^2}{b}+1}\right)}{5bx^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b*(b*B +
 A*c)*x^2*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*S
qrt[1 + (c*x^2)/b])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bcx^4+(Bb+Ac)x^2+Ab)\sqrt{cx^4+bx^2}}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

maple [A] time = 0.07, size = 427, normalized size = 1.21

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(B\,c^2x^6-7A\,c^2x^4-4Bbc\,x^4+12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+}{\sqrt{}}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x)
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)/(c*x^2+b)^2*(12*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)
^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*x^2*b*c-6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c+12*B*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*x^2*b^2-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2+B*c^2*x^6
-7*A*c^2*x^4-4*B*b*c*x^4-8*A*b*c*x^2-5*B*b^2*x^2-A*b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(13/2),x)
[Out] Timed out
```

3.242 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}} dx$

Optimal. Leaf size=204

$$\frac{4c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3Ac+7bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}}+\frac{4c\sqrt{bx^2+cx^4}\,(3Ac+7bB)}{21b\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{3/}}{21bx}$$

```
[Out] -2/21*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)-2/7*A*(c*x^4+b*x^2)^(5/2)
/b/x^(17/2)+4/21*c*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(1/2)+4/21*c^(3/4)
*(3*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/b^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2038, 2020, 2021, 2032, 329, 220}

$$\frac{4c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3Ac+7bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(3Ac+7bB)}{21bx^{9/2}}+\frac{4c\sqrt{bx^2+cx}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

```
[Out] (4*c*(7*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*Sqrt[x]) - (2*(7*b*B + 3*A*
c)*(b*x^2 + c*x^4)^(3/2))/(21*b*x^(9/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(7*b
*x^(17/2)) + (4*c^(3/4)*(7*b*B + 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(21*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{5/2}}{7bx^{17/2}} - \frac{\left(2\left(-\frac{7bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11/2}}\,dx}{7b}$$

$$= -\frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{7bx^{17/2}} + \frac{(2c(7bB+3Ac))\int \frac{\sqrt{bx}\cdots}{\cdots}}{7b}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A\left(bx^2+cx^4\right)\cdots}{7bx^{17/2}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A\left(bx^2+cx^4\right)\cdots}{7bx^{17/2}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A\left(bx^2+cx^4\right)\cdots}{7bx^{17/2}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A\left(bx^2+cx^4\right)\cdots}{7bx^{17/2}}$$

Mathematica [C] time = 0.06, size = 101, normalized size = 0.50

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(bx^2(3Ac+7bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{cx^2}{b}\right)+3A\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{21bx^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(7*b*B
 + 3*A*c)*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)]))/(21*b*x^(9
/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bcx^4 + (Bb + Ac)x^2 + Ab\right)\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

maple [A] time = 0.08, size = 254, normalized size = 1.25

$$\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(7B\,c^2x^6 - 9A\,c^2x^4 + 6\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ac\,x^3\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right.\right.}{21\left(c\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(6*A*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*x^3*c+14*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c
*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^
3*b+7*B*c^2*x^6-9*A*c^2*x^4-12*A*b*c*x^2-7*B*b^2*x^2-3*A*b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}\left(Bx^2 + A\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2 + A\right)\,\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] Timed out
```

3.243 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}} \, dx$

Optimal. Leaf size=364

$$\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+9bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+9bB)E}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/45*(A*c+9*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)-2/9*A*(c*x^4+b*x^2)^(5/2)/
b/x^(19/2)+8/15*c^(3/2)*(A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+x*c^(1/2))
/(c*x^4+b*x^2)^(1/2)-4/15*c*(A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-8/15*
c^(5/4)*(A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*c^(5/4)*(A*c+9*B*b)*x*(cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^
(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2038, 2020, 2032, 329, 305, 220, 1196}

$$\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+9bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+9bB)E}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

```
[Out] (8*c^(3/2)*(9*b*B + A*c)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) - (4*c*(9*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(3/2)
) - (2*(9*b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(45*b*x^(11/2)) - (2*A*(b*x^2 +
 c*x^4)^(5/2))/(9*b*x^(19/2)) - (8*c^(5/4)*(9*b*B + A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*c^(5/4)*(9*
b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}dx = -\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} - \frac{\left(2\left(-\frac{9bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}dx}{9b}$$

$$= -\frac{2(9bB+Ac)(bx^2+cx^4)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} + \frac{(2c(9bB+Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}}{15b}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)(bx^2+cx^4)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)(bx^2+cx^4)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)(bx^2+cx^4)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)(bx^2+cx^4)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

$$= \frac{8c^{3/2}(9bB+Ac)x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)(bx^2}{45bx^{11/2}}$$

Mathematica [C] time = 0.06, size = 100, normalized size = 0.27

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(bx^2(Ac+9bB)\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{cx^2}{b}\right)+5A(b+cx^2)^2\sqrt{\frac{cx^2}{b}+1}\right)}{45bx^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(9*b*B
 + A*c)*x^2*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b)]))/(45*b*x^(11
/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bcx^4+(Bb+Ac)x^2+Ab\right)\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

```
[Out] integral((B*c*x^4 + (B*b + A*c)*x^2 + A*b)*sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{17}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

maple [A] time = 0.07, size = 452, normalized size = 1.24

$$\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(-12A\,c^{3}x^{6}-63Bb\,c^{2}x^{6}+12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ab\,c^{2}x^{4}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*A*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c
)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*x^4*b*c^2-6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2+108*B*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))*x^4*b^2*c-54*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1
/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*
c-12*A*c^3*x^6-63*B*b*c^2*x^6-23*A*b*c^2*x^4-72*B*b^2*c*x^4-16*A*b^2*c*x^2-
9*B*b^3*x^2-5*A*b^3)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(Bx^{2}+A\right)}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^{2}+A\right)\,\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}{x^{17/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] Timed out
```

3.244 $\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=243

$$\frac{b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(13bB-15Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}-\frac{2b^2\sqrt{bx^2+cx^4}\,(13bB-15Ac)}{77c^4\sqrt{x}}+\frac{6bx^{3/2}\sqrt{bx}}{}$$

```
[Out] 6/385*b*(-15*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^3-2/165*(-15*A*c+13*
B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/15*B*x^(11/2)*(c*x^4+b*x^2)^(1/2)/c-
2/77*b^2*(-15*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+1/77*b^(11/4)*(-1
5*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2024, 2032, 329, 220}

$$-\frac{2b^2\sqrt{bx^2+cx^4}\,(13bB-15Ac)}{77c^4\sqrt{x}}+\frac{b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(13bB-15Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}-\frac{2x^{7/2}\sqrt{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b^2*(13*b*B - 15*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) + (6*b*(13*
b*B - 15*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^3) - (2*(13*b*B - 15*A*c)
*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(165*c^2) + (2*B*x^(11/2)*Sqrt[b*x^2 + c*x^4]
)/(15*c) + (b^(11/4)*(13*b*B - 15*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(77*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} - \frac{\left(2\left(\frac{13bB}{2}-\frac{15Ac}{2}\right)\right)\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c}$$

$$= -\frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{(3b(13bB-15Ac))\int \frac{x^{9/}}{\sqrt{bx^2+}}}{55c^2}$$

$$= \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+}}{15c}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac}{16}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac}{16}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac}{16}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac}{16}$$

Mathematica [C] time = 0.18, size = 143, normalized size = 0.59

$$\frac{2x^{3/2}\left(15b^3\sqrt{\frac{cx^2}{b}+1}\,(13bB-15Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\left(-9b^2c\left(25A+13Bx^2\right)+bc^2x^2\left(135A+91B\right.\right.}{1155c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-((b + c*x^2)*(195*b^3*B - 7*c^3*x^4*(15*A + 11*B*x^2) - 9*b^2*
c*(25*A + 13*B*x^2) + b*c^2*x^2*(135*A + 91*B*x^2))) + 15*b^3*(13*b*B - 15*
A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(
1155*c^4*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^6 + Ax^4\right)\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^2 + b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^6 + A*x^4)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.09, size = 298, normalized size = 1.23

$$-\frac{\left(-154B\,c^5x^9 - 210A\,c^5x^7 + 28Bb\,c^4x^7 + 60Ab\,c^4x^5 - 52B\,b^2c^3x^5 - 180A\,b^2c^3x^3 + 156B\,b^3c^2x^3 - 450A\,b^3c^2x + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/1155/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-154*B*c^5*x^9-210*A*c^5*x^7+28*B*b*c^
4*x^7+225*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c+60*A*b*c^4*x^5
-195*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^4-52*B*b^2*c^3*x^5-180*
A*b^2*c^3*x^3+156*B*b^3*c^2*x^3-450*A*b^3*c^2*x+390*B*b^4*c*x)/c^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13/2}\,\left(B\,x^2 + A\right)}{\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.245 $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=369

$$-\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}+\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB}{195c^{15/4}\sqrt{bx^2}}$$

```
[Out] -14/195*b^2*(-13*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))/
(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c^2+
2/13*B*x^(9/2)*(c*x^4+b*x^2)^(1/2)/c+14/585*b*(-13*A*c+11*B*b)*x^(1/2)*(c*x
^4+b*x^2)^(1/2)/c^3+14/195*b^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*(
(c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-7/195*b
^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2039, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{14b^2x^{3/2}\left(b+cx^2\right)(11bB-13Ac)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-13Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}+\frac{14b^{9/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-14*b^2*(11*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (14*b*(11*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 +
 c*x^4])/(585*c^3) - (2*(11*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117
*c^2) + (2*B*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(13*c) + (14*b^(9/4)*(11*b*B - 13
*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2
+ c*x^4]) - (7*b^(9/4)*(11*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{13Ac}{2}\right)\right)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{13c}$$

$$= -\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} + \frac{(7b(11bB-13Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}}{117c^2}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}$$

$$= -\frac{14b^2(11bB-13Ac)x^{3/2}\left(b+cx^2\right)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13A}{1}$$

Mathematica [C] time = 0.15, size = 122, normalized size = 0.33

$$\frac{2x^{5/2}\left(\left(b+cx^2\right)\left(-bc\left(91A+55Bx^2\right)+5c^2x^2\left(13A+9Bx^2\right)+77b^2B\right)+7b^2\sqrt{\frac{cx^2}{b}+1}\,(13Ac-11bB)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\right.\right.}{585c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*((b + c*x^2)*(77*b^2*B + 5*c^2*x^2*(13*A + 9*B*x^2) - b*c*(91*A
+ 55*B*x^2)) + 7*b^2*(-11*b*B + 13*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(585*c^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(Bx^5+Ax^3\right)\sqrt{cx^4+bx^2}\sqrt{x}}{cx^2+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^5 + A*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.09, size = 437, normalized size = 1.18

$$\frac{\left(90B\,c^4x^8+130A\,c^4x^6-20Bb\,c^3x^6-52Ab\,c^3x^4+44B\,b^2c^2x^4-182A\,b^2c^2x^2+154B\,b^3c\,x^2+546\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/585/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^4*(90*B*c^4*x^8+130*A*c^4*x^6-20*B*b*c^
3*x^6+546*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-273*A*b^3*c*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))-462*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Elliptic
E(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+231*B*b^4*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))-52*A*b*c^3*x^4+44*B*b^2*c^2*x^4-182*A*b^2*c^2*x^2+154*B
*b^3*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.246 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$-\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(9bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}+\frac{10b\sqrt{bx^2+cx^4}\,(9bB-11Ac)}{231c^3\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2}}{}$$

```
[Out] -2/77*(-11*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/11*B*x^(7/2)*(c*x^4
+b*x^2)^(1/2)/c+10/231*b*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-5/
231*b^(7/4)*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2024, 2032, 329, 220}

$$-\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(9bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}-\frac{2x^{3/2}\sqrt{bx^2+cx^4}\,(9bB-11Ac)}{77c^2}+\frac{10b\sqrt{bx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (10*b*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (2*(9*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c^2) + (2*B*x^(7/2)*Sqrt[b*x^2 +
c*x^4])/(11*c) - (5*b^(7/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{11c}$$

$$= -\frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} + \frac{(5b(9bB-11Ac))\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,d}{77c^2}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} -$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} -$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} -$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} -$$

Mathematica [C] time = 0.15, size = 122, normalized size = 0.60

$$\frac{2x^{3/2}\left(\left(b+cx^2\right)\left(-bc\left(55A+27Bx^2\right)+3c^2x^2\left(11A+7Bx^2\right)+45b^2B\right)+5b^2\sqrt{\frac{cx^2}{b}+1}\,(11Ac-9bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4}\right.}{231c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*((b + c*x^2)*(45*b^2*B + 3*c^2*x^2*(11*A + 7*B*x^2) - b*c*(55*A
+ 27*B*x^2)) + 5*b^2*(-9*b*B + 11*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(231*c^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^4+Ax^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")

[Out] integral((B*x^4 + A*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{9}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)

maple [A] time = 0.07, size = 274, normalized size = 1.34

$$\frac{\left(42B\,c^4x^7 + 66A\,c^4x^5 - 12Bb\,c^3x^5 - 44Ab\,c^3x^3 + 36B\,b^2c^2x^3 - 110A\,b^2c^2x + 90B\,b^3cx + 55\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)

[Out] 1/231/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(42*B*c^4*x^7+55*A*(-b*c)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*b^2*c+66*A*c^4*x^5-45*B*(-b*c)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*b^3-12*B*b*c^3*x^5-44*A*b*c^3*x^3+36*B*x^3*b^2*c^2-110*A*b^2*c^2*x+90*B*x*b^3*c)/c^4

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{9}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{9/2}\,\left(B\,x^2 + A\right)}{\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)

[Out] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.247 $\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=330

$$\frac{b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/15*b*(-9*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+
b*x^2)^(1/2)+2/9*B*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-2/45*(-9*A*c+7*B*b)*x^(1/2
)*(c*x^4+b*x^2)^(1/2)/c^2-2/15*b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+1/15*
b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2039, 2024, 2032, 329, 305, 220, 1196}

$$\frac{b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(7*b*B - 9*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(7*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45
*c^2) + (2*B*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (2*b^(5/4)*(7*b*B - 9*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x
^4]) + (b^(5/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9c}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{(b(7bB-9Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(b(7bB-9Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+c}}}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(2b(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subs}}{15c^2\sqrt{bx^2+c}}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(2b^{3/2}(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Su}}{15c^{5/2}\sqrt{bx^2+}}$$

$$= \frac{2b(7bB-9Ac)x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} - \cdots$$

Mathematica [C] time = 0.12, size = 97, normalized size = 0.29

$$\frac{2x^{5/2}\left(b\sqrt{\frac{cx^2}{b}+1}\,(7bB-9Ac)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\left(-9Ac+7bB-5Bcx^2\right)\right)}{45c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(-((b + c*x^2)*(7*b*B - 9*A*c - 5*B*c*x^2)) + b*(7*b*B - 9*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^
2*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^3+Ax\right)\sqrt{x}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^3 + A*x)*sqrt(x)/(c*x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.07, size = 413, normalized size = 1.25

$$-\frac{\left(-10B\,c^{3}x^{6}-18A\,c^{3}x^{4}+4Bb\,c^{2}x^{4}-18Ab\,c^{2}x^{2}+14B\,b^{2}c\,x^{2}+54\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,A\,b^{2}c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(-10*B*c^3*x^6+54*A*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1
/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*b^2*c-27*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF
(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c-42*B*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*b^3+21*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3-18*A*c^3*x^4
+4*B*b*c^2*x^4-18*A*b*c^2*x^2+14*B*b^2*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.248 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=167

$$\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(5bB-7Ac)}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] 2/7*B*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-2/21*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)
/c^2/x^(1/2)+1/21*b^(3/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2039, 2024, 2032, 329, 220}

$$\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(5bB-7Ac)}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*(5*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*B*x^(3/2)*Sq
rt[b*x^2 + c*x^4])/(7*c) + (b^(3/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{\left(2\left(\frac{5bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{(b(5bB-7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c^2}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(b(5bB-7Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b}}}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(2b(5bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}}{21c^2\sqrt{bx^2+cx}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{b^{3/4}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{\cdot}{(}}}{21c^{9/4}\sqrt{bx}}$$

Mathematica [C] time = 0.12, size = 97, normalized size = 0.58

$$\frac{2x^{3/2}\left(b\sqrt{\frac{cx^2}{b}+1}\,(5bB-7Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(b+cx^2\right)\left(-7Ac+5bB-3Bcx^2\right)\right)}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-((b + c*x^2)*(5*b*B - 7*A*c - 3*B*c*x^2)) + b*(5*b*B - 7*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^
2*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.07, size = 248, normalized size = 1.49

$$-\frac{\left(-6B\,c^3x^5 - 14A\,c^3x^3 + 4Bb\,c^2x^3 - 14Ab\,c^2x + 10B\,b^2cx + 7\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,Abc\,\mathrm{Ell}}{21\sqrt{c\,x^4 +}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(7*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^
(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)
^(1/2)*b*c-5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2-6*B*c^3*x^5-1
4*A*x^3*c^3+4*B*b*c^2*x^3-14*A*b*c^2*x+10*B*b^2*c*x)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}\left(B\,x^2 + A\right)}{\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}\left(A + Bx^2\right)}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.249 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=293

$$-\frac{\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-\cdots}{5c^{7/4}\sqrt{bx^2+cx\cdots}}$$

```
[Out] -2/5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*
x^2)^(1/2)+2/5*B*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+2/5*b^(1/4)*(-5*A*c+3*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))
*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4
+b*x^2)^(1/2)-1/5*b^(1/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2039, 2032, 329, 305, 220, 1196}

$$-\frac{2x^{3/2}\left(b+cx^2\right)(3bB-5Ac)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt[4]{b}\,x\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (2*B*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (2*b^(1/4)*
(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) - (b^(1/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\sqrt{b}\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(4\sqrt{b}\right.}{}$$

$$= -\frac{2(3bB-5Ac)x^{3/2}\left(b+cx^2\right)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} + \frac{2\sqrt[4]{b}\,(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)}{5c^{7/4}\sqrt{}}$$

Mathematica [C] time = 0.10, size = 81, normalized size = 0.28

$$\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}\,(5Ac-3bB)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+3B\left(b+cx^2\right)\right)}{15c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(3*B*(b + c*x^2) + (-3*b*B + 5*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(15*c*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}}{cx^3 + bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^3 + b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.07, size = 378, normalized size = 1.29

$$\frac{\left(2B\,c^2x^4 + 2Bbc\,x^2 + 10\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - 5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(10*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*
x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c
-5*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c
*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^
2+3*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+2*B*c^2*x^4+2*B*x^2*b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}\,\left(B\,x^2 + A\right)}{\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}} \left(A + Bx^2\right)}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.250 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=130

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/3*B*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-1/3*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*
((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2039, 2032, 329, 220}

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*B*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
```

```
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)\right)\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3c\sqrt{bx^2+cx^4}}\\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(4\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3c\sqrt{bx^2+cx^4}}\\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 80, normalized size = 0.62

$$\frac{2x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}\,(3Ac-bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+B\left(b+cx^2\right)\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (2*x^(3/2)*(B*(b + c*x^2) + (-(b*B) + 3*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^2+A\right)\sqrt{x}}{\sqrt{cx^4+bx^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^2+A\right)\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.07, size = 216, normalized size = 1.66

$$\frac{\left(2Bc^2x^3+2Bbcx+3\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-bc}\,Ac\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{3\sqrt{c\,x^4+b\,x^2}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(3*A*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-b*c)^(
1/2)*c-B*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*
c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-b*c)^(1/2)*b+2*B*c^2*x^3+2*B*x*b*c
)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)\sqrt{x}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}\,\left(B\,x^2 + A\right)}{\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}\,\left(A + Bx^2\right)}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

3.251 $\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=281

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+bB)E\left(2\tan^{-1}\left(\right.\right.}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2*(A*c+B*b)*x^(3/2)*(c*x^2+b)/b/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(
1/2)-2*A*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-2*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE
(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+(
A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3
/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2038, 2032, 329, 305, 220, 1196}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+bB)E\left(2\tan^{-1}\left(\right.\right.}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*(b*B + A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^
4]) + ((b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(
3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{(bB+Ac)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{c}\sqrt{bx^2+cx^4}} - \frac{\left(2(bB+A\right.}{}$$

$$= \frac{2(bB+Ac)x^{3/2}\left(b+cx^2\right)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c}{(\sqrt{b}+}}}{b^{3/4}c^{3/4}\sqrt{bx^2}}$$

Mathematica [C] time = 0.05, size = 82, normalized size = 0.29

$$\frac{2\sqrt{x}\left(x^2\sqrt{\frac{cx^2}{b}+1}\,(Ac+bB)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-3A\left(b+cx^2\right)\right)}{3b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[x]*(-3*A*(b + c*x^2) + (b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^5+bx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^5 + b*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

maple [A] time = 0.07, size = 377, normalized size = 1.34

$$\frac{\left(-2A\,c^2x^2+2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right)-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2
)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-A*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))*b*c+2*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)
*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-B*((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2),1/2*2^(1/2))*b^2-2*A*x^2*c^2-2*A*b*c)/c/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{\sqrt{x}\,\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

3.252 $\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=131

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

```
[Out] -2/3*A*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)+1/3*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2
)
```

Rubi [A] time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2038, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3b}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3b\sqrt{bx^2+cx^4}}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(4\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{bx^2+cx^4}}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}+\frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 82, normalized size = 0.63

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}\,(Ac-3bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+A\left(b+cx^2\right)\right)}{3b\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(A*(b + c*x^2) + (-3*b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^6+bx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^6 + b*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

maple [A] time = 0.07, size = 219, normalized size = 1.67

$$-\frac{2A\,c^2x^2+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Acx\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}}{3\sqrt{c\,x^4+b\,x^2}\;bc\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(
1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*x*c-3*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x*b+2*A*c^2*x^2+2*A*
b*c)/c/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^{3/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.253 $\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=332

$$\frac{\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-3A}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/5*(-3*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4
+b*x^2)^(1/2)-2/5*A*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)-2/5*(-3*A*c+5*B*b)*(c*x^4
+b*x^2)^(1/2)/b^2/x^(3/2)-2/5*c^(1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipti
cE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*
((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+1/5*c^(
1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2038, 2025, 2032, 329, 305, 220, 1196}

$$\frac{2\sqrt{c}\,x^{3/2}\left(b+cx^2\right)(5bB-3Ac)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(5bB-3Ac)}{5b^2x^{3/2}}+\frac{\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-3Ac)F\left(2\,\mathrm{ta}\right.}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*(5*b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (2*(5*b*
B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^(3/2)) - (2*c^(1/4)*(5*b*B - 3*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) + (c^(1/4)*(5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}dx}{5b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{(c(5bB-3Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}dx}{5b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(c(5bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2c(5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\right.}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2\sqrt{c}(5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}}{5b^{3/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{c}(5bB-3Ac)x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{2\sqrt[4]{c}}{}$$

Mathematica [C] time = 0.07, size = 83, normalized size = 0.25

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}(5bB-3Ac)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^2}{b}\right)+A\left(b+cx^2\right)\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
[Out] (-2*(A*(b + c*x^2) + (5*b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -((c*x^2)/b)]))/(5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^7+bx^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^7 + b*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

maple [A] time = 0.08, size = 413, normalized size = 1.24

$$-\frac{-6A\,c^2x^4+10Bbc\,x^4+6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(
1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c
-3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-10*B*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1
/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^2*b^2+5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2-6*A*c^2*x^4+10*B*b*c*x
^4-4*A*b*c*x^2+10*B*b^2*x^2+2*A*b^2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2 + A}{x^{5/2}\,\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^{\frac{5}{2}}\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.254 $\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=167

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(7bB-5Ac)}{21b^2x^{5/2}}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

```
[Out] -2/7*A*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)-2/21*(-5*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2
)/b^2/x^(5/2)-1/21*c^(3/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2025, 2032, 329, 220}

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(7bB-5Ac)}{21b^2x^{5/2}}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) - (2*(7*b*B - 5*A*c)*Sqrt[b*x^2 +
c*x^4])/(21*b^2*x^(5/2)) - (c^(3/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{7b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{(c(7bB-5Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(c(7bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,d}{21b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(2c(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{l}}\right.}{21b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{c^{3/4}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^{}}{(\sqrt{b}+\sqrt{c}}}}{21b^{9/4}\sqrt{bx^2+cx}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.51

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}\,(5Ac-7bB)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)-6A\left(b+cx^2\right)}{21bx^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*A*(b + c*x^2) + 2*(-7*b*B + 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometr
ic2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(21*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^8+bx^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^8 + b*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\, x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

maple [A] time = 0.07, size = 247, normalized size = 1.48

$$\frac{10A\,c^2x^4 - 14Bbc\,x^4 + 5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ac\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - 7\sqrt{-bc}}{21\sqrt{c\,x^4 + b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
[Out] 1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)
^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*x^3*c-7*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^3*b+10*A*c^2*x
^4-14*B*b*c*x^4+4*A*b*c*x^2-14*B*b^2*x^2-6*A*b^2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\, x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2 + A}{x^{7/2}\,\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^{\frac{7}{2}}\sqrt{x^2\left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.255 $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=369

$$-\frac{c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}+\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7A\dots}{15b^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/15*c^(3/2)*(-7*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)+x*c^(1/2))/(c*x
^4+b*x^2)^(1/2)-2/9*A*(c*x^4+b*x^2)^(1/2)/b/x^(11/2)-2/45*(-7*A*c+9*B*b)*(c
*x^4+b*x^2)^(1/2)/b^2/x^(7/2)+2/15*c*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3
/x^(3/2)+2/15*c^(5/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)-1/15*c^(5/4)*(-7*A*c+9
*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2
^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4
)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2038, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{2c^{3/2}x^{3/2}\left(b+cx^2\right)(9bB-7Ac)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}+\frac{2c^{5/4}x\left(\sqrt{b}\dots\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*c^(3/2)*(9*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]
*x)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) - (2*(9
*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(45*b^2*x^(7/2)) + (2*c*(9*b*B - 7*A*c)*
Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) + (2*c^(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (c^
(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{9b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{(c(9bB-7Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{(c^2(9bB -}{}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{(c^2(9bB -}{}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{(2c^2(9bB -}{}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{(2c^{3/2}(9bB}{}$$

$$= -\frac{2c^{3/2}(9bB-7Ac)x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9b}{}$$

Mathematica [C] time = 0.06, size = 84, normalized size = 0.23

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}\,(9bB-7Ac)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)+5A\left(b+cx^2\right)\right)}{45bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(5*A*(b + c*x^2) + (9*b*B - 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometr
ic2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^9+bx^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^9 + b*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\,x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)

maple [A] time = 0.08, size = 443, normalized size = 1.20

$$-42A\,c^3x^6+54Bb\,c^2x^6+42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ab\,c^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21\sqrt{\frac{cx}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x)

[Out] 1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(42*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-54*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c+27*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2*c-42*A*c^3*x^6+54*B*b*c^2*x^6-28*A*b*c^2*x^4+36*B*b^2*c*x^4+4*A*b^2*c*x^2-18*B*b^3*x^2-10*A*b^3)/b^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}\,x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^{9/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)),x)

[Out] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^{\frac{9}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral((A + B*x**2)/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)

3.256 $\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}(11bB-9Ac)}{231b^3x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{77b}$$

```
[Out] -2/11*A*(c*x^4+b*x^2)^(1/2)/b/x^(13/2)-2/77*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^(
1/2)/b^2/x^(9/2)+10/231*c*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)+5
/231*c^(7/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^
(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2025, 2032, 329, 220}

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}(11bB-9Ac)}{231b^3x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) - (2*(11*b*B - 9*A*c)*Sqrt[b*x^2
 + c*x^4])/(77*b^2*x^(9/2)) + (10*c*(11*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(
231*b^3*x^(5/2)) + (5*c^(7/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(231*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{\left(2\left(-\frac{11bB}{2}+\frac{9Ac}{2}\right)\right)\int\frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{11b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{(5c(11bB-9Ac))\int\frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}+\frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}+\frac{(5c}{}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}+\frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}+\frac{(5c}{}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}+\frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}+\frac{(10}{}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}+\frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}+\frac{5c^7}{}$$

Mathematica [C] time = 0.07, size = 84, normalized size = 0.41

$$-\frac{2\left(x^2\sqrt{\frac{cx^2}{b}+1}\,(11bB-9Ac)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)+7A\left(b+cx^2\right)\right)}{77bx^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(7*A*(b + c*x^2) + (11*b*B - 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeomet
ric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(9/2)*Sqrt[x^2*(b + c*x^2)]
)
```

fricas [F] time = 1.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{cx^{10}+bx^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c*x^10 + b*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\, x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

maple [A] time = 0.09, size = 274, normalized size = 1.34

$$-\frac{90A\,c^3x^6 - 110Bb\,c^2x^6 + 45\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,A\,c^2x^5\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - 55\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/231/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(45*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*
c)^(1/2)*x^5*c^2-55*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*x^5*b*c+90
*A*c^3*x^6-110*B*b*c^2*x^6+36*A*b*c^2*x^4-44*B*b^2*c*x^4-12*A*b^2*c*x^2+66*
B*b^3*x^2+42*A*b^3)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\, x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2 + A}{x^{11/2}\,\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^{\frac{11}{2}}\sqrt{x^2\left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(11/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.257 $\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(13bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}}+\frac{15b\sqrt{bx^2+cx^4}(13bB-11Ac)}{77c^4\sqrt{x}}-\frac{9x^{3/2}\sqrt{\ldots}}{\ldots}$$

```
[Out] -(-A*c+B*b)*x^(15/2)/b/c/(c*x^4+b*x^2)^(1/2)-9/77*(-11*A*c+13*B*b)*x^(3/2)*
(c*x^4+b*x^2)^(1/2)/c^3+1/11*(-11*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/b
/c^2+15/77*b*(-11*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)-15/154*b^(7/4
)*(-11*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^
(1/2)/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2037, 2024, 2032, 329, 220}

$$-\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(13bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}}+\frac{x^{7/2}\sqrt{bx^2+cx^4}(13bB-11Ac)}{11bc^2}-\frac{9x^{3/2}\sqrt{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(15/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (15*b*(13*b*B - 11*A*
c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) - (9*(13*b*B - 11*A*c)*x^(3/2)*Sqr
t[b*x^2 + c*x^4])/(77*c^3) + ((13*b*B - 11*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4]
)/(11*b*c^2) - (15*b^(7/4)*(13*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(154*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{17/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{13bB}{2}-\frac{11Ac}{2}\right)\int\frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2}-\frac{(9(13bB-11Ac))\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{22c^2}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}+\frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}$$

Mathematica [C] time = 0.24, size = 134, normalized size = 0.53

$$\frac{x^{3/2}\left(15b^2\sqrt{\frac{cx^2}{b}+1}\,(11Ac-13bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+b^2\left(78Bcx^2-165Ac\right)-2bc^2x^2\left(33A+13Bx^2\right)+2c^3x^4\right.}{77c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(195*b^3*B + 2*c^3*x^4*(11*A + 7*B*x^2) - 2*b*c^2*x^2*(33*A + 13*B
*x^2) + b^2*(-165*A*c + 78*B*c*x^2) + 15*b^2*(-13*b*B + 11*A*c)*Sqrt[1 + (c
```

```
*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^4*Sqrt[x^2*
(b + c*x^2)])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^6 + Ax^4\right)\sqrt{cx^4 + bx^2}\,\sqrt{x}}{c^2x^4 + 2\,bcx^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^4)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 +
 b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{17}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.15, size = 281, normalized size = 1.12

$$\frac{\left(c\,x^2 + b\right)\left(28B\,c^4x^7 + 44A\,c^4x^5 - 52Bb\,c^3x^5 - 132Ab\,c^3x^3 + 156B\,b^2c^2x^3 - 330A\,b^2c^2x + 390B\,b^3cx + 165\sqrt{-bc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/154/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(28*B*c^4*x^7+165*A*(-b*c)^(1/2
)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*b^2*c+44*A*c^4*x^5-195*B*(-b*c)^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1
/(-b*c)^(1/2)*c*x)^(1/2)*b^3-52*B*b*c^3*x^5-132*A*b*c^3*x^3+156*B*b^2*c^2*x
^3-330*A*b^2*c^2*x+390*B*x*b^3*c)/c^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{17}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{17/2}\,\left(B\,x^2 + A\right)}{\left(c\,x^4 + b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

3.258 $\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=377

$$\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}-\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9.}{15c^{15/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(13/2)/b/c/(c*x^4+b*x^2)^(1/2)+7/15*b*(-9*A*c+11*B*b)*x^(3/2)
*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/9*(-9*A*c+11*B
*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/b/c^2-7/45*(-9*A*c+11*B*b)*x^(1/2)*(c*x^4+b
*x^2)^(1/2)/c^3-7/15*b^(5/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)+7/30*b^(5/4)*(
-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2037, 2024, 2032, 329, 305, 220, 1196}

$$\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}-\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9.}{15c^{15/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(13/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (7*b*(11*b*B - 9*A*c)
*x^(3/2)*(b + c*x^2))/(15*c^(7/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]
) - (7*(11*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^3) + ((11*b*B -
9*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*b*c^2) - (7*b^(5/4)*(11*b*B - 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(15/4)*Sqrt[b*x^2 + c*x^
4]) + (7*b^(5/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(30*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{15/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{11bB}{2}-\frac{9Ac}{2}\right)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} - \frac{(7(11bB-9Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{18c^2}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{(7b\cdots}{\cdots}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{\left(7b\cdots\right.}{\cdots}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{\left(7b\cdots\right.}{\cdots}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{\left(7b\cdots\right.}{\cdots}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{7b(11bB-9Ac)x^{3/2}\left(b+cx^2\right)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(\cdots}{\cdots}$$

Mathematica [C] time = 0.18, size = 110, normalized size = 0.29

$$\frac{2x^{5/2}\left(7b\sqrt{\frac{cx^2}{b}+1}\,(9Ac-11bB)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-bc\left(63A+11Bx^2\right)+c^2x^2\left(9A+5Bx^2\right)+77b^2B\right)}{45c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(77*b^2*B + c^2*x^2*(9*A + 5*B*x^2) - b*c*(63*A + 11*B*x^2) + 7*
b*(-11*b*B + 9*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -(
(c*x^2)/b)]))/(45*c^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^5+Ax^3\right)\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^5 + A*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 +
 b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{15}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.12, size = 420, normalized size = 1.11

$$-\frac{\left(c\,x^2+b\right)\left(-20B\,c^3x^6-36A\,c^3x^4+44Bb\,c^2x^4-126Ab\,c^2x^2+154B\,b^2c\,x^2+378\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/90/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(-20*B*c^3*x^6+378*A*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*b^2*c-189*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*E
llipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c-462*B*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))*b^3+231*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^
(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^3-36
*A*c^3*x^4+44*B*b*c^2*x^4-126*A*b*c^2*x^2+154*B*b^2*c*x^2)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{15}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{15/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.259 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}\,(9bB-7Ac)}{21c^3\sqrt{x}}+\frac{x^{3/2}\sqrt{bx^2+cx^4}\,(\ldots}{7bc^2\ldots}$$

```
[Out] -(-A*c+B*b)*x^(11/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/7*(-7*A*c+9*B*b)*x^(3/2)*(c*
x^4+b*x^2)^(1/2)/b/c^2-5/21*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)+
5/42*b^(3/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2037, 2024, 2032, 329, 220}

$$\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}\sqrt{bx^2+cx^4}\,(9bB-7Ac)}{7bc^2}-\frac{5\sqrt{bx^2+cx^4}\,(\ldots}{21c^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(11/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (5*(9*b*B - 7*A*c)*Sq
rt[b*x^2 + c*x^4])/(21*c^3*Sqrt[x]) + ((9*b*B - 7*A*c)*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(7*b*c^2) + (5*b^(3/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(42*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{13/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{9bB}{2}-\frac{7Ac}{2}\right)\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2}-\frac{(5(9bB-7Ac))\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{14c^2}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}-\frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2}+\frac{(5b(9b\ldots}{\ldots}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}-\frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2}+\frac{\left(5b(9b\ldots\right.}{\ldots}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}-\frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2}+\frac{\left(5b(9b\ldots\right.}{\ldots}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}-\frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2}+\frac{5b^{3/4}(\ldots}{\ldots}$$

Mathematica [C] time = 0.17, size = 110, normalized size = 0.51

$$\frac{x^{3/2}\left(5b\sqrt{\frac{cx^2}{b}+1}\,(9bB-7Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+bc\left(35A-18Bx^2\right)+2c^2x^2\left(7A+3Bx^2\right)-45b^2B\right)}{21c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-45*b^2*B + b*c*(35*A - 18*B*x^2) + 2*c^2*x^2*(7*A + 3*B*x^2) + 5
*b*(9*b*B - 7*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
c*x^2)/b)]))/(21*c^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^4+Ax^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 +
 b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{13}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.09, size = 255, normalized size = 1.19

$$-\frac{\left(c\,x^2+b\right)\left(-12B\,c^3x^5-28A\,c^3x^3+36Bb\,c^2x^3-70Ab\,c^2x+90B\,b^2cx+35\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\right.}{42\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/42/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(35*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^
(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(-b*c)^(1/2)*b*c-45*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2-1
2*B*c^3*x^5-28*A*c^3*x^3+36*B*b*c^2*x^3-70*A*b*c^2*x+90*B*b^2*c*x)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{13}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

3.260 $\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=340

$$-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-5A}{5c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(9/2)/b/c/(c*x^4+b*x^2)^(1/2)-3/5*(-5*A*c+7*B*b)*x^(3/2)*(c*x
^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-5*A*c+7*B*b)*x^
(1/2)*(c*x^4+b*x^2)^(1/2)/b/c^2+3/5*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*E
llipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(
1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)-
3/10*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2037, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{3x^{3/2}\left(b+cx^2\right)(7bB-5Ac)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{\sqrt{x}\,\sqrt{bx^2+cx^4}\,(7bB-5Ac)}{5bc^2}-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-5Ac)F\left(2\right.}{10c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(9/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(7*b*B - 5*A*c)*x^(
3/2)*(b + c*x^2))/(5*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (
(7*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b*c^2) + (3*b^(1/4)*(7*b*B
- 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(11/4)*Sqrt[b*x^2
 + c*x^4]) - (3*b^(1/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{11/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{7bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{(3(7bB-5Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10c^2}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \right.}{5c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3\sqrt{b}(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\right.}{5c^{5/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} - \frac{3(7bB-5Ac)x^{3/2}\left(b+cx^2\right)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} + \frac{3\sqrt[4]{b}}{}$$

Mathematica [C] time = 0.13, size = 85, normalized size = 0.25

$$\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}\,(7bB-5Ac)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)+5Ac-7bB+Bcx^2\right)}{5c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(-7*b*B + 5*A*c + B*c*x^2 + (7*b*B - 5*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2
)])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^3+Ax\right)\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^3 + A*x)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 + b
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{11}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)

maple [A] time = 0.09, size = 394, normalized size = 1.16

$$\left(c x^2+b\right)\left(4B c^2 x^4-10A c^2 x^2+14Bbc x^2+30\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)

[Out] 1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(30*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-15*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-42*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+21*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2+4*B*c^2*x^4-10*A*c^2*x^2+14*B*b*c*x^2)/c^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{11}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)

[Out] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)

[Out] Timed out

3.261 $\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{b}\,c^{9/4}\sqrt{bx^2+cx^4}}+\frac{\sqrt{bx^2+cx^4}\,(5bB-3Ac)}{3bc^2\sqrt{x}}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(7/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/3*(-3*A*c+5*B*b)*(c*x^4+b*x^2
)^(1/2)/b/c^2/x^(1/2)-1/6*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2037, 2024, 2032, 329, 220}

$$\frac{\sqrt{bx^2+cx^4}\,(5bB-3Ac)}{3bc^2\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{b}\,c^{9/4}\sqrt{bx^2+cx^4}}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(7/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((5*b*B - 3*A*c)*Sqrt[
b*x^2 + c*x^4])/(3*b*c^2*Sqrt[x]) - ((5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(6*b^(1/4)*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{9/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{5bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}-\frac{(5bB-3Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6c^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}-\frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{6c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}-\frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+c}}\right.}{3c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}-\frac{(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}}{6\sqrt[4]{b}\,c^{9/4}\sqrt{bx^2+cx}}$$

Mathematica [C] time = 0.13, size = 86, normalized size = 0.48

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}\,(3Ac-5bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-3Ac+5bB+2Bcx^2\right)}{3c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(5*b*B - 3*A*c + 2*B*c*x^2 + (-5*b*B + 3*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2
)])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 + b^2
), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{9}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)

maple [A] time = 0.08, size = 230, normalized size = 1.29

$$\frac{\left(c\,x^2 + b\right)\left(4B\,c^2x^3 - 6A\,c^2x + 10Bbcx + 3\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-bc}\;Ac\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \right.\right.}{6\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)

[Out] 1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(3*A*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-b*c)^(1/2)*c-5*B*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(
-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-b*c)^(1/2)*b+4*B*c^2*x
^3-6*A*x*c^2+10*B*b*c*x)/c^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^{\frac{9}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}\,\left(B\,x^2 + A\right)}{\left(c\,x^4 + b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)

[Out] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)

[Out] Timed out

3.262 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=299

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)E\left(2\tan\right.}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(5/2)/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+3*B*b)*x^(3/2)*(c*x^2+b)/
b/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-(-A*c+3*B*b)*x*(cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x
*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x^4+b*
x^2)^(1/2)+1/2*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2037, 2032, 329, 305, 220, 1196}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)E\left(2\tan\right.}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(5/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - A*c)*x^(3/2)
*(b + c*x^2))/(b*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - ((3*b
*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]
*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(2*b^(3/4)*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{3bB}{2}-\frac{Ac}{2}\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b}\,c^{3/2}\sqrt{bx^2+cx^4}}-\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)\right.}{}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-Ac)x^{3/2}\left(b+cx^2\right)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right.}}}{b^{3/4}c^{7/4}\sqrt{bx^2+c}}$$

Mathematica [C] time = 0.11, size = 78, normalized size = 0.26

$$-\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}\,(3bB-Ac)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-3bB\right)}{3bc\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x^(5/2)*(-3*b*B + (3*b*B - A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3
/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b*c*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,(Bx^2+A)\sqrt{x}}{c^2x^5+2\,bcx^3+b^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^5 + 2*b*c*x^3 + b^2
*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2+A)x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.08, size = 388, normalized size = 1.30

$$-\frac{(c\,x^2+b)\left(-2A\,c^2x^2+2Bbc\,x^2+2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*A*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1
/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b*c-A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/
2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*b^2+3*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-2*A*c^2*x^2+2*B*b*c*x^2)/c^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2+A)x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}\,\left(B\,x^2+A\right)}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

3.263 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(3/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(A*c+B*b)*x*(cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1
/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(5/4)/(c*x^4+b*x^2)^
(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2037, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(3/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((b*B + A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2037

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[(e^(j - 1)*(b*c - a*d)*(e*x)^(m - j +
 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(e^j*(a*d*(m
+ j*p + 1) - b*c*(m + n + p*(j + n) + 1)))/(a*b*n*(p + 1)), Int[(e*x)^(m -
```

```
j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n},
 x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &
& GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{2bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.08, size = 76, normalized size = 0.55

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}\,(Ac+bB)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+Ac-bB\right)}{bc\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-(b*B) + A*c + (b*B + A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[
1/4, 1/2, 5/4, -((c*x^2)/b)]))/(b*c*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^6+2\,bcx^4+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^6 + 2*b*c*x^4 + b^2
*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.09, size = 222, normalized size = 1.62

$$\frac{\left(c\,x^{2}+b\right)\left(2A\,c^{2}x-2Bbcx+\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-bc}\,Ac\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+\sqrt{2}\right.}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(A*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-b*c)^(1/2)*c+B*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c
)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-b*c)^(1/2)*b+2*A*c^2*x-2*B
*b*c*x)/b/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{2}+A\right)x^{\frac{5}{2}}}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}\,\left(B\,x^{2}+A\right)}{\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.264 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)E\left(2\tan^{-1}\right.}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (-3*A*c+B*b)*x^(5/2)/b^2/(c*x^4+b*x^2)^(1/2)-(-3*A*c+B*b)*x^(3/2)*(c*x^2+b)
/b^2/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*A*x^(1/2)/b/(c*x^4+b
*x^2)^(1/2)+(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2
))^2)^(1/2)/b^(7/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)-1/2*(-3*A*c+B*b)*x*(cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)
+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/c^(3/4)/(c*x^4+
b*x^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2038, 2023, 2032, 329, 305, 220, 1196}

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)E\left(2\tan^{-1}\right.}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A*Sqrt[x])/(b*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x^(5/2))/(b^2*Sqrt[
b*x^2 + c*x^4]) - ((b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(b^2*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(b^(7/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - ((b*B - 3*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(7/4)*c^(3/4)*Sqrt[b*x^2
+ c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{b}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b^{3/2}\sqrt{c}\,\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x^{3/2}\left(b+cx^2\right)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x\left(\sqrt{b}+\right.}{}$$

Mathematica [C] time = 0.08, size = 77, normalized size = 0.24

$$\frac{2\sqrt{x}\left(x^2\sqrt{\frac{cx^2}{b}+1}\,(bB-3Ac)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-3Ab\right)}{3b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(-3*A*b + (b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^7+2\,bcx^5+b^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^7 + 2*b*c*x^5 + b^2
*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.10, size = 392, normalized size = 1.23

$$\frac{\left(c\,x^2+b\right)\left(-6A\,c^2x^2+2Bbc\,x^2+6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/
2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*b*c-3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c-2*B*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1
/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b^2+B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-6*A*c^2*x^2+2*B*b*c*x^2-4*A*b*c)
/c/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}\left(A+Bx^2\right)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(3/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.265 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{x^{3/2}(3bB-5Ac)}{3b^2\sqrt{bx^2+cx^4}}-\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}}$$

```
[Out] 1/3*(-5*A*c+3*B*b)*x^(3/2)/b^2/(c*x^4+b*x^2)^(1/2)-2/3*A/b/x^(1/2)/(c*x^4+b
*x^2)^(1/2)+1/6*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*
c^(1/2))^2)^(1/2)/b^(9/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2038, 2023, 2032, 329, 220}

$$\frac{x^{3/2}(3bB-5Ac)}{3b^2\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}-\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A)/(3*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x^(3/2))/(3*b^2
*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(6*b^(9/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6b^2}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,d\right.}{3b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\right.}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.07, size = 92, normalized size = 0.55

$$\frac{x^2\sqrt{\frac{cx^2}{b}+1}\,(3bB-5Ac)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-2Ab-5Acx^2+3bBx^2}{3b^2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A*b + 3*b*B*x^2 - 5*A*c*x^2 + (3*b*B - 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*H
ypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)])/(3*b^2*Sqrt[x]*Sqrt[x^2*(b +
 c*x^2)])
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^8+2\,bcx^6+b^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^8 + 2*b*c*x^6 + b^2
*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)\sqrt{x}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.09, size = 235, normalized size = 1.41

$$-\frac{\left(c\,x^2+b\right)\left(10A\,c^2x^2-6Bbc\,x^2+5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Acx\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\right.}{6\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*A*(-b*c)^(1/2)*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*x*c-3*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1
/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x*b+10*A
*c^2*x^2-6*B*b*c*x^2+4*A*b*c)/c/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)\sqrt{x}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(x)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.266 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=368

$$\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-7Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-7Ac}{5b^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/5*A/b/x^(3/2)/(c*x^4+b*x^2)^(1/2)+3/5*(-7*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c
^(1/2)/b^3/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-7*A*c+5*B*b)*x^(1/
2)/b^2/(c*x^4+b*x^2)^(1/2)-3/5*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(3/
2)-3/5*c^(1/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c
^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)+3/10*c^(1/4)*(-7*A*c+5*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))
*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^
4+b*x^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt{c}\,x^{3/2}\left(b+cx^2\right)(5bB-7Ac)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{\sqrt{x}\,(5bB-7Ac)}{5b^2\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}\,(5bB-7Ac)}{5b^3x^{3/2}}+\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5}{10b^{11/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(5*b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) + ((5*b*B - 7*A*c)*Sqrt[x])/(5*b^2
*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*(5*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(5*
b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*(5*b*B - 7*A*c)*Sqrt[b*
x^2 + c*x^4])/(5*b^3*x^(3/2)) - (3*c^(1/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) + (3*c^(1/4)*(
5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqr
t[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{5b}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{(3(5bB-7Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{10b^2}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{(3c(5bB-7Ac)}{10}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{(3c(5bB-7Ac)}{10b}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{(3c(5bB-7Ac)}{}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{(3\sqrt{c}(5bB-7}{}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}(5bB-7Ac)x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{3(5bB-7}{5}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.21

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}\,(7Ac-5bB)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{cx^2}{b}\right)-2Ab}{5b^2x^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A*b + 2*(-5*b*B + 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4
, 3/2, 3/4, -((c*x^2)/b)])/(5*b^2*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^9+2\,bcx^7+b^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^9 + 2*b*c*x^7 + b^2
*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

maple [A] time = 0.09, size = 420, normalized size = 1.14

$$-\frac{\left(c\,x^2+b\right)\left(-42A\,c^2x^4+30Bbc\,x^4+42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Abc\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^
(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*b*c-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-30*B*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*x^2*b^2+15*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b^2-42*A*c^2
*x^4+30*B*b*c*x^4-28*A*b*c*x^2+20*B*b^2*x^2+4*A*b^2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{\sqrt{x}\,\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Timed out
```

3.267 $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}(7bB-9Ac)}{21b^3x^{5/2}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+}}$$

```
[Out] -2/7*A/b/x^(5/2)/(c*x^4+b*x^2)^(1/2)+1/7*(-9*A*c+7*B*b)/b^2/x^(1/2)/(c*x^4+
b*x^2)^(1/2)-5/21*(-9*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)-5/42*c^(3/
4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(
1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(
1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2038, 2023, 2025, 2032, 329, 220}

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}(7bB-9Ac)}{21b^3x^{5/2}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(7*b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) + (7*b*B - 9*A*c)/(7*b^2*Sqrt[x]*S
qrt[b*x^2 + c*x^4]) - (5*(7*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b^3*x^(5/
2)) - (5*c^(3/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(42*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{7b}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(5(7bB-9Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{14b^2}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{(5c(7bB-}{}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{\left(5c(7bB-\right.}{}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{\left(5c(7bB-\right.}{}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{5c^{3/4}(7bB}{}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 0.39

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}\,(9Ac-7bB)\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{cx^2}{b}\right)-6Ab}{21b^2x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-6*A*b + 2*(-7*b*B + 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4
, 3/2, 1/4, -((c*x^2)/b)])/(21*b^2*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(Bx^2 + A\right)\sqrt{x}}{c^2x^{10} + 2\,bcx^8 + b^2x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^10 + 2*b*c*x^8 + b^
2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

maple [A] time = 0.09, size = 254, normalized size = 1.25

$$\frac{\left(c\,x^2 + b\right)\left(90A\,c^2x^4 - 70Bbc\,x^4 + 45\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ac\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right)\right.}{42\left(c\,x^4 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/42/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(45*A*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*x^3*c-35*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^3*
b+90*A*c^2*x^4-70*B*b*c*x^4+36*A*b*c*x^2-28*B*b^2*x^2-12*A*b^2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^{3/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x)
[Out] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

3.268 $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=405

$$-\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}+\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-1}{15b^{15/4}\sqrt{bx^2+c}}$$

```
[Out] -2/9*A/b/x^(7/2)/(c*x^4+b*x^2)^(1/2)+1/9*(-11*A*c+9*B*b)/b^2/x^(3/2)/(c*x^4
+b*x^2)^(1/2)-7/15*c^(3/2)*(-11*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^4/(b^(1/2)+x
*c^(1/2))/(c*x^4+b*x^2)^(1/2)-7/45*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/
x^(7/2)+7/15*c*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^(3/2)+7/15*c^(5/4)
*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1
/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)-7/30*c^(5/4)*(-11*A*c+9*B*b)*x*(cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*
c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(15/4)/(c*x^4+b*x^2)^(1/
2)
```

Rubi [A] time = 0.52, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{7c^{3/2}x^{3/2}\left(b+cx^2\right)(9bB-11Ac)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-11Ac)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}+\frac{7c^{5/4}x\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(9*b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (9*b*B - 11*A*c)/(9*b^2*x^(3/2)*
Sqrt[b*x^2 + c*x^4]) - (7*c^(3/2)*(9*b*B - 11*A*c)*x^(3/2)*(b + c*x^2))/(15
*b^4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*(9*b*B - 11*A*c)*Sqrt[
b*x^2 + c*x^4])/(45*b^3*x^(7/2)) + (7*c*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4
])/(15*b^4*x^(3/2)) + (7*c^(5/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) - (7*c^(5/4)*(9*b*B -
11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2
 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx}{9b}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} + \frac{(7(9bB-11Ac))\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{18b^2}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{(7c(9bB-1}{}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-1}{1}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-1}{1}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-1}{1}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-1}{1}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7c^{3/2}(9bB-11Ac)x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{7(9bB}{}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.20

$$\frac{2x^2\sqrt{\frac{cx^2}{b}+1}\,(11Ac-9bB)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)-10Ab}{45b^2x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-10*A*b + 2*(-9*b*B + 11*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5
/4, 3/2, -1/4, -((c*x^2)/b)])/(45*b^2*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}}{c^2x^{11}+2\,bcx^9+b^2x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + b^
2*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

maple [A] time = 0.09, size = 450, normalized size = 1.11

$$\frac{\left(c\,x^2+b\right)\left(-462A\,c^3x^6+378Bb\,c^2x^6+462\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,Ab\,c^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^
(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*x^4*b*c^2-231*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-378*B*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))*x^4*b^2*c+189*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b^2
*c-462*A*c^3*x^6+378*B*b*c^2*x^6-308*A*b*c^2*x^4+252*B*b^2*c*x^4+44*A*b^2*c
*x^2-36*B*b^3*x^2-20*A*b^3)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^{5/2}\,\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.269 $\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=96

$$\frac{Ab^3x^{m+7}}{m+7}+\frac{b^2x^{m+9}(3Ac+bB)}{m+9}+\frac{c^2x^{m+13}(Ac+3bB)}{m+13}+\frac{3bcx^{m+11}(Ac+bB)}{m+11}+\frac{Bc^3x^{m+15}}{m+15}$$

```
[Out] A*b^3*x^(7+m)/(7+m)+b^2*(3*A*c+B*b)*x^(9+m)/(9+m)+3*b*c*(A*c+B*b)*x^(11+m)/
(11+m)+c^2*(A*c+3*B*b)*x^(13+m)/(13+m)+B*c^3*x^(15+m)/(15+m)
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{b^2x^{m+9}(3Ac+bB)}{m+9}+\frac{Ab^3x^{m+7}}{m+7}+\frac{c^2x^{m+13}(Ac+3bB)}{m+13}+\frac{3bcx^{m+11}(Ac+bB)}{m+11}+\frac{Bc^3x^{m+15}}{m+15}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (A*b^3*x^(7 + m))/(7 + m) + (b^2*(b*B + 3*A*c)*x^(9 + m))/(9 + m) + (3*b*c*
(b*B + A*c)*x^(11 + m))/(11 + m) + (c^2*(3*b*B + A*c)*x^(13 + m))/(13 + m)
+ (B*c^3*x^(15 + m))/(15 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx &= \int x^{6+m} \left(A+Bx^2\right) \left(b+cx^2\right)^3 \, dx\\
&= \int \left(Ab^3x^{6+m}+b^2(bB+3Ac)x^{8+m}+3bc(bB+Ac)x^{10+m}+c^2(3bB+Ac)x^{12+m}\right.\\
&= \frac{Ab^3x^{7+m}}{7+m}+\frac{b^2(bB+3Ac)x^{9+m}}{9+m}+\frac{3bc(bB+Ac)x^{11+m}}{11+m}+\frac{c^2(3bB+Ac)x^{13+m}}{13+m}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 89, normalized size = 0.93

$$x^{m+7}\left(\frac{Ab^3}{m+7}+\frac{b^2x^2(3Ac+bB)}{m+9}+\frac{c^2x^6(Ac+3bB)}{m+13}+\frac{3bcx^4(Ac+bB)}{m+11}+\frac{Bc^3x^8}{m+15}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^(7 + m)*((A*b^3)/(7 + m) + (b^2*(b*B + 3*A*c)*x^2)/(9 + m) + (3*b*c*(b*B
+ A*c)*x^4)/(11 + m) + (c^2*(3*b*B + A*c)*x^6)/(13 + m) + (B*c^3*x^8)/(15 +
 m))
```

fricas [B] time = 0.91, size = 381, normalized size = 3.97

$$\frac{\left(\left(Bc^3m^4 + 40\,Bc^3m^3 + 590\,Bc^3m^2 + 3800\,Bc^3m + 9009\,Bc^3\right)x^{15} + \left(\left(3\,Bbc^2 + Ac^3\right)m^4 + 31185\,Bbc^2 + 10395\,A\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] ((B*c^3*m^4 + 40*B*c^3*m^3 + 590*B*c^3*m^2 + 3800*B*c^3*m + 9009*B*c^3)*x^1
5 + ((3*B*b*c^2 + A*c^3)*m^4 + 31185*B*b*c^2 + 10395*A*c^3 + 42*(3*B*b*c^2
+ A*c^3)*m^3 + 644*(3*B*b*c^2 + A*c^3)*m^2 + 4278*(3*B*b*c^2 + A*c^3)*m)*x^
13 + 3*((B*b^2*c + A*b*c^2)*m^4 + 12285*B*b^2*c + 12285*A*b*c^2 + 44*(B*b^2
*c + A*b*c^2)*m^3 + 706*(B*b^2*c + A*b*c^2)*m^2 + 4884*(B*b^2*c + A*b*c^2)*
m)*x^11 + ((B*b^3 + 3*A*b^2*c)*m^4 + 15015*B*b^3 + 45045*A*b^2*c + 46*(B*b^
3 + 3*A*b^2*c)*m^3 + 776*(B*b^3 + 3*A*b^2*c)*m^2 + 5666*(B*b^3 + 3*A*b^2*c)
*m)*x^9 + (A*b^3*m^4 + 48*A*b^3*m^3 + 854*A*b^3*m^2 + 6672*A*b^3*m + 19305*
A*b^3)*x^7)*x^m/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

giac [B] time = 0.22, size = 603, normalized size = 6.28

$$\frac{Bc^3m^4x^{15}x^m + 40\,Bc^3m^3x^{15}x^m + 3\,Bbc^2m^4x^{13}x^m + Ac^3m^4x^{13}x^m + 590\,Bc^3m^2x^{15}x^m + 126\,Bbc^2m^3x^{13}x^m + 42}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (B*c^3*m^4*x^15*x^m + 40*B*c^3*m^3*x^15*x^m + 3*B*b*c^2*m^4*x^13*x^m + A*c^
3*m^4*x^13*x^m + 590*B*c^3*m^2*x^15*x^m + 126*B*b*c^2*m^3*x^13*x^m + 42*A*c
^3*m^3*x^13*x^m + 3800*B*c^3*m*x^15*x^m + 3*B*b^2*c*m^4*x^11*x^m + 3*A*b*c^
2*m^4*x^11*x^m + 1932*B*b*c^2*m^2*x^13*x^m + 644*A*c^3*m^2*x^13*x^m + 9009*
B*c^3*x^15*x^m + 132*B*b^2*c*m^3*x^11*x^m + 132*A*b*c^2*m^3*x^11*x^m + 1283
4*B*b*c^2*m*x^13*x^m + 4278*A*c^3*m*x^13*x^m + B*b^3*m^4*x^9*x^m + 3*A*b^2*
c*m^4*x^9*x^m + 2118*B*b^2*c*m^2*x^11*x^m + 2118*A*b*c^2*m^2*x^11*x^m + 311
85*B*b*c^2*x^13*x^m + 10395*A*c^3*x^13*x^m + 46*B*b^3*m^3*x^9*x^m + 138*A*b
^2*c*m^3*x^9*x^m + 14652*B*b^2*c*m*x^11*x^m + 14652*A*b*c^2*m*x^11*x^m + A*
b^3*m^4*x^7*x^m + 776*B*b^3*m^2*x^9*x^m + 2328*A*b^2*c*m^2*x^9*x^m + 36855*
B*b^2*c*x^11*x^m + 36855*A*b*c^2*x^11*x^m + 48*A*b^3*m^3*x^7*x^m + 5666*B*b
^3*m*x^9*x^m + 16998*A*b^2*c*m*x^9*x^m + 854*A*b^3*m^2*x^7*x^m + 15015*B*b^
3*x^9*x^m + 45045*A*b^2*c*x^9*x^m + 6672*A*b^3*m*x^7*x^m + 19305*A*b^3*x^7*
x^m)/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

maple [B] time = 0.05, size = 474, normalized size = 4.94

$$\frac{\left(B\,c^3m^4x^8 + 40B\,c^3m^3x^8 + A\,c^3m^4x^6 + 3Bb\,c^2m^4x^6 + 590B\,c^3m^2x^8 + 42A\,c^3m^3x^6 + 126Bb\,c^2m^3x^6 + 3800B\,c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x)
```

```
[Out] x^(m+7)*(B*c^3*m^4*x^8+40*B*c^3*m^3*x^8+A*c^3*m^4*x^6+3*B*b*c^2*m^4*x^6+590
*B*c^3*m^2*x^8+42*A*c^3*m^3*x^6+126*B*b*c^2*m^3*x^6+3800*B*c^3*m*x^8+3*A*b*
c^2*m^4*x^4+644*A*c^3*m^2*x^6+3*B*b^2*c*m^4*x^4+1932*B*b*c^2*m^2*x^6+9009*B
*c^3*x^8+132*A*b*c^2*m^3*x^4+4278*A*c^3*m*x^6+132*B*b^2*c*m^3*x^4+12834*B*b
*c^2*m*x^6+3*A*b^2*c*m^4*x^2+2118*A*b*c^2*m^2*x^4+10395*A*c^3*x^6+B*b^3*m^4
*x^2+2118*B*b^2*c*m^2*x^4+31185*B*b*c^2*x^6+138*A*b^2*c*m^3*x^2+14652*A*b*c
^2*m*x^4+46*B*b^3*m^3*x^2+14652*B*b^2*c*m*x^4+A*b^3*m^4+2328*A*b^2*c*m^2*x^
2+36855*A*b*c^2*x^4+776*B*b^3*m^2*x^2+36855*B*b^2*c*x^4+48*A*b^3*m^3+16998*
A*b^2*c*m*x^2+5666*B*b^3*m*x^2+854*A*b^3*m^2+45045*A*b^2*c*x^2+15015*B*b^3*
x^2+6672*A*b^3*m+19305*A*b^3)/(m+15)/(m+13)/(m+11)/(m+9)/(m+7)
```

maxima [A] time = 1.37, size = 129, normalized size = 1.34

$$\frac{Bc^3x^{m+15}}{m+15}+\frac{3\,Bbc^2x^{m+13}}{m+13}+\frac{Ac^3x^{m+13}}{m+13}+\frac{3\,Bb^2cx^{m+11}}{m+11}+\frac{3\,Abc^2x^{m+11}}{m+11}+\frac{Bb^3x^{m+9}}{m+9}+\frac{3\,Ab^2cx^{m+9}}{m+9}+\frac{Ab^3x^{m+7}}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] B*c^3*x^(m + 15)/(m + 15) + 3*B*b*c^2*x^(m + 13)/(m + 13) + A*c^3*x^(m + 13
)/(m + 13) + 3*B*b^2*c*x^(m + 11)/(m + 11) + 3*A*b*c^2*x^(m + 11)/(m + 11)
+ B*b^3*x^(m + 9)/(m + 9) + 3*A*b^2*c*x^(m + 9)/(m + 9) + A*b^3*x^(m + 7)/(
m + 7)
```

mupad [B] time = 0.48, size = 291, normalized size = 3.03

$$\frac{A\,b^3\,x^m\,x^7\,\left(m^4+48\,m^3+854\,m^2+6672\,m+19305\right)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}+\frac{B\,c^3\,x^m\,x^{15}\,\left(m^4+40\,m^3+590\,m^2+3800\,m+9009\right)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+13513}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (A*b^3*x^m*x^7*(6672*m + 854*m^2 + 48*m^3 + m^4 + 19305))/(66009*m + 12650*
m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (B*c^3*x^m*x^15*(3800*m + 590*m^2
 + 40*m^3 + m^4 + 9009))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 1
35135) + (b^2*x^m*x^9*(3*A*c + B*b)*(5666*m + 776*m^2 + 46*m^3 + m^4 + 1501
5))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (c^2*x^m*x^1
3*(A*c + 3*B*b)*(4278*m + 644*m^2 + 42*m^3 + m^4 + 10395))/(66009*m + 12650
*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (3*b*c*x^m*x^11*(A*c + B*b)*(488
4*m + 706*m^2 + 44*m^3 + m^4 + 12285))/(66009*m + 12650*m^2 + 1190*m^3 + 55
*m^4 + m^5 + 135135)
```

sympy [A] time = 9.05, size = 2077, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise((-A*b**3/(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c
**3/(2*x**2) - B*b**3/(6*x**6) - 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2)
+ B*c**3*log(x), Eq(m, -15)), (-A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3*A
*b*c**2/(2*x**2) + A*c**3*log(x) - B*b**3/(4*x**4) - 3*B*b**2*c/(2*x**2) +
3*B*b*c**2*log(x) + B*c**3*x**2/2, Eq(m, -13)), (-A*b**3/(4*x**4) - 3*A*b**
2*c/(2*x**2) + 3*A*b*c**2*log(x) + A*c**3*x**2/2 - B*b**3/(2*x**2) + 3*B*b*
*2*c*log(x) + 3*B*b*c**2*x**2/2 + B*c**3*x**4/4, Eq(m, -11)), (-A*b**3/(2*x
**2) + 3*A*b**2*c*log(x) + 3*A*b*c**2*x**2/2 + A*c**3*x**4/4 + B*b**3*log(x
) + 3*B*b**2*c*x**2/2 + 3*B*b*c**2*x**4/4 + B*c**3*x**6/6, Eq(m, -9)), (A*b
**3*log(x) + 3*A*b**2*c*x**2/2 + 3*A*b*c**2*x**4/4 + A*c**3*x**6/6 + B*b**3
*x**2/2 + 3*B*b**2*c*x**4/4 + B*b*c**2*x**6/2 + B*c**3*x**8/8, Eq(m, -7)),
(A*b**3*m**4*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m +
 135135) + 48*A*b**3*m**3*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**
2 + 66009*m + 135135) + 854*A*b**3*m**2*x**7*x**m/(m**5 + 55*m**4 + 1190*m*
*3 + 12650*m**2 + 66009*m + 135135) + 6672*A*b**3*m*x**7*x**m/(m**5 + 55*m*
*4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 19305*A*b**3*x**7*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b**2*c*m**
4*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
138*A*b**2*c*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 2328*A*b**2*c*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 +
12650*m**2 + 66009*m + 135135) + 16998*A*b**2*c*m*x**9*x**m/(m**5 + 55*m**4
```

```
 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 45045*A*b**2*c*x**9*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b*c**2*m**
4*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 132*A*b*c**2*m**3*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 2118*A*b*c**2*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3
 + 12650*m**2 + 66009*m + 135135) + 14652*A*b*c**2*m*x**11*x**m/(m**5 + 55*
m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 36855*A*b*c**2*x**11*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + A*c**3*m*
*4*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
+ 42*A*c**3*m**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 644*A*c**3*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12
650*m**2 + 66009*m + 135135) + 4278*A*c**3*m*x**13*x**m/(m**5 + 55*m**4 + 1
190*m**3 + 12650*m**2 + 66009*m + 135135) + 10395*A*c**3*x**13*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*b**3*m**4*x**9*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 46*B*b**3
*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135
) + 776*B*b**3*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 5666*B*b**3*m*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 126
50*m**2 + 66009*m + 135135) + 15015*B*b**3*x**9*x**m/(m**5 + 55*m**4 + 1190
*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b**2*c*m**4*x**11*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 132*B*b**2*c*m**3*x
**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 21
18*B*b**2*c*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 14652*B*b**2*c*m*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1
2650*m**2 + 66009*m + 135135) + 36855*B*b**2*c*x**11*x**m/(m**5 + 55*m**4 +
 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b*c**2*m**4*x**13*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 126*B*b*c**2*m
**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
 + 1932*B*b*c**2*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 +
 66009*m + 135135) + 12834*B*b*c**2*m*x**13*x**m/(m**5 + 55*m**4 + 1190*m**
3 + 12650*m**2 + 66009*m + 135135) + 31185*B*b*c**2*x**13*x**m/(m**5 + 55*m
**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*c**3*m**4*x**15*x**m/(
m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 40*B*c**3*m**
3*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 590*B*c**3*m**2*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 3800*B*c**3*m*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1265
0*m**2 + 66009*m + 135135) + 9009*B*c**3*x**15*x**m/(m**5 + 55*m**4 + 1190*
m**3 + 12650*m**2 + 66009*m + 135135), True))
```

3.270 $\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=71

$$\frac{Ab^2x^{m+5}}{m+5}+\frac{bx^{m+7}(2Ac+bB)}{m+7}+\frac{cx^{m+9}(Ac+2bB)}{m+9}+\frac{Bc^2x^{m+11}}{m+11}$$

```
[Out] A*b^2*x^(5+m)/(5+m)+b*(2*A*c+B*b)*x^(7+m)/(7+m)+c*(A*c+2*B*b)*x^(9+m)/(9+m)
+B*c^2*x^(11+m)/(11+m)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1584, 448}

$$\frac{Ab^2x^{m+5}}{m+5}+\frac{bx^{m+7}(2Ac+bB)}{m+7}+\frac{cx^{m+9}(Ac+2bB)}{m+9}+\frac{Bc^2x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^(5 + m))/(5 + m) + (b*(b*B + 2*A*c)*x^(7 + m))/(7 + m) + (c*(2*b*B
 + A*c)*x^(9 + m))/(9 + m) + (B*c^2*x^(11 + m))/(11 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx &= \int x^{4+m} \left(A+Bx^2\right) \left(b+cx^2\right)^2 \, dx\\
&= \int \left(Ab^2x^{4+m}+b(bB+2Ac)x^{6+m}+c(2bB+Ac)x^{8+m}+Bc^2x^{10+m}\right) dx\\
&= \frac{Ab^2x^{5+m}}{5+m}+\frac{b(bB+2Ac)x^{7+m}}{7+m}+\frac{c(2bB+Ac)x^{9+m}}{9+m}+\frac{Bc^2x^{11+m}}{11+m}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.93

$$x^{m+5}\left(\frac{Ab^2}{m+5}+\frac{cx^4(Ac+2bB)}{m+9}+\frac{bx^2(2Ac+bB)}{m+7}+\frac{Bc^2x^6}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(5 + m)*((A*b^2)/(5 + m) + (b*(b*B + 2*A*c)*x^2)/(7 + m) + (c*(2*b*B + A*
c)*x^4)/(9 + m) + (B*c^2*x^6)/(11 + m))
```

fricas [B] time = 0.97, size = 217, normalized size = 3.06

$$\frac{\left(\left(Bc^{2}m^{3}+21\,Bc^{2}m^{2}+143\,Bc^{2}m+315\,Bc^{2}\right)x^{11}+\left(\left(2\,Bbc+Ac^{2}\right)m^{3}+770\,Bbc+385\,Ac^{2}+23\left(2\,Bbc+Ac^{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((B*c^2*m^3 + 21*B*c^2*m^2 + 143*B*c^2*m + 315*B*c^2)*x^11 + ((2*B*b*c + A*
c^2)*m^3 + 770*B*b*c + 385*A*c^2 + 23*(2*B*b*c + A*c^2)*m^2 + 167*(2*B*b*c
+ A*c^2)*m)*x^9 + ((B*b^2 + 2*A*b*c)*m^3 + 495*B*b^2 + 990*A*b*c + 25*(B*b^
2 + 2*A*b*c)*m^2 + 199*(B*b^2 + 2*A*b*c)*m)*x^7 + (A*b^2*m^3 + 27*A*b^2*m^2
 + 239*A*b^2*m + 693*A*b^2)*x^5)*x^m/(m^4 + 32*m^3 + 374*m^2 + 1888*m + 346
5)
```

giac [B] time = 0.19, size = 340, normalized size = 4.79

$$\frac{Bc^{2}m^{3}x^{11}x^{m}+21\,Bc^{2}m^{2}x^{11}x^{m}+2\,Bbcm^{3}x^{9}x^{m}+Ac^{2}m^{3}x^{9}x^{m}+143\,Bc^{2}mx^{11}x^{m}+46\,Bbcm^{2}x^{9}x^{m}+23\,Ac^{2}m^{2}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] (B*c^2*m^3*x^11*x^m + 21*B*c^2*m^2*x^11*x^m + 2*B*b*c*m^3*x^9*x^m + A*c^2*m
^3*x^9*x^m + 143*B*c^2*m*x^11*x^m + 46*B*b*c*m^2*x^9*x^m + 23*A*c^2*m^2*x^9
*x^m + 315*B*c^2*x^11*x^m + B*b^2*m^3*x^7*x^m + 2*A*b*c*m^3*x^7*x^m + 334*B
*b*c*m*x^9*x^m + 167*A*c^2*m*x^9*x^m + 25*B*b^2*m^2*x^7*x^m + 50*A*b*c*m^2*
x^7*x^m + 770*B*b*c*x^9*x^m + 385*A*c^2*x^9*x^m + A*b^2*m^3*x^5*x^m + 199*B
*b^2*m*x^7*x^m + 398*A*b*c*m*x^7*x^m + 27*A*b^2*m^2*x^5*x^m + 495*B*b^2*x^7
*x^m + 990*A*b*c*x^7*x^m + 239*A*b^2*m*x^5*x^m + 693*A*b^2*x^5*x^m)/(m^4 +
32*m^3 + 374*m^2 + 1888*m + 3465)
```

maple [B] time = 0.05, size = 262, normalized size = 3.69

$$\frac{\left(B\,c^{2}m^{3}x^{6}+21B\,c^{2}m^{2}x^{6}+A\,c^{2}m^{3}x^{4}+2Bbc\,m^{3}x^{4}+143B\,c^{2}m\,x^{6}+23A\,c^{2}m^{2}x^{4}+46Bbc\,m^{2}x^{4}+315B\,c^{2}x^{6}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x)
```

```
[Out] x^(m+5)*(B*c^2*m^3*x^6+21*B*c^2*m^2*x^6+A*c^2*m^3*x^4+2*B*b*c*m^3*x^4+143*B
*c^2*m*x^6+23*A*c^2*m^2*x^4+46*B*b*c*m^2*x^4+315*B*c^2*x^6+2*A*b*c*m^3*x^2+
167*A*c^2*m*x^4+B*b^2*m^3*x^2+334*B*b*c*m*x^4+50*A*b*c*m^2*x^2+385*A*c^2*x^
4+25*B*b^2*m^2*x^2+770*B*b*c*x^4+A*b^2*m^3+398*A*b*c*m*x^2+199*B*b^2*m*x^2+
27*A*b^2*m^2+990*A*b*c*x^2+495*B*b^2*x^2+239*A*b^2*m+693*A*b^2)/(m+11)/(m+9
)/(m+7)/(m+5)
```

maxima [A] time = 1.37, size = 91, normalized size = 1.28

$$\frac{Bc^{2}x^{m+11}}{m+11}+\frac{2\,Bbcx^{m+9}}{m+9}+\frac{Ac^{2}x^{m+9}}{m+9}+\frac{Bb^{2}x^{m+7}}{m+7}+\frac{2\,Abcx^{m+7}}{m+7}+\frac{Ab^{2}x^{m+5}}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] B*c^2*x^(m + 11)/(m + 11) + 2*B*b*c*x^(m + 9)/(m + 9) + A*c^2*x^(m + 9)/(m
+ 9) + B*b^2*x^(m + 7)/(m + 7) + 2*A*b*c*x^(m + 7)/(m + 7) + A*b^2*x^(m + 5
)/(m + 5)
```

mupad [B] time = 0.34, size = 179, normalized size = 2.52

$$x^m\left(\frac{A\,b^2\,x^5\,\left(m^3+27\,m^2+239\,m+693\right)}{m^4+32\,m^3+374\,m^2+1888\,m+3465}+\frac{B\,c^2\,x^{11}\,\left(m^3+21\,m^2+143\,m+315\right)}{m^4+32\,m^3+374\,m^2+1888\,m+3465}+\frac{b\,x^7\,\left(2\,A\,c+B\,b\right)\,\left(m^3\right.}{m^4+32\,m^3+374\,m^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^m*((A*b^2*x^5*(239*m + 27*m^2 + m^3 + 693))/(1888*m + 374*m^2 + 32*m^3 +
m^4 + 3465) + (B*c^2*x^11*(143*m + 21*m^2 + m^3 + 315))/(1888*m + 374*m^2 +
 32*m^3 + m^4 + 3465) + (b*x^7*(2*A*c + B*b)*(199*m + 25*m^2 + m^3 + 495))/
(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465) + (c*x^9*(A*c + 2*B*b)*(167*m + 23
*m^2 + m^3 + 385))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))
```

sympy [A] time = 4.42, size = 1051, normalized size = 14.80

$$\begin{cases} -\frac{Ab^2}{6x^6}-\frac{Abc}{2x^4}-\frac{Ac^2}{2x^2}-\frac{Bb^2}{4x^4}-\frac{Bbc}{x^2}+Bc^2\log(x) \\ -\frac{Ab^2}{4x^4}-\frac{Abc}{x^2}+Ac^2\log(x)-\frac{Bb^2}{2x^2}+2Bbc\log(x)+\frac{Bc^2x^2}{2} \\ -\frac{Ab^2}{2x^2}+2Abc\log(x)+\frac{Ac^2x^2}{2}+Bb^2\log(x)+Bbcx^2+\frac{Bc^2x^4}{4} \\ Ab^2\log(x)+Abcx^2+\frac{Ac^2x^4}{4}+\frac{Bb^2x^2}{2}+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6} \\ \frac{Ab^2m^3x^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{27Ab^2m^2x^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{239Ab^2mx^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{693Ab^2x^5x^m}{m^4+32m^3+374m^2+1888m+3465}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise((-A*b**2/(6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*b**2/(4*
x**4) - B*b*c/x**2 + B*c**2*log(x), Eq(m, -11)), (-A*b**2/(4*x**4) - A*b*c/
x**2 + A*c**2*log(x) - B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2, Eq
(m, -9)), (-A*b**2/(2*x**2) + 2*A*b*c*log(x) + A*c**2*x**2/2 + B*b**2*log(x
) + B*b*c*x**2 + B*c**2*x**4/4, Eq(m, -7)), (A*b**2*log(x) + A*b*c*x**2 + A
*c**2*x**4/4 + B*b**2*x**2/2 + B*b*c*x**4/2 + B*c**2*x**6/6, Eq(m, -5)), (A
*b**2*m**3*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 27*A*b**
2*m**2*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 239*A*b**2*m
*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 693*A*b**2*x**5*x*
*m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*A*b*c*m**3*x**7*x**m/(m*
*4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 50*A*b*c*m**2*x**7*x**m/(m**4 +
32*m**3 + 374*m**2 + 1888*m + 3465) + 398*A*b*c*m*x**7*x**m/(m**4 + 32*m**3
 + 374*m**2 + 1888*m + 3465) + 990*A*b*c*x**7*x**m/(m**4 + 32*m**3 + 374*m*
*2 + 1888*m + 3465) + A*c**2*m**3*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 18
88*m + 3465) + 23*A*c**2*m**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 167*A*c**2*m*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 346
5) + 385*A*c**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + B*b
**2*m**3*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 25*B*b**2*
m**2*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 199*B*b**2*m*x
**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 495*B*b**2*x**7*x**m
/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*B*b*c*m**3*x**9*x**m/(m**4
 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 46*B*b*c*m**2*x**9*x**m/(m**4 + 32
*m**3 + 374*m**2 + 1888*m + 3465) + 334*B*b*c*m*x**9*x**m/(m**4 + 32*m**3 +
 374*m**2 + 1888*m + 3465) + 770*B*b*c*x**9*x**m/(m**4 + 32*m**3 + 374*m**2
 + 1888*m + 3465) + B*c**2*m**3*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 188
8*m + 3465) + 21*B*c**2*m**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 143*B*c**2*m*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 34
65) + 315*B*c**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465), Tr
ue))
```

3.271 $\int x^m (A+Bx^2)(bx^2+cx^4)\,dx$

Optimal. Leaf size=45

$$\frac{x^{m+5}(Ac+bB)}{m+5}+\frac{Abx^{m+3}}{m+3}+\frac{Bcx^{m+7}}{m+7}$$

```
[Out] A*b*x^(3+m)/(3+m)+(A*c+B*b)*x^(5+m)/(5+m)+B*c*x^(7+m)/(7+m)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 448}

$$\frac{x^{m+5}(Ac+bB)}{m+5}+\frac{Abx^{m+3}}{m+3}+\frac{Bcx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^(3 + m))/(3 + m) + ((b*B + A*c)*x^(5 + m))/(5 + m) + (B*c*x^(7 + m))
/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^m (A+Bx^2)(bx^2+cx^4)\,dx &= \int x^{2+m}(A+Bx^2)(b+cx^2)\,dx\\ &= \int \left(Abx^{2+m}+(bB+Ac)x^{4+m}+Bcx^{6+m}\right)dx\\ &= \frac{Abx^{3+m}}{3+m}+\frac{(bB+Ac)x^{5+m}}{5+m}+\frac{Bcx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.93

$$x^{m+3}\left(\frac{x^2(Ac+bB)}{m+5}+\frac{Ab}{m+3}+\frac{Bcx^4}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] x^(3 + m)*((A*b)/(3 + m) + ((b*B + A*c)*x^2)/(5 + m) + (B*c*x^4)/(7 + m))
```

fricas [B] time = 1.28, size = 94, normalized size = 2.09

$$\frac{\left(\left(Bcm^2+8\,Bcm+15\,Bc\right)x^7+\left((Bb+Ac)m^2+21\,Bb+21\,Ac+10\,(Bb+Ac)m\right)x^5+\left(Abm^2+12\,Abm+35\,A\right.}{m^3+15\,m^2+71\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] ((B*c*m^2 + 8*B*c*m + 15*B*c)*x^7 + ((B*b + A*c)*m^2 + 21*B*b + 21*A*c + 10
*(B*b + A*c)*m)*x^5 + (A*b*m^2 + 12*A*b*m + 35*A*b)*x^3)*x^m/(m^3 + 15*m^2
+ 71*m + 105)
```

giac [B] time = 0.16, size = 149, normalized size = 3.31

$$\frac{Bcm^2x^7x^m + 8\,Bcmx^7x^m + Bbm^2x^5x^m + Acm^2x^5x^m + 15\,Bcx^7x^m + 10\,Bbmx^5x^m + 10\,Acmx^5x^m + Abm^2x^3x^m + 2}{m^3 + 15\,m^2 + 71\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*c*m^2*x^7*x^m + 8*B*c*m*x^7*x^m + B*b*m^2*x^5*x^m + A*c*m^2*x^5*x^m + 15
*B*c*x^7*x^m + 10*B*b*m*x^5*x^m + 10*A*c*m*x^5*x^m + A*b*m^2*x^3*x^m + 21*B
*b*x^5*x^m + 21*A*c*x^5*x^m + 12*A*b*m*x^3*x^m + 35*A*b*x^3*x^m)/(m^3 + 15*
m^2 + 71*m + 105)
```

maple [B] time = 0.05, size = 110, normalized size = 2.44

$$\frac{\left(Bc\,m^2x^4 + 8Bcm\,x^4 + Ac\,m^2x^2 + Bb\,m^2x^2 + 15Bc\,x^4 + 10Acm\,x^2 + 10Bbm\,x^2 + Ab\,m^2 + 21Ac\,x^2 + 21Bb\,x^2 + 1}{(m+7)\,(m+5)\,(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2),x)
```

```
[Out] x^(m+3)*(B*c*m^2*x^4+8*B*c*m*x^4+A*c*m^2*x^2+B*b*m^2*x^2+15*B*c*x^4+10*A*c*
m*x^2+10*B*b*m*x^2+A*b*m^2+21*A*c*x^2+21*B*b*x^2+12*A*b*m+35*A*b)/(m+7)/(m+
5)/(m+3)
```

maxima [A] time = 1.30, size = 53, normalized size = 1.18

$$\frac{Bcx^{m+7}}{m+7} + \frac{Bbx^{m+5}}{m+5} + \frac{Acx^{m+5}}{m+5} + \frac{Abx^{m+3}}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] B*c*x^(m + 7)/(m + 7) + B*b*x^(m + 5)/(m + 5) + A*c*x^(m + 5)/(m + 5) + A*b
*x^(m + 3)/(m + 3)
```

mupad [B] time = 0.25, size = 97, normalized size = 2.16

$$x^m\,\left(\frac{x^5\,\left(A\,c+B\,b\right)\,\left(m^2+10\,m+21\right)}{m^3+15\,m^2+71\,m+105}+\frac{A\,b\,x^3\,\left(m^2+12\,m+35\right)}{m^3+15\,m^2+71\,m+105}+\frac{B\,c\,x^7\,\left(m^2+8\,m+15\right)}{m^3+15\,m^2+71\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] x^m*((x^5*(A*c + B*b)*(10*m + m^2 + 21))/(71*m + 15*m^2 + m^3 + 105) + (A*b
*x^3*(12*m + m^2 + 35))/(71*m + 15*m^2 + m^3 + 105) + (B*c*x^7*(8*m + m^2 +
 15))/(71*m + 15*m^2 + m^3 + 105))
```

sympy [A] time = 1.62, size = 415, normalized size = 9.22

$$\begin{cases} -\frac{Ab}{4x^4} - \frac{Ac}{2x^2} - \frac{Bb}{2x^2} + Bc\log(x) \\ -\frac{Ab}{2x^2} + Ac\log(x) + Bb\log(x) + \frac{Bcx^2}{2} \\ Ab\log(x) + \frac{Acx^2}{2} + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} \\ \frac{Abm^2x^3x^m}{m^3+15m^2+71m+105} + \frac{12Abmx^3x^m}{m^3+15m^2+71m+105} + \frac{35Abx^3x^m}{m^3+15m^2+71m+105} + \frac{Acm^2x^5x^m}{m^3+15m^2+71m+105} + \frac{10Acmx^5x^m}{m^3+15m^2+71m+105} + \frac{21Acx^5x^m}{m^3+15m^2+71m\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((-A*b/(4*x**4) - A*c/(2*x**2) - B*b/(2*x**2) + B*c*log(x), Eq(m,
-7)), (-A*b/(2*x**2) + A*c*log(x) + B*b*log(x) + B*c*x**2/2, Eq(m, -5)), (A
*b*log(x) + A*c*x**2/2 + B*b*x**2/2 + B*c*x**4/4, Eq(m, -3)), (A*b*m**2*x**
3*x**m/(m**3 + 15*m**2 + 71*m + 105) + 12*A*b*m*x**3*x**m/(m**3 + 15*m**2 +
 71*m + 105) + 35*A*b*x**3*x**m/(m**3 + 15*m**2 + 71*m + 105) + A*c*m**2*x*
*5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*A*c*m*x**5*x**m/(m**3 + 15*m**2
+ 71*m + 105) + 21*A*c*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*b*m**2*x
**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*B*b*m*x**5*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 21*B*b*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*c*m**2*
x**7*x**m/(m**3 + 15*m**2 + 71*m + 105) + 8*B*c*m*x**7*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 15*B*c*x**7*x**m/(m**3 + 15*m**2 + 71*m + 105), True))
```

3.272 $\int \frac{x^m(A+Bx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=71

$$\frac{x^{m-1}(bB-Ac)\,_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}-\frac{Bx^{m-1}}{c(1-m)}$$

```
[Out] -B*x^(-1+m)/c/(1-m)+(-A*c+B*b)*x^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2*
m],-c*x^2/b)/b/c/(1-m)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 459, 364}

$$\frac{x^{m-1}(bB-Ac)\,_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}-\frac{Bx^{m-1}}{c(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((B*x^(-1 + m))/(c*(1 - m))) + ((b*B - A*c)*x^(-1 + m)*Hypergeometric2F1[1
, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*c*(1 - m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m\left(A+Bx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{x^{-2+m}\left(A+Bx^2\right)}{b+cx^2}\,dx\\
&= -\frac{Bx^{-1+m}}{c(1-m)}-\frac{(bB(-1+m)-Ac(-1+m))\int \frac{x^{-2+m}}{b+cx^2}\,dx}{c(-1+m)}\\
&= -\frac{Bx^{-1+m}}{c(1-m)}+\frac{(bB-Ac)x^{-1+m}\,_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.77

$$\frac{x^{m-1}\left((Ac-bB)\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)+bB\right)}{bc(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (x^(-1 + m)*(b*B + (-(b*B) + A*c)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/
2, -((c*x^2)/b)]))/(b*c*(-1 + m))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^2+A\right)x^m}{cx^4+bx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^2+A\right)x^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int\frac{\left(B\,x^2+A\right)x^m}{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
```

```
[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^2+A\right)x^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^m\,\left(B\,x^2+A\right)}{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(A + Bx^2\right)}{x^2 \left(b + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2),x)
[Out] Integral(x**m*(A + B*x**2)/(x**2*(b + c*x**2)), x)
```

3.273 $\int \frac{x^m\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{x^{m-3}(bB(3-m)-Ac(5-m))\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{2b^2c(3-m)}-\frac{x^{m-3}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

```
[Out] -1/2*(-A*c+B*b)*x^(-3+m)/b/c/(c*x^2+b)+1/2*(b*B*(3-m)-A*c*(5-m))*x^(-3+m)*h
ypergeom([1, -3/2+1/2*m],[-1/2+1/2*m],-c*x^2/b)/b^2/c/(3-m)
```

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1584, 457, 364}

$$\frac{x^{m-3}\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{2b^2}-\frac{x^{m-3}(bB-Ac)}{2bc\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - A*c)*x^(-3 + m))/(2*b*c*(b + c*x^2)) + (((b*B)/c - (A*(5 - m))/(3
- m))*x^(-3 + m)*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)]
)/(2*b^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^m (A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{-4+m}(A+Bx^2)}{(b+cx^2)^2}\,dx$$
$$= -\frac{(bB-Ac)x^{-3+m}}{2bc(b+cx^2)} + \frac{(-Ac(-5+m)+bB(-3+m))\int \frac{x^{-4+m}}{b+cx^2}\,dx}{2bc}$$
$$= -\frac{(bB-Ac)x^{-3+m}}{2bc(b+cx^2)} + \frac{\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)x^{-3+m}\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{2b^2}$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.82

$$\frac{x^{m-3}\left((Ac-bB)\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)+bB\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)\right)}{b^2c(m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(-3 + m)*(b*B*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)]
 + (-(b*B) + A*c)*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)
]))/(b^2*c*(-3 + m))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bx^2+A)x^m}{c^2x^8+2\,bcx^6+b^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2+A)x^m}{(cx^4+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(B\,x^2+A)\,x^m}{(c\,x^4+b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2 + A\right)x^m}{\left(cx^4 + bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(A + Bx^2\right)}{x^4\left(b + cx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Integral(x**m*(A + B*x**2)/(x**4*(b + c*x**2)**2), x)
```

3.274 $\int x^m \left(A+Bx^2\right)\left(bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=140

$$\frac{Bx^{m-1}\left(bx^2+cx^4\right)^{p+1}}{c(m+4p+3)}-\frac{x^{m+1}\left(\frac{cx^2}{b}+1\right)^{-p}\left(bx^2+cx^4\right)^p\left(bB(m+2p+1)-Ac(m+4p+3)\right){}_2F_1\left(-p,\frac{1}{2}(m+2p+1);\right.}{c(m+2p+1)(m+4p+3)}$$

```
[Out] B*x^(-1+m)*(c*x^4+b*x^2)^(1+p)/c/(3+m+4*p)-(b*B*(1+m+2*p)-A*c*(3+m+4*p))*x^
(1+m)*(c*x^4+b*x^2)^p*hypergeom([-p, 1/2+1/2*m+p],[3/2+1/2*m+p],-c*x^2/b)/c
/(1+m+2*p)/(3+m+4*p)/((1+c*x^2/b)^p)
```

Rubi [A] time = 0.14, antiderivative size = 126, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2039, 2032, 365, 364}

$$x^{m+1}\left(\frac{cx^2}{b}+1\right)^{-p}\left(bx^2+cx^4\right)^p\left(\frac{A}{m+2p+1}-\frac{bB}{c(m+4p+3)}\right){}_2F_1\left(-p,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\frac{cx^2}{b}\right)+\frac{Bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (B*x^(-1 + m)*(b*x^2 + c*x^4)^(1 + p))/(c*(3 + m + 4*p)) + ((A/(1 + m + 2*p
) - (b*B)/(c*(3 + m + 4*p)))*x^(1 + m)*(b*x^2 + c*x^4)^p*Hypergeometric2F1[
-p, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)])/(1 + (c*x^2)/b)^p
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)\int x^m\left(bx^2+cx^4\right)^p\,dx$$
$$=\frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(b+cx^2\right)^{-p}\left(bx^2+\right.\right.$$
$$=\frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx^2\right.\right.$$
$$=\frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}+\left(\frac{A}{1+m+2p}-\frac{bB}{c(3+m+4p)}\right)x^{1+m}\left(1+\frac{cx^2}{b}\right)^{-p}$$

Mathematica [A] time = 0.12, size = 135, normalized size = 0.96

$$\frac{x^{m+1}\left(x^2\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(A(m+2p+3)\,{}_2F_1\left(-p,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\frac{cx^2}{b}\right)+Bx^2(m+2p+\right.}{(m+2p+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^(1 + m)*(x^2*(b + c*x^2))^p*(A*(3 + m + 2*p)*Hypergeometric2F1[-p, (1 +
m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)] + B*(1 + m + 2*p)*x^2*Hypergeome
tric2F1[-p, (3 + m + 2*p)/2, (5 + m + 2*p)/2, -((c*x^2)/b)]))/((1 + m + 2*p
)*(3 + m + 2*p)*(1 + (c*x^2)/b)^p)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bx^2+A\right)\left(cx^4+bx^2\right)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^2+A\right)\left(cx^4+bx^2\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int\left(B\,x^2+A\right)x^m\left(c\,x^4+b\,x^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

```
[Out] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^2 + A\right)\left(cx^4 + bx^2\right)^p x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(B\,x^2 + A\right)\left(c\,x^4 + b\,x^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(x^2\left(b + cx^2\right)\right)^p \left(A + Bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**p,x)
```

```
[Out] Integral(x**m*(x**2*(b + c*x**2))**p*(A + B*x**2), x)
```

3.275 $\int x^{-1+n-jp}\left(c+dx^n\right)\left(ax^j+bx^{j+n}\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}-\frac{x^{-j(p+1)}(ad-bc(p+2))\left(ax^j+bx^{j+n}\right)^{p+1}}{b^2n(p+1)(p+2)}$$

```
[Out] -(a*d-b*c*(2+p))*(a*x^j+b*x^(j+n))^(1+p)/b^2/n/(p^2+3*p+2)/(x^(j*(1+p)))+d*
x^(n-j*(1+p))*(a*x^j+b*x^(j+n))^(1+p)/b/n/(2+p)
```

Rubi [A] time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2039, 2014}

$$\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}-\frac{x^{-j(p+1)}(ad-bc(p+2))\left(ax^j+bx^{j+n}\right)^{p+1}}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] -(((a*d - b*c*(2 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b^2*n*(1 + p)*(2 + p
)*x^(j*(1 + p)))) + (d*x^(n - j*(1 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b*
n*(2 + p))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int x^{-1+n-jp}\left(c+dx^n\right)\left(ax^j+bx^{j+n}\right)^p\,dx &= \frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}-\left(-c+\frac{ad}{b(2+p)}\right)\int x^{-1+n-jp}\left(ax^j+bx^{j+n}\right.\\ &= \frac{\left(c-\frac{ad}{b(2+p)}\right)x^{-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(1+p)}+\frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.66

$$\frac{x^{-jp}\left(a+bx^n\right)\left(x^j\left(a+bx^n\right)\right)^p\left(-ad+bc(p+2)+bd(p+1)x^n\right)}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] ((a + b*x^n)*(x^j*(a + b*x^n))^p*(-(a*d) + b*c*(2 + p) + b*d*(1 + p)*x^n))/
(b^2*n*(1 + p)*(2 + p)*x^(j*p))
```

fricas [A] time = 0.92, size = 140, normalized size = 1.47

$$\frac{\left(\left(b^{2} d p + b^{2} d\right) x x^{-j p + n - 1} x^{2 n} + \left(2 b^{2} c + \left(b^{2} c + a b d\right) p\right) x x^{-j p + n - 1} x^{n} + \left(a b c p + 2 a b c - a^{2} d\right) x x^{-j p + n - 1}\right) \left(\frac{\left(b x^{n} + a\right) x^{j + n}}{x^{n}}\right)^{p}}{\left(b^{2} n p^{2} + 3 b^{2} n p + 2 b^{2} n\right) x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

```
[Out] ((b^2*d*p + b^2*d)*x*x^(-j*p + n - 1)*x^(2*n) + (2*b^2*c + (b^2*c + a*b*d)*
p)*x*x^(-j*p + n - 1)*x^n + (a*b*c*p + 2*a*b*c - a^2*d)*x*x^(-j*p + n - 1))
*((b*x^n + a)*x^(j + n)/x^n)^p/((b^2*n*p^2 + 3*b^2*n*p + 2*b^2*n)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d x^{n} + c\right) \left(b x^{j + n} + a x^{j}\right)^{p} x^{-j p + n - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)*(b*x^(j + n) + a*x^j)^p*x^(-j*p + n - 1), x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \left(d\,x^{n}+c\right) x^{-j p +n -1} \left(a\,x^{j}+b\,x^{j +n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-j*p+n-1)*(d*x^n+c)*(a*x^j+b*x^(j+n))^p,x)
```

```
[Out] int(x^(-j*p+n-1)*(d*x^n+c)*(a*x^j+b*x^(j+n))^p,x)
```

maxima [A] time = 2.04, size = 112, normalized size = 1.18

$$\frac{{\left(b x^{n} + a\right)} c e^{\left(-j p \log\left(x\right) + p \log\left(b x^{n} + a\right) + p \log\left(x^{j}\right)\right)}}{b n {\left(p + 1\right)}} + \frac{{\left(b^{2} {\left(p + 1\right)} x^{2 \, n} + a b p x^{n} - a^{2}\right)} d e^{\left(-j p \log\left(x\right) + p \log\left(b x^{n} + a\right) + p \log\left(x^{j}\right)\right)}}{{\left(p^{2} + 3 \, p + 2\right)} b^{2} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
```

```
[Out] (b*x^n + a)*c*e^(-j*p*log(x) + p*log(b*x^n + a) + p*log(x^j))/(b*n*(p + 1))
 + (b^2*(p + 1)*x^(2*n) + a*b*p*x^n - a^2)*d*e^(-j*p*log(x) + p*log(b*x^n +
 a) + p*log(x^j))/((p^2 + 3*p + 2)*b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{n-j\,p-1}\,{\left(a\,x^j+b\,x^{j+n}\right)}^p\,\left(c+d\,x^n\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n),x)
```

```
[Out] int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-j*p+n-1)*(c+d*x**n)*(a*x**j+b*x**(j+n))**p,x)
[Out] Timed out
```

3.276 $\int (ex)^m (c + dx^n)^q \left(ax^j + bx^{j+n}\right)^p dx$

Optimal. Leaf size=113

$$\frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{m+jp+1}{n};-p,-q;\frac{m+n+jp+1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

```
[Out] x*(e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p*AppellF1((j*p+m+1)/n,-p,-q,(j*p+m
+n+1)/n,-b*x^n/a,-d*x^n/c)/(j*p+m+1)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Rubi [A] time = 0.22, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2042, 511, 510}

$$\frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{m+jp+1}{n};-p,-q;\frac{m+n+jp+1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] (x*(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p*AppellF1[(1 + m + j*p)/n,
-p, -q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1
 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p dx = \left(x^{-m-jp}(ex)^m (a+bx^n)^{-p} \left(ax^j+bx^{j+n}\right)^p\right) \int x^{m+jp} (a+bx^n)^p (c+dx^n)^q$$
$$= \left(x^{-m-jp}(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} \left(ax^j+bx^{j+n}\right)^p\right) \int x^{m+jp} \left(1+\frac{bx^n}{a}\right)^p (c+dx^n)$$
$$= \left(x^{-m-jp}(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(ax^j+bx^{j+n}\right)^p\right) \int x^{m-}$$
$$= \frac{x(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{1+m+jp}{n};-p,\right.}{1+m+jp}$$

Mathematica [A] time = 0.18, size = 111, normalized size = 0.98

$$\frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(x^j (a+bx^n)\right)^p F_1\left(\frac{m+jp+1}{n};-p,-q;\frac{m+n+jp+1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] (x*(e*x)^m*(x^j*(a + b*x^n))^p*(c + d*x^n)^q*AppellF1[(1 + m + j*p)/n, -p,
-q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1 + (
b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^{j+n}+ax^j\right)^p (dx^n+c)^q (ex)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{j+n}+ax^j\right)^p (dx^n+c)^q (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

maple [F] time = 2.31, size = 0, normalized size = 0.00

$$\int (ex)^m (d\,x^n+c)^q \left(a\,x^j+b\,x^{j+n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(d*x^n+c)^q*(a*x^j+b*x^(j+n))^p,x)
```

```
[Out] int((e*x)^m*(d*x^n+c)^q*(a*x^j+b*x^(j+n))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{j+n}+ax^j\right)^p (dx^n+c)^q (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,x^j+b\,x^{j+n}\right)^p\,{\left(e\,x\right)}^m\,{\left(c+d\,x^n\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q,x)
```

```
[Out] int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**p,x)
```

```
[Out] Timed out
```

3.277 $\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} \, dx$

Optimal. Leaf size=129

$$\frac{12ae(ex)^{3/4}x^{j+2}\left(ax^j+bx^{j+n}\right)^{2/3}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}F_1\left(\frac{20j+33}{12n};-\frac{5}{3},-q;\frac{20j+12n+33}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(20j+33)\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

```
[Out] 12*a*e*x^(2+j)*(e*x)^(3/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(2/3)*AppellF1(1/1
2*(33+20*j)/n,-5/3,-q,1+(11/4+5/3*j)/n,-b*x^n/a,-d*x^n/c)/(33+20*j)/(1+b*x^
n/a)^(2/3)/((1+d*x^n/c)^q)
```

Rubi [A] time = 0.36, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2042, 511, 510}

$$\frac{12ae(ex)^{3/4}x^{j+2}\left(ax^j+bx^{j+n}\right)^{2/3}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}F_1\left(\frac{20j+33}{12n};-\frac{5}{3},-q;\frac{20j+12n+33}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(20j+33)\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

```
[Out] (12*a*e*x^(2 + j)*(e*x)^(3/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(2/3)*App
ellF1[(33 + 20*j)/(12*n), -5/3, -q, (33 + 20*j + 12*n)/(12*n), -((b*x^n)/a)
, -((d*x^n)/c)])/((33 + 20*j)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \frac{\left(ex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}(a+bx^n)^{5/3}(c+dx^n)^q\,dx}{(a+bx^n)^{2/3}}$$

$$= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(1+\frac{bx^n}{a}\right)^{5/3}(c+dx^n)^q\,dx}{\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

$$= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(1+\frac{bx^n}{a}\right.}{\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

$$= \frac{12aex^{2+j}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}F_1\left(\frac{33+20j}{12n};-\frac{5}{3},-q;\right.}{(33+20j)\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

Mathematica [A] time = 0.25, size = 210, normalized size = 1.63

$$\frac{12(ex)^{7/4}x^{j+1}\left(x^j(a+bx^n)\right)^{2/3}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\left(b(20j+33)x^nF_1\left(\frac{20j+12n+33}{12n};-\frac{2}{3},-q;\frac{20j+24n+33}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+\right.}{(20j+33)(20j+12n+33)\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

```
[Out] (12*x^(1 + j)*(e*x)^(7/4)*(x^j*(a + b*x^n))^(2/3)*(c + d*x^n)^q*(a*(33 + 20
*j + 12*n)*AppellF1[(33 + 20*j)/(12*n), -2/3, -q, (11/4 + (5*j)/3 + n)/n, -
((b*x^n)/a), -((d*x^n)/c)] + b*(33 + 20*j)*x^n*AppellF1[(33 + 20*j + 12*n)/
(12*n), -2/3, -q, (33 + 20*j + 24*n)/(12*n), -((b*x^n)/a), -((d*x^n)/c)]))/
((33 + 20*j)*(33 + 20*j + 12*n)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bexx^{j+n}+aexx^j\right)\left(bx^{j+n}+ax^j\right)^{\frac{2}{3}}(ex)^{\frac{3}{4}}(dx^n+c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="fri
cas")
```

```
[Out] integral((b*e*x*x^(j + n) + a*e*x*x^j)*(b*x^(j + n) + a*x^j)^(2/3)*(e*x)^(3
/4)*(d*x^n + c)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}}(ex)^{\frac{7}{4}}(dx^n+c)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{7}{4}} \left(a\,x^{j}+b\,x^{j+n}\right)^{\frac{5}{3}} (d\,x^{n}+c)^{q}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/4)*(d*x^n+c)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

```
[Out] int((e*x)^(7/4)*(d*x^n+c)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{j+n}+ax^{j}\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^{n}+c)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,x^{j}+b\,x^{j+n}\right)^{5/3} (e\,x)^{7/4} (c+d\,x^{n})^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q,x)
```

```
[Out] int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/4)*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**(5/3),x)
```

```
[Out] Timed out
```

3.278 $\int \frac{4+3x^4}{5x+2x^5}\,dx$

Optimal. Leaf size=19

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

```
[Out] 4/5*ln(x)+7/40*ln(2*x^4+5)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 446, 72}

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

```
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+3x^4}{5x+2x^5}\,dx &= \int \frac{4+3x^4}{x\left(5+2x^4\right)}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{4+3x}{x(5+2x)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{4}{5x}+\frac{7}{5(5+2x)}\right)dx,x,x^4\right)\\
&= \frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^4)/(5*x + 2*x^5),x]
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

fricas [A] time = 0.86, size = 15, normalized size = 0.79

$$\frac{7}{40} \log\left(2 x^4 + 5\right) + \frac{4}{5} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="fricas")
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

giac [A] time = 0.16, size = 17, normalized size = 0.89

$$\frac{7}{40} \log\left(2 x^4 + 5\right) + \frac{1}{5} \log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="giac")
[Out] 7/40*log(2*x^4 + 5) + 1/5*log(x^4)
```

maple [A] time = 0.05, size = 16, normalized size = 0.84

$$\frac{4 \ln(x)}{5} + \frac{7 \ln\left(2x^4 + 5\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+4)/(2*x^5+5*x),x)
[Out] 4/5*ln(x)+7/40*ln(2*x^4+5)
```

maxima [A] time = 2.91, size = 15, normalized size = 0.79

$$\frac{7}{40} \log\left(2 x^4 + 5\right) + \frac{4}{5} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="maxima")
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

mupad [B] time = 0.17, size = 13, normalized size = 0.68

$$\frac{7 \ln\left(x^4 + \frac{5}{2}\right)}{40} + \frac{4 \ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 4)/(5*x + 2*x^5),x)
[Out] (7*log(x^4 + 5/2))/40 + (4*log(x))/5
```

sympy [A] time = 0.11, size = 17, normalized size = 0.89

$$\frac{4 \log(x)}{5} + \frac{7 \log\left(2x^4 + 5\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/(2*x**5+5*x),x)
[Out] 4*log(x)/5 + 7*log(2*x**4 + 5)/40
```

3.279 $\int \frac{1+x^6}{x-x^7} \, dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{3} \log\left(1 - x^6\right)$$

[Out] ln(x)-1/3*ln(-x^6+1)

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 446, 72}

$$\log(x) - \frac{1}{3} \log\left(1 - x^6\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^6)/(x - x^7),x]

[Out] Log[x] - Log[1 - x^6]/3

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^6}{x-x^7} \, dx &= \int \frac{1+x^6}{x\left(1-x^6\right)} \, dx \\
&= \frac{1}{6} \text{Subst}\left(\int \frac{1+x}{(1-x)x} \, dx, x, x^6\right) \\
&= \frac{1}{6} \text{Subst}\left(\int \left(-\frac{2}{-1+x} + \frac{1}{x}\right) \, dx, x, x^6\right) \\
&= \log(x) - \frac{1}{3} \log\left(1-x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{3} \log\left(1 - x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x - x^7),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

fricas [A] time = 0.93, size = 11, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^6-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^6 - 1) + log(x)
```

giac [A] time = 0.15, size = 16, normalized size = 1.07

$$\frac{1}{6}\log\left(x^6\right)-\frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="giac")
```

```
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

maple [B] time = 0.05, size = 36, normalized size = 2.40

$$\ln(x)-\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}-\frac{\ln\left(x^2-x+1\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(-x^7+x),x)
```

```
[Out] -1/3*ln(x-1)-1/3*ln(x^2+x+1)-1/3*ln(x+1)+ln(x)-1/3*ln(x^2-x+1)
```

maxima [B] time = 3.01, size = 35, normalized size = 2.33

$$-\frac{1}{3}\log\left(x^2+x+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^2 + x + 1) - 1/3*log(x^2 - x + 1) - 1/3*log(x + 1) - 1/3*log(x -
 1) + log(x)
```

mupad [B] time = 0.15, size = 11, normalized size = 0.73

$$\ln(x)-\frac{\ln\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x - x^7),x)
```

```
[Out] log(x) - log(x^6 - 1)/3
```

sympy [A] time = 0.12, size = 10, normalized size = 0.67

$$\log(x)-\frac{\log\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(-x**7+x),x)
```

```
[Out] log(x) - log(x**6 - 1)/3
```

3.280 $\int \frac{8+5x^{10}}{2x-x^{11}} \, dx$

Optimal. Leaf size=17

$$4\log(x) - \frac{9}{10}\log\left(2-x^{10}\right)$$

```
[Out] 4*ln(x)-9/10*ln(-x^10+2)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 446, 72}

$$4\log(x) - \frac{9}{10}\log\left(2-x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 5*x^10)/(2*x - x^11),x]
```

```
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{8+5x^{10}}{2x-x^{11}} \, dx &= \int \frac{8+5x^{10}}{x\left(2-x^{10}\right)} \, dx \\
&= \frac{1}{10}\operatorname{Subst}\left(\int \frac{8+5x}{(2-x)x} \, dx, x, x^{10}\right) \\
&= \frac{1}{10}\operatorname{Subst}\left(\int \left(-\frac{9}{-2+x}+\frac{4}{x}\right) dx, x, x^{10}\right) \\
&= 4\log(x) - \frac{9}{10}\log\left(2-x^{10}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$4\log(x) - \frac{9}{10}\log\left(2-x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 5*x^10)/(2*x - x^11),x]
```

```
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

fricas [A] time = 1.27, size = 13, normalized size = 0.76

$$-\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="fricas")
```

```
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

giac [A] time = 0.19, size = 16, normalized size = 0.94

$$\frac{2}{5}\,\log\left(x^{10}\right)-\frac{9}{10}\,\log\left(\left|x^{10}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="giac")
```

```
[Out] 2/5*log(x^10) - 9/10*log(abs(x^10 - 2))
```

maple [A] time = 0.05, size = 14, normalized size = 0.82

$$4\ln(x)-\frac{9\ln\left(x^{10}-2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^10+8)/(-x^11+2*x),x)
```

```
[Out] -9/10*ln(x^10-2)+4*ln(x)
```

maxima [A] time = 2.94, size = 13, normalized size = 0.76

$$-\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="maxima")
```

```
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

mupad [B] time = 0.14, size = 13, normalized size = 0.76

$$4\,\ln(x)-\frac{9\,\ln\left(x^{10}-2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^10 + 8)/(2*x - x^11),x)
```

```
[Out] 4*log(x) - (9*log(x^10 - 2))/10
```

sympy [A] time = 0.15, size = 14, normalized size = 0.82

$$4\log\left(x\right)-\frac{9\log\left(x^{10}-2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**10+8)/(-x**11+2*x),x)
```

```
[Out] 4*log(x) - 9*log(x**10 - 2)/10
```

3.281 $\int \frac{-3+2x}{-x^2+x^3} \, dx$

Optimal. Leaf size=16

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

[Out] -3/x-ln(1-x)+ln(x)

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 77}

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(-3 + 2*x)/(-x^2 + x^3),x]

[Out] -3/x - Log[1 - x] + Log[x]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+2x}{-x^2+x^3} \, dx &= \int \frac{-3+2x}{(-1+x)x^2} \, dx\\ &= \int \left(\frac{1}{1-x}+\frac{3}{x^2}+\frac{1}{x}\right) dx\\ &= -\frac{3}{x}-\log(1-x)+\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 2*x)/(-x^2 + x^3),x]

[Out] -3/x - Log[1 - x] + Log[x]

fricas [A] time = 0.69, size = 18, normalized size = 1.12

$$-\frac{x\log(x-1)-x\log(x)+3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] -(x*log(x - 1) - x*log(x) + 3)/x
```

giac [A] time = 0.18, size = 16, normalized size = 1.00

$$-\frac{3}{x} - \log\left(|x - 1|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="giac")
```

```
[Out] -3/x - log(abs(x - 1)) + log(abs(x))
```

maple [A] time = 0.05, size = 15, normalized size = 0.94

$$\ln(x) - \ln\left(x - 1\right) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-3)/(x^3-x^2),x)
```

```
[Out] -ln(x-1)+ln(x)-3/x
```

maxima [A] time = 1.62, size = 14, normalized size = 0.88

$$-\frac{3}{x} - \log\left(x - 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] -3/x - log(x - 1) + log(x)
```

mupad [B] time = 0.13, size = 14, normalized size = 0.88

$$2\,\mathrm{atanh}\left(2\,x - 1\right) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 3)/(x^2 - x^3),x)
```

```
[Out] 2*atanh(2*x - 1) - 3/x
```

sympy [A] time = 0.10, size = 10, normalized size = 0.62

$$\log\left(x\right) - \log\left(x - 1\right) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x**3-x**2),x)
```

```
[Out] log(x) - log(x - 1) - 3/x
```

3.282 $\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx$

Optimal. Leaf size=54

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}+\frac{ax}{c}$$

```
[Out] a*x/c+(-a*d+b*c)*x*hypergeom([1, 1/(m-n)],[1+1/(m-n)],-c*x^(m-n)/d)/c/d
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1593, 1584, 388, 245}

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*x*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1
), -((c*x^(m - n))/d)])/(c*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx &= \int \frac{x^n\left(b+ax^{m-n}\right)}{cx^m+dx^n}\,dx\\
&= \int \frac{b+ax^{m-n}}{d+cx^{m-n}}\,dx\\
&= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{d+cx^{m-n}}\,dx}{c}\\
&= \frac{ax}{c}+\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.96

$$
\frac{x\left((bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)+ad\right)}{cd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1),
-((c*x^(m - n))/d)]))/(c*d)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{ax^m+bx^n}{cx^m+dx^n},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="fricas")
[Out] integral((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="giac")
[Out] integrate((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$
\int \frac{a\,x^m+b\,x^n}{c\,x^m+d\,x^n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^n)/(c*x^m+d*x^n),x)
[Out] int((a*x^m+b*x^n)/(c*x^m+d*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$
-(bc-ad)\int \frac{x^m}{cdx^m+d^2x^n}\,dx+\frac{bx}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(x^m/(c*d*x^m + d^2*x^n), x) + b*x/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x^m+b\,x^n}{c\,x^m+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^n)/(c*x^m + d*x^n),x)
```

```
[Out] int((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**n)/(c*x**m+d*x**n),x)
```

```
[Out] Integral((a*x**m + b*x**n)/(c*x**m + d*x**n), x)
```

3.283 $\int x^m (a+bx^n)^p \left(a(1+m+q)x^q + b(1+m+n(1+p)\right.$

Optimal. Leaf size=18

$$x^{m+q+1}(a+bx^n)^{p+1}$$

```
[Out] x^(1+m+q)*(a+b*x^n)^(1+p)
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1584, 449}

$$x^{m+q+1}(a+bx^n)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*x^(n +
 q)),x]
```

```
[Out] x^(1 + m + q)*(a + b*x^n)^(1 + p)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^m (a+bx^n)^p \left(a(1+m+q)x^q + b(1+m+n(1+p)+q)x^{n+q}\right) dx = \int x^{m+q} (a+bx^n)^p \left(a(1+m+q) + b(1\right.$$
$$= x^{1+m+q}(a+bx^n)^{1+p}$$

Mathematica [C] time = 0.22, size = 116, normalized size = 6.44

$$x^{m+q+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(\frac{bx^n(m+np+n+q+1)\,{}_2F_1\left(-p,\frac{m+n+q+1}{n};\frac{m+2n+q+1}{n};-\frac{bx^n}{a}\right)}{m+n+q+1} + a\,{}_2F_1\left(-p,\frac{m+q}{n}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*
x^(n + q)),x]
```

```
[Out] (x^(1 + m + q)*(a + b*x^n)^p*(a*Hypergeometric2F1[-p, (1 + m + q)/n, (1 + m
 + n + q)/n, -((b*x^n)/a)] + (b*(1 + m + n + n*p + q)*x^n*Hypergeometric2F1
[-p, (1 + m + n + q)/n, (1 + m + 2*n + q)/n, -((b*x^n)/a)])/(1 + m + n + q)
))/(1 + (b*x^n)/a)^p
```

fricas [B] time = 0.79, size = 43, normalized size = 2.39

$$(bxx^m x^{n+q} + axx^m x^q)\left(\frac{bx^{n+q}+ax^q}{x^q}\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="fricas")
```

```
[Out] (b*x*x^m*x^(n + q) + a*x*x^m*x^q)*((b*x^(n + q) + a*x^q)/x^q)^p
```

giac [B] time = 0.37, size = 48, normalized size = 2.67

$$(bx^n + a)^p bxx^n e^{(m\log(x)+q\log(x))} + (bx^n + a)^p axe^{(m\log(x)+q\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="giac")
```

```
[Out] (b*x^n + a)^p*b*x*x^n*e^(m*log(x) + q*log(x)) + (b*x^n + a)^p*a*x*e^(m*log(
x) + q*log(x))
```

maple [F] time = 1.10, size = 0, normalized size = 0.00

$$\int \left(\left(m+q+1\right) a\, x^{q} + \left(m+\left(p+1\right) n+q+1\right) b\, x^{n+q}\right) x^{m} \left(b\, x^{n}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n+a)^p*(a*(1+m+q)*x^q+b*(1+m+(p+1)*n+q)*x^(n+q)),x)
```

```
[Out] int(x^m*(b*x^n+a)^p*(a*(1+m+q)*x^q+b*(1+m+(p+1)*n+q)*x^(n+q)),x)
```

maxima [B] time = 2.15, size = 37, normalized size = 2.06

$$\left(axx^m + bxe^{(m\log(x)+n\log(x))}\right)e^{(p\log(bx^n+a)+q\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="maxima")
```

```
[Out] (a*x*x^m + b*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + q*log(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int x^m\,\left(a\,x^q\,\left(m+q+1\right)+b\,x^{n+q}\,\left(m+q+n\,\left(p+1\right)+1\right)\right)\,{\left(a+b\,x^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x^
n)^p,x)
```

```
[Out] int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x^
n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**p*(a*(1+m+q)*x**q+b*(1+m+n*(1+p)+q)*x**(n+q)),x)
```

```
[Out] Timed out
```

3.284 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{x^m\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

```
[Out] (a+b/x)^n*x^m*AppellF1(-m,-n,1,1-m,-b/a/x,-c/d/x)/d/m/((1+b/a/x)^n)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {514, 497, 135, 133}

$$\frac{x^m\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^m)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*x^m*AppellF1[-m, -n, 1, 1 - m, -(b/(a*x)), -(c/(d*x))])/(d*m*(
1 + b/(a*x))^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 497

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[((a + b/x^n)^p*(c
+ d/x^n)^q)/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}, x
] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^{-1+m}}{d+\frac{c}{x}}\,dx$$
$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}\left(1+\frac{bx}{a}\right)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n} x^m F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x),x]
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x), x]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m\left(\frac{ax+b}{x}\right)^n}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="fricas")
[Out] integral(x^m*((a*x + b)/x)^n/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^m/(d*x+c),x)
```

```
[Out] int((a+b/x)^n*x^m/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x^{m}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m}\left(a+\frac{b}{x}\right)^{n}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b/x)^n)/(c + d*x),x)
```

```
[Out] int((x^m*(a + b/x)^n)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(a+\frac{b}{x}\right)^{n}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**m/(d*x+c),x)
```

```
[Out] Integral(x**m*(a + b/x)**n/(c + d*x), x)
```

3.285 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx$

Optimal. Leaf size=195

$$-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{2a^2d^2}+\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right){}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{2a^3d^3(n+1)}-\frac{c^3\left(a\right.}{}$$

```
[Out] -1/2*(2*a*c+b*d*(1-n))*(a+b/x)^(1+n)*x/a^2/d^2+1/2*(a+b/x)^(1+n)*x^2/a/d-c^
3*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^3/(a*c-b*d)
/(1+n)+1/2*(2*a^2*c^2-2*a*b*c*d*n-b^2*d^2*(1-n)*n)*(a+b/x)^(1+n)*hypergeom(
[1, 1+n],[2+n],1+b/a/x)/a^3/d^3/(1+n)
```

Rubi [A] time = 0.22, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {514, 446, 103, 151, 156, 65, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right){}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{2a^3d^3(n+1)}-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{2a^2d^2}-\frac{c^3\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] -((2*a*c + b*d*(1 - n))*(a + b/x)^(1 + n)*x)/(2*a^2*d^2) + ((a + b/x)^(1 +
n)*x^2)/(2*a*d) - (c^3*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c - b*d)*(1 + n)) + ((2*a^2*c^2 - 2*a*
b*c*d*n - b^2*d^2*(1 - n)*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, 1 + b/(a*x)])/(2*a^3*d^3*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{d+\frac{c}{x}}\,dx$$

$$= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^3(d+cx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(2ac+bd(1-n)+bc(1-n)x)}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right)}{2ad}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n\left(2a^2c^2-2abcdn-b^2d^2(1-n)n-\right.}{x(d+cx)}\right.}{2a^2d^2}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} + \frac{c^3\,\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^3} - \frac{\left(2a^2c^2\right.}{}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{c^3\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\right.}{ac-\right.}\right.}{d^3(ac-bd)(1+n)}$$

Mathematica [A] time = 0.17, size = 157, normalized size = 0.81

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left((ac-bd)\left(\left(2a^2c^2-2abcdn+b^2d^2(n-1)n\right) {}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)+ad(n+1)x(a(dx-\right.\right.\right.}{2a^3d^3(n+1)x(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(-2*a^3*c^3*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
 + b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x*(b*d*(-1 + n) + a*(-2*c
+ d*x)) + (2*a^2*c^2 - 2*a*b*c*d*n + b^2*d^2*(-1 + n)*n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])))/(2*a^3*d^3*(a*c - b*d)*(1 + n)*x)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\left(\frac{ax+b}{x}\right)^n}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x + b)/x)^n/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^2/(d*x+c),x)
```

```
[Out] int((a+b/x)^n*x^2/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{c+d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x)^n)/(c + d*x),x)
[Out] int((x^2*(a + b/x)^n)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a + \frac{b}{x}\right)^n}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**2/(d*x+c),x)
[Out] Integral(x**2*(a + b/x)**n/(c + d*x), x)
```

3.286 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx} \, dx$

Optimal. Leaf size=131

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^2(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

```
[Out] (a+b/x)^(1+n)*x/a/d+c^2*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a
*c-b*d))/d^2/(a*c-b*d)/(1+n)-(-b*d*n+a*c)*(a+b/x)^(1+n)*hypergeom([1, 1+n],
[2+n],1+b/a/x)/a^2/d^2/(1+n)
```

Rubi [A] time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {514, 375, 103, 156, 65, 68}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^2(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^(1 + n)*x)/(a*d) + (c^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^2*(a*c - b*d)*(1 + n)) - ((a*c
- b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])
/(a^2*d^2*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{d+\frac{c}{x}}\,dx\\ &= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right)\\ &= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(ac-bdn-bcnx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{ad}\\ &= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} - \frac{c^2\,\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(ac-bdn)\,\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{ad^2}\\ &= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} + \frac{c^2\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)(1+n)} - \frac{(ac-bdn)\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1\right.}{a^2d^2(1+n)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.91

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(a^2c^2\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left((bdn-ac)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)+ad(n\right.\right.}{a^2d^2(n+1)x(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a^2*c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a +
b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x + (-(a*c) + b*d*n)*Hypergeo
metric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^2*(a*c - b*d)*(1 + n)*x)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\left(\frac{ax+b}{x}\right)^n}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + b)/x)^n/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^{n}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x/(d*x+c),x)
[Out] int((a+b/x)^n*x/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a+\frac{b}{x}\right)^{n}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x)^n)/(c + d*x),x)
[Out] int((x*(a + b/x)^n)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^{n}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x/(d*x+c),x)
[Out] Integral(x*(a + b/x)**n/(c + d*x), x)
```

3.287 $\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$

Optimal. Leaf size=101

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad(n+1)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}$$

```
[Out] -c*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)/
(1+n)+(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d/(1+n)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 86, 65, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad(n+1)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + n))) + ((a + b/x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{d} + \frac{c\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d} \\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+n)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{ad(1+n)}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.96

$$
\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(ac\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(bd-ac)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)\right)}{ad(n+1)x(bd-ac)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + n)*x)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx+c}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(c + d*x),x)
```

```
[Out] int((a + b/x)^n/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(c + d*x), x)
```

3.288 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

```
[Out] (a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {514, 444, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.00

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)),x]
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^2+cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^2 + c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x/(d*x+c),x)
[Out] int((a+b/x)^n/x/(d*x+c),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{(dx + c)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x\, (c + d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x*(c + d*x)),x)
```

```
[Out] int((a + b/x)^n/(x*(c + d*x)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x\,(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(x*(c + d*x)), x)
```

3.289 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{d\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

```
[Out] -(a+b/x)^(1+n)/b/c/(1+n)-d*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)
/(a*c-b*d))/c/(a*c-b*d)/(1+n)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {514, 446, 80, 68}

$$-\frac{d\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)),x]
```

```
[Out] -((a + b/x)^(1 + n)/(b*c*(1 + n))) - (d*(a + b/x)^(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c*(a*c - b*d)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^3}\,dx$$
$$= -\text{Subst}\left(\int \frac{x(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} + \frac{d\,\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} - \frac{d\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)(1+n)}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.92

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(bd\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+ac-bd\right)}{bc(n+1)x(bd-ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a*c - b*d + b*d*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)]))/(b*c*(-(a*c) + b*d)*(1 + n)*x)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^3+cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^3 + c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^2/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/x^2/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{(dx + c)x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x^{2}\, \left(c + d\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^2*(c + d*x)),x)
```

```
[Out] int((a + b/x)^n/(x^2*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x^{2} \left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**2/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(x**2*(c + d*x)), x)
```

3.290 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{n+1}}{b^2c^2(n+1)}-\frac{\left(a+\frac{b}{x}\right)^{n+2}}{b^2c(n+2)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)}$$

```
[Out] (a*c+b*d)*(a+b/x)^(1+n)/b^2/c^2/(1+n)-(a+b/x)^(2+n)/b^2/c/(2+n)+d^2*(a+b/x)
^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^2/(a*c-b*d)/(1+n)
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {514, 446, 88, 68}

$$\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{n+1}}{b^2c^2(n+1)}-\frac{\left(a+\frac{b}{x}\right)^{n+2}}{b^2c(n+2)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a + b/x)^(1 + n))/(b^2*c^2*(1 + n)) - (a + b/x)^(2 + n)/(b^2*
c*(2 + n)) + (d^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(
a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^4}\,dx$$
$$=-\operatorname{Subst}\left(\int \frac{x^2(a+bx)^n}{d+cx}\,dx,x,\frac{1}{x}\right)$$
$$=-\operatorname{Subst}\left(\int \left(\frac{(-ac-bd)(a+bx)^n}{bc^2}+\frac{(a+bx)^{1+n}}{bc}+\frac{d^2(a+bx)^n}{c^2(d+cx)}\right)dx,x,\frac{1}{x}\right)$$
$$=\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}-\frac{d^2\operatorname{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx,x,\frac{1}{x}\right)}{c^2}$$
$$=\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)(1+n)}$$

Mathematica [A] time = 0.14, size = 112, normalized size = 0.97

$$-\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(b^2d^2(n+2)x\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)(acx-bc(n+1)+bd(n+2)x)\right)}{b^2c^2(n+1)(n+2)x^2(bd-ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(-(b*c*(1 + n)) + a*c*x + b*d*(2 + n)
*x) + b^2*d^2*(2 + n)*x*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a
*c - b*d)]))/(b^2*c^2*(-(a*c) + b*d)*(1 + n)*(2 + n)*x^2))
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^4+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^4 + c*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^3/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/x^3/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(dx + c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^3\,\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^3*(c + d*x)),x)
```

```
[Out] int((a + b/x)^n/(x^3*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^3\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**3/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(x**3*(c + d*x)), x)
```

3.291 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^4c^4(n+1)}-\frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+2}}{b^4c^3(n+2)}+\frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{n+3}}{b^4c^2(n+3)}-\frac{\left(a+\frac{b}{x}\right)^{n+4}}{b^4c(n+4)}+$$

```
[Out] (a*c+b*d)*(a^2*c^2+b^2*d^2)*(a+b/x)^(1+n)/b^4/c^4/(1+n)-(3*a^2*c^2+2*a*b*c*
d+b^2*d^2)*(a+b/x)^(2+n)/b^4/c^3/(2+n)+(3*a*c+b*d)*(a+b/x)^(3+n)/b^4/c^2/(3
+n)-(a+b/x)^(4+n)/b^4/c/(4+n)+d^4*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*
(a+b/x)/(a*c-b*d))/c^4/(a*c-b*d)/(1+n)
```

Rubi [A] time = 0.16, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {514, 446, 88, 68}

$$\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^4c^4(n+1)}-\frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+2}}{b^4c^3(n+2)}+\frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{n+3}}{b^4c^2(n+3)}-\frac{\left(a+\frac{b}{x}\right)^{n+4}}{b^4c(n+4)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^5*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a^2*c^2 + b^2*d^2)*(a + b/x)^(1 + n))/(b^4*c^4*(1 + n)) - ((3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x)^(2 + n))/(b^4*c^3*(2 + n)) + ((3*
a*c + b*d)*(a + b/x)^(3 + n))/(b^4*c^2*(3 + n)) - (a + b/x)^(4 + n)/(b^4*c*
(4 + n)) + (d^4*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
+ b/x))/(a*c - b*d)])/(c^4*(a*c - b*d)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
```

```
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^6}\,dx$$
$$= -\text{Subst}\left(\int \frac{x^4(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int \left(\frac{(ac+bd)\left(-a^2c^2-b^2d^2\right)(a+bx)^n}{b^3c^4} + \frac{\left(3a^2c^2+2abcd+b^2d^2\right)(a+bx)^{1+n}}{b^3c^3} + \frac{(-3a\right.$$
$$= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)}{b^4c^2(3+n)}$$
$$= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)}{b^4c^2(3+n)}$$

Mathematica [A] time = 0.24, size = 184, normalized size = 0.89

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(-\frac{c\left(a+\frac{b}{x}\right)\left(3a^2c^2+2abcd+b^2d^2\right)}{b^4(n+2)} + \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)}{b^4(n+1)} - \frac{c^3\left(a+\frac{b}{x}\right)^3}{b^4(n+4)} + \frac{c^2\left(a+\frac{b}{x}\right)^2(3ac+bd)}{b^4(n+3)} + \frac{d^4\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^5*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*(((a*c + b*d)*(a^2*c^2 + b^2*d^2))/(b^4*(1 + n)) - (c*(3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x))/(b^4*(2 + n)) + (c^2*(3*a*c + b*
d)*(a + b/x)^2)/(b^4*(3 + n)) - (c^3*(a + b/x)^3)/(b^4*(4 + n)) + (d^4*Hype
rgeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c - b*d)*(1
+ n))))/c^4
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{dx^6+cx^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^6 + c*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{\left(dx+c\right)x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^5/(d*x+c),x)
[Out] int((a+b/x)^n/x^5/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(dx+c)x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{5}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^5*(c + d*x)),x)
[Out] int((a + b/x)^n/(x^5*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{5}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**5/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x**5*(c + d*x)), x)
```

3.292 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{x^{m-1}\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

```
[Out] -(a+b/x)^n*x^(-1+m)*AppellF1(1-m,-n,2,2-m,-b/a/x,-c/d/x)/d^2/(1-m)/((1+b/a/
x)^n)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {514, 497, 135, 133}

$$-\frac{x^{m-1}\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^m)/(c + d*x)^2,x]
```

```
[Out] -(((a + b/x)^n*x^(-1 + m)*AppellF1[1 - m, -n, 2, 2 - m, -(b/(a*x)), -(c/(d*
x))])/(d^2*(1 - m)*(1 + b/(a*x))^n))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 497

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[((a + b/x^n)^p*(c
+ d/x^n)^q)/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}, x
] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^{-2+m}}{\left(d+\frac{c}{x}\right)^2}\,dx$$
$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\mathrm{Subst}\left(\int \frac{x^{-m}(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}\left(\frac{1}{x}\right)^m x^m\right)\mathrm{Subst}\left(\int \frac{x^{-m}\left(1+\frac{bx}{a}\right)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n} x^{-1+m} F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2,x]
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2, x]
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^m\left(\frac{ax+b}{x}\right)^n}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(x^m*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^m/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n*x^m/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n} x^{m}}{(dx+c)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m} \left(a+\frac{b}{x}\right)^{n}}{(c+d\,x)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x^m*(a + b/x)^n)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a+\frac{b}{x}\right)^{n}}{(c+dx)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**m/(d*x+c)**2,x)
[Out] Integral(x**m*(a + b/x)**n/(c + d*x)**2, x)
```

3.293 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2} \, dx$

Optimal. Leaf size=202

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^3(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(2-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bc\right.}}{d^3(n+1)(ac-bd)^2}$$

```
[Out] c*(2*a*c-b*d)*(a+b/x)^(1+n)/a/d^2/(a*c-b*d)/(d+c/x)+(a+b/x)^(1+n)*x/a/d/(d+
c/x)+c^2*(2*a*c-b*d*(2-n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)
/(a*c-b*d))/d^3/(a*c-b*d)^2/(1+n)-(-b*d*n+2*a*c)*(a+b/x)^(1+n)*hypergeom([1
, 1+n],[2+n],1+b/a/x)/a^2/d^3/(1+n)
```

Rubi [A] time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {514, 375, 103, 151, 156, 65, 68}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^3(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(2-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bc\right.}}{d^3(n+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

```
[Out] (c*(2*a*c - b*d)*(a + b/x)^(1 + n))/(a*d^2*(a*c - b*d)*(d + c/x)) + ((a + b
/x)^(1 + n)*x)/(a*d*(d + c/x)) + (c^2*(2*a*c - b*d*(2 - n))*(a + b/x)^(1 +
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c
 - b*d)^2*(1 + n)) - ((2*a*c - b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a^2*d^3*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2}\,dx$$

$$= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(2ac-bdn+bc(1-n)x)}{x(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{ad}$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n((ac-bd)(2ac-bdn)-bc(2ac-bd)nx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{ad^2(ac-bd)}$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} - \frac{\left(c^2(2ac-bd(2-n))\right)\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^3(ac-bd)} + \cdots$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{c^2(2ac-bd(2-n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\cdots\right.}{d^3(ac-bd)^2(1+n)}$$

Mathematica [A] time = 0.24, size = 179, normalized size = 0.89

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left((c+dx)\left(a^2c^2(2ac+bd(n-2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)-(ac-bd)^2(2ac-bdn)\,{}_2F_1\left(1,n+1;n\right.\right.\right.}{a^2d^3(n+1)(c+dx)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^(1 + n)*(a*c*d*(a*c - b*d)*(2*a*c - b*d)*(1 + n)*x + a*d^2*(a*c
- b*d)^2*(1 + n)*x^2 + (c + d*x)*(a^2*c^2*(2*a*c + b*d*(-2 + n))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)] - (a*c - b*d)^2*(2*a*c
- b*d*n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^3*(a*c -
 b*d)^2*(1 + n)*(c + d*x))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\left(\frac{ax+b}{x}\right)^n}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(a+\frac{b}{x}\right)^n}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x^2*(a + b/x)^n)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**2/(d*x+c)**2,x)
[Out] Integral(x**2*(a + b/x)**n/(c + d*x)**2, x)
```

3.294 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{c\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(1-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)^2}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{ad^2(n+1)}$$

```
[Out] -c*(a+b/x)^(1+n)/d/(a*c-b*d)/(d+c/x)-c*(a*c-b*d*(1-n))*(a+b/x)^(1+n)*hyperg
eom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^2/(a*c-b*d)^2/(1+n)+(a+b/x)^(1+n)
*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d^2/(1+n)
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {514, 446, 103, 156, 65, 68}

$$-\frac{c\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(1-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)^2}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{ad^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x)/(c + d*x)^2,x]
```

```
[Out] -((c*(a + b/x)^(1 + n))/(d*(a*c - b*d)*(d + c/x))) - (c*(a*c - b*d*(1 - n))
*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/(d^2*(a*c - b*d)^2*(1 + n)) + ((a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d^2*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x}\,dx$$

$$= -\text{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n(ac-bd-bcnx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{d(ac-bd)}$$

$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(c(ac-bd(1-n)))\,\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right.}{d^2(ac-bd)}$$

$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{c(ac-bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)^2(1+n)} + \frac{\left(a+\frac{b}{x}\right)^{1+n}}{}$$

Mathematica [A] time = 0.15, size = 120, normalized size = 0.80

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(-\frac{c(ac+bd(n-1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2} - \frac{cdx}{(c+dx)(ac-bd)} + \frac{{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a(n+1)}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^(1 + n)*(-((c*d*x)/((a*c - b*d)*(c + d*x))) - (c*(a*c + b*d*(-1
+ n))*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c
- b*d)^2*(1 + n)) + Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)]/(a*(1 +
 n))))/d^2
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\left(\frac{ax+b}{x}\right)^n}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n*x/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n*x/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a+\frac{b}{x}\right)^n}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x)^n)/(c + d*x)^2,x)
```

```
[Out] int((x*(a + b/x)^n)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x/(d*x+c)**2,x)
```

```
[Out] Integral(x*(a + b/x)**n/(c + d*x)**2, x)
```

3.295 $\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2}$$

```
[Out] -b*(a+b/x)^(1+n)*hypergeom([2, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2/
(1+n)
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {434, 444, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + n)))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^2}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{b\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.02

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(2,n+1;n+2;-\frac{c\left(a+\frac{b}{x}\right)}{bd-ac}\right)}{(n+1)(bd-ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + n)))
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/(d*x + c)^2, x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(c + d*x)^2,x)
```

```
[Out] int((a + b/x)^n/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/(d*x+c)**2,x)
```

```
[Out] Integral((a + b/x)**n/(c + d*x)**2, x)
```

3.296 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx$

Optimal. Leaf size=105

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(n+1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)^2}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

```
[Out] -d*(a+b/x)^(1+n)/c/(a*c-b*d)/(d+c/x)+(a*c-b*d*(1+n))*(a+b/x)^(1+n)*hypergeo
m([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^2/(1+n)
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {514, 446, 78, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(n+1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)^2}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x*(c + d*x)^2),x]
```

```
[Out] -((d*(a + b/x)^(1 + n))/(c*(a*c - b*d)*(d + c/x))) + ((a*c - b*d*(1 + n))*(
a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*
d)])/(c*(a*c - b*d)^2*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^3}\,dx\\
&= -\text{Subst}\left(\int \frac{x(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{(ac-bd(1+n))\,\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c(ac-bd)}\\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(ac-bd(1+n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^2(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 0.84

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(\frac{(ac-bd(n+1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{n+1}+\frac{dx(bd-ac)}{c+dx}\right)}{c(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)^2),x]
```

```
[Out] ((a + b/x)^(1 + n)*((d*(-(a*c) + b*d)*x)/(c + d*x) + ((a*c - b*d*(1 + n))*H
ypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(1 + n)))/(c*
(a*c - b*d)^2)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{d^2x^3+2\,cdx^2+c^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^3 + 2*c*d*x^2 + c^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{\left(dx+c\right)^{2} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/x/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{\left(dx+c\right)^{2} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x\,\left(c+d\,x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x*(c + d*x)^2),x)
```

```
[Out] int((a + b/x)^n/(x*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x\left(c+dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x/(d*x+c)**2,x)
```

```
[Out] Integral((a + b/x)**n/(x*(c + d*x)**2), x)
```

3.297 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx$

Optimal. Leaf size=133

$$\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(n+2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc^2(n+1)}$$

```
[Out] -(a+b/x)^(1+n)/b/c^2/(1+n)+d^2*(a+b/x)^(1+n)/c^2/(a*c-b*d)/(d+c/x)-d*(2*a*c
-b*d*(2+n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^2
/(a*c-b*d)^2/(1+n)
```

Rubi [A] time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {514, 446, 89, 80, 68}

$$\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(n+2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

```
[Out] -((a + b/x)^(1 + n)/(b*c^2*(1 + n))) + (d^2*(a + b/x)^(1 + n))/(c^2*(a*c -
b*d)*(d + c/x)) - (d*(2*a*c - b*d*(2 + n))*(a + b/x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2*(1 + n)
)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^4}\,dx \\
&= -\mathrm{Subst}\left(\int \frac{x^2(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\mathrm{Subst}\left(\int \frac{(a+bx)^n(-d(ac-bd(1+n))+c(ac-bd)x)}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(d(2ac-bd(2+n)))\,\mathrm{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{d(2ac-bd(2+n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\right.}{ac-b}\right.}{c^2(ac-bd)^2(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 125, normalized size = 0.94

$$-\frac{(ax+b)\left(a+\frac{b}{x}\right)^n\left(bd(c+dx)(2ac-bd(n+2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)(ac(c+dx)-bd(c+d(n+\right.}{bc^2(n+1)x(c+dx)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

```
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(a*c*(c + d*x) - b*d*(c + d*(2 + n)*x
)) + b*d*(2*a*c - b*d*(2 + n))*(c + d*x)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)]))/(b*c^2*(a*c - b*d)^2*(1 + n)*x*(c + d*x)))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^n}{d^2x^4+2\,cdx^3+c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^4 + 2*c*d*x^3 + c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^2/(d*x+c)^2,x)
[Out] int((a+b/x)^n/x^2/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^2*(c + d*x)^2),x)
[Out] int((a + b/x)^n/(x^2*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**2/(d*x+c)**2,x)
[Out] Integral((a + b/x)**n/(x**2*(c + d*x)**2), x)
```

3.298 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$

Optimal. Leaf size=217

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\frac{c(ac-bd)(ac+bd(n+3))}{x}\right)}{b^2c^3(n+1)(n+2)\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}(3ac-bd(n}{c^3(n}$$

```
[Out] -(a+b/x)^(1+n)*(d*(b*d*(2+n)*(a*c+b*d*(3+n))-a*c*(a*c+b*d*(5+3*n)))-c*(a*c-
b*d)*(a*c+b*d*(3+n))/x)/b^2/c^3/(a*c-b*d)/(1+n)/(2+n)/(d+c/x)-(a+b/x)^(1+n)
/b/c/(2+n)/(d+c/x)/x^2+d^2*(3*a*c-b*d*(3+n))*(a+b/x)^(1+n)*hypergeom([1, 1+
n],[2+n],c*(a+b/x)/(a*c-b*d))/c^3/(a*c-b*d)^2/(1+n)
```

Rubi [A] time = 0.26, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {514, 446, 100, 146, 68}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\frac{c(ac-bd)(ac+bd(n+3))}{x}\right)}{b^2c^3(n+1)(n+2)\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}(3ac-bd(n}{c^3(n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] -(((a + b/x)^(1 + n)*(d*(b*d*(2 + n)*(a*c + b*d*(3 + n)) - a*c*(a*c + b*d*(
5 + 3*n))) - (c*(a*c - b*d)*(a*c + b*d*(3 + n)))/x))/(b^2*c^3*(a*c - b*d)*(
1 + n)*(2 + n)*(d + c/x))) - (a + b/x)^(1 + n)/(b*c*(2 + n)*(d + c/x)*x^2)
+ (d^2*(3*a*c - b*d*(3 + n))*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^3*(a*c - b*d)^2*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
```

```
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^5}\,dx \\
&= -\text{Subst}\left(\int \frac{x^3(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} - \frac{\text{Subst}\left(\int \frac{x(a+bx)^n(-2ad+(-ac-bd(3+n))x)}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{bc(2+n)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} - \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} -
\end{aligned}$$

Mathematica [A] time = 0.35, size = 182, normalized size = 0.84

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(\frac{-a^2c^2(c+dx)-abcd(c(n+2)+d(2n+3)x)+b^2d^2(n+3)(c+d(n+2)x)}{bc^2(n+1)(c+dx)(bd-ac)} - \frac{bd^2(n+2)(bd(n+3)-3ac)\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2} - \frac{1}{x(c+dx)}\right)}{bc(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] ((a + b/x)^(1 + n)*(-(1/(x*(c + d*x))) + (-(a^2*c^2*(c + d*x)) + b^2*d^2*(3
 + n)*(c + d*(2 + n)*x) - a*b*c*d*(c*(2 + n) + d*(3 + 2*n)*x))/(b*c^2*(-(a*
c) + b*d)*(1 + n)*(c + d*x)) - (b*d^2*(2 + n)*(-3*a*c + b*d*(3 + n))*Hyperg
eometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2
*(1 + n))))/(b*c*(2 + n))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^{n}}{d^{2}x^{5}+2\,cdx^{4}+c^{2}x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d^2*x^5 + 2*c*d*x^4 + c^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(dx+c)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(dx+c)^{2}\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^n/x^3/(d*x+c)^2,x)
[Out] int((a+b/x)^n/x^3/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(dx+c)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{3}\,(c+d\,x)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^3*(c + d*x)^2),x)
[Out] int((a + b/x)^n/(x^3*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{3}\,(c+dx)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**3/(d*x+c)**2,x)
```

```
[Out] Integral((a + b/x)**n/(x**3*(c + d*x)**2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```